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Interactive Comment

Interactive comment on "A linear algorithm for solving non-linear isothermal ice-shelf equations" *by* A. Sargent and J. L. Fastook

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This paper develops methods for solving the MacAyeal/Morland (MM) or shallow-shelf approximation stress balance equations that avoid the usual nonlinear elliptic equations in the velocity and instead solve first-order linear equations for the stress field, then a second set of first-order linear equations for the velocity field. It presents first a one-dimensional case (which is correct, but quite well known) and then a two-dimensional case (which I think is incorrect, or at least applies only to very limited and atypical cases)

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Interactive Discussion

Discussion Paper

1 1D case

The authors note that the 1D SSA model can be written in the form

$$\frac{\partial \tau}{\partial x} = ch\frac{\partial s}{\partial x} \tag{1}$$

with a boundary value of the form

$$\tau(x_L) = bh^2 \tag{2}$$

and these can be integrated to give $\tau(x)$. This is correct and discussed elsewhere, for example it is used, with the PDE rewritten in as

$$\frac{\partial \tau}{\partial x} = \frac{bc}{2}\frac{\partial h^2}{\partial x} \tag{3}$$

so that

$$\tau(x) = \frac{bc}{2}h^2 \tag{4}$$

in the Schoof 2007 papers that derive expressions for grounding line mass flux.

Having found $\tau(x)$, $u(x)$ can be found by solving

$$h\frac{\partial u}{\partial x} = \tau^n \tag{5}$$

with an appropriate boundary value for u. I think this result is fairly well known because it is straightforward, but the 1D case is useful as a preliminary to the 2D discussion.

2 2D case

The authors then examine the 2D equations. If the 1D case could be extended to 2D, that would be very useful indeed, but to do so it is necessary to reduce the three stress components ($\tau_{xx}, \tau_{xy} = \tau_{yx}, \tau_{yy}$) to two in some way (as there are only two PDEs). The authors use the relation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

to this end, but this expression is not valid for typical ice shelves. It is not the correct form of the incompressibility condition for the MM model (the authors claim that it is), because the flow is not 2D, it is 3D and the vertical component of velocity (w) is not constant in z. The correct equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

which is usually integrated vertically (and surface conditions imposed) to give the mass transport equation

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} = a \tag{8}$$

so that

$$h(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = a - \frac{\partial h}{\partial t} - u\frac{\partial h}{\partial x} - v\frac{\partial h}{\partial y}. \tag{9}$$

Equation 6 limits the discussion to a class of flows that cannot accelerate in the plane without changing direction in the plane: they would look like magnetic field lines in plane geometries.

As a result of the assumption 6 only ice shelves that make the right hand side of 9 vanish can be treated. That is true of the test case, which assumes the incorrect incompressibility condition 6 and hence an incorrect mass transport equation. It is not true for realistic ice shelves. It would apply to uniformly thick ice shelves in steady state, or to ice shelves where the surface gradient was perpendicular to the velocity field, but that is not normally the case, and often the velocity is parallel to the surface gradient. Notice that 6 does not apply to the 1D case - if it did, u would have to be constant, and it is not.

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The rest of the paper is devoted to solving the resulting equations, and I think that the methods presented might be of interest. Ultimately though, I think that they are not applicable to ice shelf dynamics unless the equations can somehow be constructed to use 9, which would be a major change because something needs to be done with the u and v that get introduced. Alternatively, the authors would need to construct their methods without assuming 6, or make some convincing case that they could be useful despite the limitation I have described.

3 Presentation

I found the paper easy enough to follow for the most part, though there are a few places with a staccato style (every sentence in paragraph 1 begins 'The', 3.4 is similar) or spelling (e.g 'Furier' rather than 'Fourier'). These are minor issues.

Interactive comment on Geosci. Model Dev. Discuss., 7, 1829, 2014.

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