Supplement of Geosci. Model Dev. Discuss., 7, 8649–8701, 2014
http://www.geosci-model-dev-discuss.net/7/8649/2014/
doi:10.5194/gmdd-7-8649-2014-supplement

Geoscientific
Model Development
Discussions
Open Access

Supplement of

A multi-layer land surface energy budget model for implicit coupling with global atmospheric simulations

J. Ryder et al.

Correspondence to: J. Ryder (jryder@lsce.ipsl.fr)

Supplementary material

Notes

- **Numbering of equations:** the three 'key' equations (and variations as a result of substitution) are labelled a), b) and c) on the left hand side throughout the document. The assumptions are labelled in Roman numerals on the left hand side. All equations are numbered conventionally on the right hand side for ease of reference.

- **Potential enthalpy:** the present ORCHIDEE uses the term 'surface static energy' as the potential for calculating sensible heat flux. This is defined in the model (for the surface layer) as:

  $$ps_{surf} = \Theta_{p,a} T_{surf}$$

  where ps_{surf} is the surface static energy, $\Theta_{p,a}$ is the mass specific heat capacity of air and T_{surf} the surface temperature.

  Now the enthalpy of a system (H) is defined $H = U + pV$, but over the height of a surface model (< 30m approx), change in p and V is negliable, so:

  $$\begin{aligned} \delta H &= \delta U + p\delta V + V\delta p \\ &= (\delta Q + \delta W + \delta W') + p\delta V + V\delta p \end{aligned}$$

  now $\delta W = -p\delta V$, so we can say:

  $$\begin{aligned} \delta H &= \delta Q + \delta W' + V\delta p \\ &= Q + W' + \int_{p_0}^{p} V\delta p \approx Q \\ &\approx \Theta_{p,a} T \end{aligned}$$

  So here we can also assume a proportional relationship between enthalpy and temperature over the vertical range of the model.

- **Sign convention:** For latent and sensible heat fluxes, an upward flux is positive (so a positive flux from the ground is cooling the ground)

S1 Parameters

S1.1 Derivation of the leaf layer resistances (R_i and R_i')

The variables R_i and R_i' represent the leaf layer resistance to the sensible and latent heat flux, respectively. R_i is calculated based upon the leaf boundary layer resistance, and is described at present according to the following expression from Baldocchi (1988):

$$R_b(z) = \frac{l}{df(z)D_s Sh(z)} \tag{S1.1}$$

where R_b denotes the boundary layer resistance (=R_i), l is the characteristic length of leaves, D_s is the molecular diffusivity of the entity in question and Sh is the Sherwood number.

R_i' is the stomatal resistance of the leaf that may be calculated using the model of (Ball et al., 1987) as in ORCHIDEE at present, or potentially the refinement of Medlyn et al. (2011).

S1.2 Derivation of the eddy-diffusivity coefficient (k_i)

The transport term k_i is calculated using the 1D second-order closure model of Massman & Weil (1999) which makes use of the LAI profile of the stand.

$$\frac{u(z)}{u(h)} = e^{-n\left(1-\frac{\zeta(z)}{\zeta(h)}\right)} \tag{S1.2}$$

$$\frac{-\overline{u'w'}(z)}{u_*^2} = e^{-2n\left(1-\frac{\zeta(z)}{\zeta(h)}\right)} \tag{S1.3}$$

$$\zeta(z) = \int_0^z \left[\frac{C_d(z')a(z')}{P_m(z')}\right] dz' \tag{S1.4}$$

$$n = \frac{\zeta(h)}{2u_*^2/u(h)^2} \tag{S1.5}$$

$$\frac{u_*}{u(h)} = c_1 - c_2 e^{c_3\zeta(h)} \tag{S1.6}$$

from Massman (1997), where u(z) is the horizontal wind speed, $\overline{u'w'}(z)$ is the turbulent shear stress, $\zeta(z)$ is the cumulative leaf drag area per unit 'planform' (projected) area, z is the height above the soil surface, h is the canopy height, u_* is the friction velocity above the canopy (assuming a constant sheer layer above the canopy i.e. u_*^2 = - $\overline{u'w'}$, C_d is the drag coefficient of the foliage elements, a(z) is the foliage area density as a function of height and P_m is the momentum shelter factor.

The constants c_1=0.320, c_2=0.246 and c_3=15.1 are model constants that are related to the bulk surface drag coefficient:

$$c_{surf} = \frac{2u_*^2}{u(h)^2} \tag{S1.7}$$

$\zeta(h)$ is a generalisation of the more commonly used C_dLAI, where LAI is the one-sided leaf area index:

$$\zeta(h) = \int_0^h a(z')dz' \tag{S1.8}$$

Thus the canopy structure is accounted for by:

$$\frac{C_d(z)a(z)}{P_m(z)} \tag{S1.9}$$

and the foliage area density is described by:

$\zeta(h)$ or C_d(LAI)

ω_u^2, ω_v^2, and ω_w^2 are each assumed proportional to ω_e^2, and following Mellor (1973) and Wilson & Shaw (1977), the constants μ_i are chosen consistent with constant stress layer and a logarithmic wind profile above the stress layer.

This gives the following relations for ν_u:

$$\nu_1 = (\gamma_1^2 + \gamma_2^2 + \gamma_3^2)^{-1/2} \tag{S1.10}$$

$$\nu_3 = (\gamma_1^2 + \gamma_2^2 + \gamma_3^2)^{-3/2} \tag{S1.11}$$

$$\nu_2 = \frac{\nu_3}{6} - \frac{\gamma_3^2}{2\nu_1} \tag{S1.12}$$

from general comparisons with ensembles of data: $\gamma_1 = 2.40$, $\gamma_2 = 1.90$ and $\gamma_3 = 1.25$ (Raupach et al., 1991) solved analytically:

$$\frac{\omega_e(z)}{u_*} = \left[\nu_3 e^{\Lambda\zeta(h)\left(1-\frac{\zeta(z)}{\zeta(h)}\right)} + B_1\left(e^{-3n\left(1-\frac{\zeta(z)}{\zeta(h)}\right)} - e^{-\Lambda\zeta(h)\left(1-\frac{\zeta(z)}{\zeta(h)}\right)}\right)\right]^{1/3} \tag{S1.13}$$

where:

$$B_1 = \frac{-\left(\frac{9u_*}{u(h)}\right)}{2\alpha\nu_1\left(\frac{9}{4} - \frac{\Lambda^2 u_*^4}{u(h)^4}\right)} \tag{S1.14}$$

solving for ω_i in terms of ω_e yields:

$$\frac{\omega_i(z)}{u_*} = \gamma_i\nu_i\frac{\omega_e(z)}{u_*} \tag{S1.15}$$

S1.3 Relationship between LAI and θ_i at each level

The heat capacity of each vegetation layer (θ_i) is assumed equal to that of water. The vegetation density is sourced from the Leaf Mass Area (LMA) (g/m^2) in the TRY database and the leaf area density profile.

S2 The numerical solution

S2.1 Introduction

The structure of the derivation here outlined is based on that of the LMDz transport scheme (Dufresne & Ghattas, 2009), but extended to include interactions with the vegetation layer at each level.

S2.2 Leaf vapour pressure assumption

The air within leaf level cavities is assumed completely saturated. This means that the vapour pressure of the leaf can be calculated as the saturated vapour pressure at that leaf temperature. Therefore the change in pressure within the leaf is assumed proportional to the difference in temperature between the present timestep and next timestep, multiplied by the rate of change in saturated pressure against temperature.

$$q_0 \equiv q_{L,i}^{t+1} = q_{sat}^{T_{L,i}^t} + \frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t}(T_{L,i}^{t+1} - T_{L,i}^t) \tag{S2.1}$$

$$= \frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t}(T_{L,i}^{t+1}) + \left(q_{sat}^{T_{L,i}^t} - T_{L,i}^t \frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t} \right) \tag{S2.2}$$

$$= \alpha_i T_{L,i}^{t+1} + \beta_i \tag{S2.3}$$

where α_i and β_i are regarded as constants for each particular level and timestep so $\alpha_i = \frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t}$ and $\beta_i = \left(q_{sat}^{T_{L,i}^t} - T_{L,i}^t \frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t} \right)$

But to find a solution we still need to find an expression for the terms $q_{sat}^{T_{L,i}^t}$ and $\frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t}$ in α_i and β_i above.

Using the empirical approximation of Tetens (e.g. as in Monteith & Unsworth (2.1), 2008) and the specific humidity vapour pressure relationship we can describe the saturation vapour pressure to within 1 Pa up to a temperature of about 35 °C.

$$e_{sat}(T) = e_{sat}(T^*)exp[A(T - T^*)/(T - T')] \tag{S2.4}$$

where A = 17.27, T^* = 273K, $e_{sat}(T^*)$ = 0.611 kPa, T' = 36K

Now, specific humidity is related to vapour pressure by the relationship: (e.g. Monteith & Unsworth (2.1), 2008):

$$q = \frac{\left(\frac{M_W}{M_A}\right) e}{(p-e) + \left(\frac{M_W}{M_A}\right) e} \tag{S2.5}$$

where q = specific humidity (kg/kg), e = vapour pressure (kPa), (M_W/M_A) = (ratio of molecular weight of water to air) = 0.622, and p = atmospheric pressure (kPa)

So, to find $q_{sat}^{T_{L,i}^t}$, we substitute $e_{sat}(T_L)$ derived from (S2.4) for e in (S2.5):

$$q_{sat}^{T_L} = \frac{\left(\frac{M_W}{M_A}\right) e_{sat}(T_L)}{(p-e_{sat}(T_L)) + \left(\frac{M_W}{M_A}\right) e_{sat}(T_L)} \tag{S2.6}$$

To calculate $\frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t}$, we use the expression for the saturated humidity curve against temperature (as derived using the method of Monteith & Unsworth, 2008):

$$q_{sat}^{T_{L,i}^t} = q_0 e^{-\lambda M_W/RT} \tag{S2.7}$$

The derivative of this expression is:

$$\frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t} = \frac{\lambda M_W q_{sat}(T)}{R(T_{L,i}^t)^2} \tag{S2.8}$$

So $\frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t}$ can be determined by substitution of the expression for $q_{sat}(T_L)$ from (S2.6) into (S2.8), as below:

$$\frac{\delta q_{sat}}{\delta T}|_{T_{L,i}^t} = \frac{\lambda M_W}{R(T_{L,i}^t)^2} \left(\frac{\left(\frac{M_W}{M_A}\right) e_{sat}(T_L)}{(p-e_{sat}(T_L)) + \left(\frac{M_W}{M_A}\right) e_{sat}(T_L)} \right) \tag{S2.9}$$

S2.3 Physical and biophysical parameters:

We here concentrate on the formulation of an implicit solution that assumes a parameterisation for R_i (the resistance to sensible heat flux at each level), R_i' (resistance to latent heat flux at each level) and k_i (transport coefficients at each level). The derivation of these coefficients, based on literature study, will be described in a seperate document.

S2.4 The leaf energy balance equation for each layer

Now at the leaf level, we assume the energy balance for each layer. It is assumed that (for a leaf layer of volume ΔV_i, area ΔA_i and thickness Δh_i):

$$\Delta V_i \theta_i \rho_v \frac{\delta T_{L,i}}{\delta t} = (H_i + LE_i + R_{SW,i} + R_{LW,i})\Delta A_i \tag{S2.10}$$

Dividing (S2.10) by ΔV_i:

$$\theta_i \rho_v \frac{\delta T_{L,i}}{\delta t} = (H_i + LE_i + R_{SW,i} + R_{LW,i}) \left(\frac{1}{\Delta h_i}\right) \tag{S2.11}$$

The source sensible heat flux from the leaf at level $'i'$ is the difference between the leaf temperature and that above it, divided by R_i which is the leaf resistance to sensible heat flux (a combination of stomatal and boundary layer resistance)). Similarly, the source latent heat flux from the leaf at level $'i'$ is the difference between the leaf temperature and that above it, divided by R'_i which is the leaf resistance to sensible heat flux. So the terms of (S2.11) are defined (in units W/m^2):

$$H_i = \Theta_{p,a} \rho_a \frac{(T_{L,i} - T_{a,i})}{R_i} \tag{S2.12}$$

$$LE_i = \lambda \rho_a \frac{(q_{L,i} - q_{a,i})}{R'_i} \tag{S2.13}$$

- $R_{LW(tot),i}$ is the sum total of long wave radiation - that is: downwelling LW radiation from above the canopy, the LW radiation emitted from vegetation layer $'i'$ and the LW radiation reflected from the vegetation layers $'i+1'$ and $'i-1'$.

- R_{SW} is the downwelling short radiation. So we express the sensible and latent heat fluxs between the leaf and the atmosphere respectively as:

$$a) \quad \boxed{\theta_i \rho_v \frac{\delta T_{L,i}}{\delta t} = \left(\Theta_{p,a} \rho_a \frac{(T_{L,i} - T_{a,i})}{R_i} + \lambda \rho_a \frac{(q_{L,i} - q_{a,i})}{R'_i} + R_{SW,i} + R_{LW(tot),i}\right) \left(\frac{1}{\Delta h_i}\right)} \tag{S2.14}$$

S2.5 Vertical transport within a column

The transport equation may be stated as:

$$\frac{\delta(\rho\chi)}{\delta t} + div(\rho\chi u) = div(\Gamma grad(\chi)) + S_\chi \tag{S2.15}$$

div is the operator that calculates the divergence of the vector field, χ is the property under question, ρ is the fluid density, u is the horizontal wind speed vector (assumed negligable here), S_χ is the concentration for the property in question and Γ is a parameter that will in this case be the diffusion coefficient k(z) henceforth.

To derive from this the conservation of scalars equation as might be applied to vertical air columns, we proceed according to the Finite Volume Method as outlined in e.g. Vieno (2006). Integrate over dV (a unit volume):

$$\int_V \frac{\delta(\rho\chi)}{\delta t} dV + \int_V (\rho\chi u) dV = \int_V (\Gamma grad(\chi) dV) + \int_V S_\chi dV \tag{S2.16}$$

Using Gauss' theorem, integrating over a time Δt and re-writing in one-dimension we ultimately obtain the expression below:

$$\overbrace{\int_{\Delta t} \frac{\delta}{\delta t} \int_V \chi dV \int_{\Delta t} dt}^{time\ dependent\ term\ (PART\ A)} + \overbrace{\int_{\Delta t} \int_{AreaV} n \cdot (\chi u) dA dt}^{horizontal\ advection\ term\ (assume\ zero\ for\ now)} = \overbrace{\int_{\Delta t} \int_{AreaV} n \cdot (k grad \chi) dA dt}^{gradient\ term\ (PART\ B)} + \overbrace{\int_{\Delta t} \int_V S_\chi dV dt}^{source\ term\ (PART\ C)} \tag{S2.17}$$

Now, the diffusion is considered only along the z-axis, and only the top and bottom of the boxes of volume ΔV have non-zero flux (n.b. for our set-up, concentration is uniform within each layer), so we can say:

$$(\chi_n^{t+\Delta t} - \chi_n^t)\Delta V = \int_t^{t+\Delta t} \left(\left(k_{top} \frac{\delta \chi_{top}}{\delta z} \Delta A \right) - \left(k_{bottom} \frac{\delta \chi_{bottom}}{\delta z} \Delta A \right) \right) dt + \int_t^{t+\Delta t} S_\chi \Delta V dt \tag{S2.18}$$

where ΔA is the area of the box in question In the non-differenced form, this equation becomes:

$$\overbrace{\frac{\delta\chi}{\delta t}\Delta V}^{PART\ A} = \overbrace{k(z)\frac{d^2\chi}{dz^2}\Delta A}^{PART\ B} + \overbrace{S(z)\Delta V}^{PART\ C} \tag{S2.19}$$

where F is the vertical flux density, U the mean windspeed, z represents coordinates in the vertical and x coordinates in the streamwise direction. χ may represent the concentration of any constituent that may include water vapour or heat, but also gas or aerosol phase concentration of particular species. S represents the source density of that constituent.

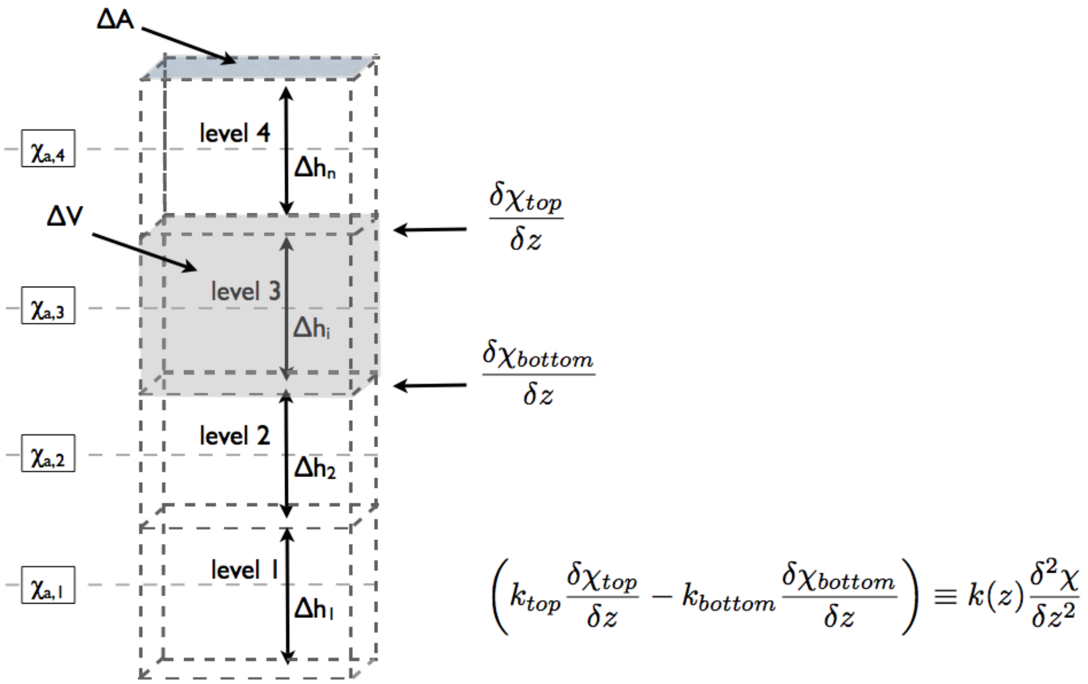

Figure 1: *Schematic of the finite volume method for differencing the column, as in* (S2.18)

This eddy diffusivity parameter k(z) is used in the formulae outlined above, and throughout this document. Raupach (1989) develops a model that is a more realistic Lagrangian description of canopy transport, an approach which is impractical to apply directly to a linear, non-iterative model such as ORCHIDEE. However, it is hoped to develop a form of the eddy diffusivity coefficient k(z) that accommodates the counter-gradient fluxes also observed in canopies (as in the method of Makar et al, 1999).

S2.6 Sensible heat transport between each atmospheric layer

We re-write the scalar conservation equation, as applied to canopies (equation (S2.19)) in terms of the sensible heat flux, temperature and source sensible heat from the vegetation at each layer. (so, comparing with (S2.19), $\chi \equiv T$, $F \equiv H$ and $S \equiv$ (the source sensible heat flux at each vegetation layer)).

The sensible heat flux profile is not constant over the height of the canopy. The rate of change of $T_{a,i}$ (the temperature of the atmosphere surrounding the leaf at level i) is proportional to the rate of change of sensible heat flux with height and the source sensible heat flux from the leaf at that level as in (S2.12) above (n.b. overbraces refer to (S2.19)):

$$b) \quad \overbrace{\Theta_{p,a}\rho_a \frac{\delta T_{a,i}}{\delta t}\Delta V_i}^{PART\ A} = \overbrace{-\frac{\delta H_{a,i}}{\delta z}\Delta A_i}^{PART\ B} + \overbrace{\left(\frac{T_{L,i} - T_{a,i}}{R_i}\right)\left(\frac{\Theta_{p,a}\rho_a}{\Delta h_i}\right)\Delta V_i}^{PART\ C} \tag{S2.20}$$

now $H_{a,i} = -(\rho_a \Theta_{p,a}) k_i \frac{\delta T_{a,i}}{\delta z}$ (if the flux-gradient relation is assumed) so we can say:

$$b) \quad \boxed{\frac{\delta T_{a,i}}{\delta t}\Delta V_i = k_i \frac{\delta^2 T_{a,i}}{\delta z^2}\Delta A_i + \left(\frac{T_{L,i} - T_{a,i}}{R_i}\right)\left(\frac{1}{\Delta h_i}\right)\Delta V_i} \tag{S2.21}$$

S2.7 Latent heat transport between each atmospheric layer

We re-write the simplified scalar conservation equation, as applied to canopies (equation (S2.19)) in terms of the sensible heat flux, temperature and source sensible heat from the vegetation at each layer. (so, comparing with (S2.19), $\chi \equiv q$, $F \equiv E$ and $S \equiv$ (the source latent heat flux at each vegetation layer)).

The latent heat flux profile is also not constant over the height of the canopy. The rate of change of $q_{a,i}$ (the specific humidity of the atmosphere surrounding the leaf at level i) is proportional to the rate of change of latent heat flux with height and the source latent heat flux from the leaf as in (S2.13) (n.b. overbraces refer to (S2.19)):

$$c) \quad \overbrace{\lambda\rho_a \frac{\delta q_{a,i}}{\delta t}\Delta V_i}^{PART\ A} = \overbrace{-\frac{\delta(LE)_{a,i}}{\delta z}\Delta A_i}^{PART\ B} + \overbrace{\left(\frac{q_{L,i} - q_{a,i}}{R'_i}\right)\left(\frac{\lambda\rho_a}{\Delta h_i}\right)\Delta V_i}^{PART\ C} \tag{S2.22}$$

$$= -\frac{\delta(LE)_{a,i}}{\delta z}\Delta A_i + \left(\frac{(\alpha T_{L,i} + \beta_i) - q_{a,i}}{R'_i}\right)\left(\frac{\lambda\rho_a}{\Delta h_i}\right)\Delta V_i \tag{S2.23}$$

now $(LE)_{a,i} = -(\lambda\rho_a)k_i\frac{\delta q_{a,i}}{\delta z}$ (again assuming the flux-gradient relation) so...

$$c) \quad \boxed{\frac{\delta q_{a,i}}{\delta t}\Delta V_i = k_i\frac{\delta^2 q_{a,i}}{\delta z^2}\Delta A_i + \left(\frac{(\alpha T_{L,i} + \beta_i) - q_{a,i}}{R'_i}\right)\left(\frac{1}{\Delta h_i}\Delta V_i\right)} \tag{S2.24}$$

S2.8 The 'zero-leaf' scenario

Canopy layers that do not contain foliage may be accounted for at a level by assuming that $R_i = R'_i = \infty$ for that level (i.e. an open circuit), and that the various coefficients that relate to the leaf interactions at that level (R_{SW}, R_{LW}, $C_{T,i}$, $C_{T,i+1}$, $C_{q,i}$, $C_{q,i+1}$, D_i, E_i, F_i, D_{i+1}, E_{i+1}, F_{i+1}) are zero.

S2.9 Write equations in implicit format

To maintain the implicit coupling between the atmospheric model (i.e. LMDZ) and the land surface model (i.e. ORCHIDEE) we need to express the relationships that are outlined above in terms of a linear relationship between the 'present' timestep 't' and the 'next' timestep 't+1'.

We therefore re-cast equations a), b) and c) in implicit form (i.e. in terms of the 'next' timestep, which is 't+1', as below.

S2.9.1 Radiation scheme

The radiation approach is the application of the Longwave Radiation Transfer Matrix (LRTM) (Gu, 1988; Gu et al. 1999), as applied in Ogée et al. (2003). This approach seperates the calculation of the radiation distribution completely from the implicit expression. Instead a single source term for the long wave radiation is added at each level. This means that the distribution of radiation is now completely explicit (i.e. makes use of information only from the 'present' and not the 'next' time step. However, an advantage of the approach is that it accounts for a higher order of reflections from adjacent levels that the single order that is assumed in the process above.

The components for longwave radiation are abbreviated as:

$$R_{LW,i} = \eta_1 R_{LW}^{down} + \eta_2 T_{L,i}^{t+1} + \eta_3 \tag{S2.25}$$

The shortwave radiation component is abbreviated as:

$$R_{SW,i} = \eta_4 R_{SW}^{down} \tag{S2.26}$$

where η_1, η_2, η_3 and η_4 are components of the radiation scheme. η_1 is the component of the LW downwelling radiation, η_2 the components relating to emission and absorption of LW radiation from the vegetation at level i, η_3 the radiation components relating to radiation emitted from vegetation at all other levels incident on the vegetation at level i.

η_4 is the component of the SW radiation scheme, where a version of the Beer-Lambert law is used to calculate the incident radiation at each vegetation level (modified to allow for the radiation reflected):

$$\eta_4 = (exp(-K_{SW} \sum_{x=i}^{n} PAD_i)((1 - \rho_{albedo})) \tag{S2.27}$$

S2.9.2 Implicit form of the energy balance equation

We substitute the expressions (S2.25) and (S2.26) to the energy balance equation (S2.11), which we rewrite in implicit form:

$$a) \quad \theta_i \rho_v \frac{(T_{L,i}^{t+1} - T_{L,i}^{t})}{\Delta t} = \left(\frac{1}{\Delta h_i}\right) \Bigg(\Theta_{p,a}\rho_a \frac{(T_{L,i}^{t+1} - T_{a,i}^{t+1})}{R_i} + \lambda \rho_a \frac{(\alpha_i T_{L,i}^{t+1} + \beta_i - q_{a,i}^{t+1})}{R_i'} + \eta_1 R_{LW}^{down} + \eta_2 T_{L,i}^{t+1} + \eta_3 + \eta_4 R_{SW}^{down} \Bigg) \tag{S2.28}$$

Rearranging to isolate the state variables terms (temperature and specific humidity) at the 'next' timestep:

$$a) \quad T_{L,i}^{t+1} - T_{L,i}^{t} = \frac{\lambda\rho_a \Delta t \beta_i}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\eta_4 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_i)\theta_i} + \frac{\eta_1 R_{LW}^{down} \Delta t}{(\rho_v \Delta h_i)\theta_i} + \frac{\eta_3 \Delta t}{(\rho_v \Delta h_i)\theta_i}$$
$$+ T_{L,i}^{t+1} \left(\Theta_{p,a}\rho_a \frac{\Delta t}{(\rho_v \Delta h_i) R_i \theta_i} + \lambda \rho_a \frac{\Delta t \alpha_i}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_i)\theta_i} \right)$$
$$- T_{a,i}^{t+1} \Theta_{p,a}\rho_a \left(\frac{\Delta t}{R_i \theta_i (\rho_v \Delta h_i)} \right) - q_{a,i}^{t+1} \lambda \rho_a \left(\frac{\Delta t}{(\rho_v \Delta h_i) R_i' \theta_i} \right) \quad \text{(S2.29)}$$

S2.9.3 Implicit form of the sensible heat flux transport equation

We difference (S2.21) according to the finite volume method (S2.18), and divide by ΔV_i:

$$b) \quad \frac{T_{a,i}^{t+1} - T_{a,i}^{t}}{\Delta t} = k_i \left(\frac{(T_{a,i+1}^{t+1} - T_{a,i}^{t+1})}{\Delta z_i \Delta h_i} \right) - k_{i-1} \left(\frac{(T_{a,i}^{t+1} - T_{a,i-1}^{t+1})}{\Delta z_{i-1} \Delta h_i} \right)$$
$$+ \left(\frac{1}{\Delta h_i} \right) \frac{(T_{L,i}^{t+1} - T_{a,i}^{t+1})}{R_i} \quad \text{(S2.30)}$$

S2.9.4 Implicit form of the latent heat flux transport equation

We difference (S2.24) according to the finite volume method (S2.18), and divide by ΔV_i:

$$c) \quad \frac{q_{a,i}^{t+1} - q_{a,i}^{t}}{\Delta t} = k_i \left(\frac{(q_{a,i+1}^{t+1} - q_{a,i}^{t+1})}{\Delta z_i \Delta h_i} \right) - k_{i-1} \left(\frac{(q_{a,i}^{t+1} - q_{a,i-1}^{t+1})}{\Delta z_{i-1} \Delta h_i} \right)$$
$$+ \left(\frac{1}{\Delta h_i} \right) \frac{(\alpha_i T_{L,i}^{t+1} + \beta_i - q_{a,i}^{t+1})}{R_i'} \quad \text{(S2.31)}$$

S2.10 Solving the leaf energy balance equation by induction

We determine to solve these equations by assuming a solution of a particular form and finding the coefficients that are introduced in terms of the coefficients of the layer above. This is 'proof by induction'. Now, for (S2.29) we want to express $T_{a,i}^{t+1}$ in terms of values further down the column, to allow the equation to solved by 'moving up' the column, as in Richtmyer & Morton (1967) and Dufresne & Gattas (2009)

We assume that:

$$i) \quad \boxed{T_{a,i}^{t+1} = A_{T,i}T_{a,i-1}^{t+1} + B_{T,i} + C_{T,i}T_{L,i}^{t+1} + D_{T,i}q_{a,i-1}^{t+1}} \tag{S2.32}$$

$$ii) \quad \boxed{q_{a,i}^{t+1} = A_{q,i}q_{a,i-1}^{t+1} + B_{q,i} + C_{q,i}T_{L,i}^{t+1} + D_{q,i}T_{a,i-1}^{t+1}} \tag{S2.33}$$

These two expressions are the equivalent of (??) (from Richtmyer, 1967) for the present system.

We also re-write these expressions in terms of the values of the next level:

$$i) \quad \boxed{T_{a,i+1}^{t+1} = A_{T,i+1}T_{a,i}^{t+1} + B_{T,i+1} + C_{T,i+1}T_{L,i+1}^{t+1} + D_{T,i+1}q_{a,i}^{t+1}} \tag{S2.34}$$

$$ii) \quad \boxed{q_{a,i+1}^{t+1} = A_{q,i+1}q_{a,i}^{t+1} + B_{q,i+1} + C_{q,i+1}T_{L,i+1}^{t+1} + D_{q,i+1}T_{a,i}^{t+1}} \tag{S2.35}$$

where $A_{T,i}, B_{T,i}, C_{T,i}, D_{T,i}, A_{q,i}, B_{q,i}, C_{q,i}$ and $D_{q,i}$ are constants for that particular level and timestep but are (as yet) unknown. We thus substitute (S2.32) and (S2.33) into (S2.29) to eliminate $T_{a,i}^{t+1}$

$$\begin{aligned} a) \quad T_{L,i}^{t+1} - T_{L,i}^{t} = {} & \lambda\rho_a \frac{\Delta t \beta_i}{(\rho_v \Delta h_i) R'_i \theta_i} + \frac{\eta_3 \Delta t}{(\rho_v \Delta h_i)\theta_i} \\ & + \frac{\eta_4 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_i)\theta_i} + \frac{\eta_1 R_{LW}^{down} \Delta t}{(\rho_v \Delta h_i)\theta_i} \\ & + \frac{T_{L,i}^{t+1}}{(\rho_v \Delta h_i)} \left(\frac{\lambda \rho_a \Delta t \alpha_i}{R'_i \theta_i} + \frac{\Theta_{p,a}\rho_a \Delta t}{\theta_i R_i} + \frac{\eta_2 \Delta t}{\theta_i} \right) \\ & - \left(\frac{\Theta_{p,a}\rho_a \Delta t}{(\rho_v \Delta h_i) R_i \theta_i} \right) (A_{T,i}T_{a,i-1}^{t+1} + B_{T,i} + C_{T,i}T_{L,i}^{t+1} + D_{T,i}q_{a,i-1}^{t+1}) \\ & - \left(\frac{\lambda \rho_a \Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} \right) (A_{q,i}q_{a,i-1}^{t+1} + B_{q,i} + C_{q,i}(T_{L,i}^{t+1}) + D_{q,i}T_{a,i-1}^{t+1}) \end{aligned} \tag{S2.36}$$

or, to rearrange again in terms of the unknown state variables (left hand side) and the know variables (right hand side):

$$a)\quad T_{L,i}^{t+1}\left(1+\frac{\Delta t\Theta_{p,a}}{(\rho_v\Delta h_i)R_i\theta_i}C_{T,i}-\frac{\lambda\rho_a\alpha_i\Delta t}{(\rho_v\Delta h_i)R_i'\theta_i}-\frac{\eta_2\Delta t}{(\rho_v\Delta h_i)\theta_i}\right.$$
$$\left.-\frac{\Theta_{p,a}\rho_a\Delta t}{(\rho_v\Delta h_i)\theta_i R_i}+\frac{\lambda\rho_a\Delta t}{(\rho_v\Delta h_i)R_i'\theta_i}C_{q,i}\right)=$$
$$T_{L,i}^{t}+q_{a,i-1}^{t+1}\left(-\frac{\lambda\rho_a\Delta t A_{q,i}}{(\rho_v\Delta h_i)R_i'\theta_i}-\frac{\Theta_{p,a}\rho_a\Delta t D_{T,i}}{(\rho_v\Delta h_i)R_i\theta_i}\right)$$
$$+T_{a,i-1}^{t+1}\left(-\frac{\Theta_{p,a}\rho_a\Delta t A_{T,i}}{(\rho_v\Delta h_i)R_i\theta_i}-\frac{\lambda\rho_a\Delta t D_{q,i}}{(\rho_v\Delta h_i)R_i\theta_i}\right)+\frac{\eta_3\Delta t}{(\rho_v\Delta h_i)\theta_i}$$
$$+\frac{\eta_1 R_{LW}^{down}\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\eta_4 R_{SW,i}^{down}\Delta t}{(\rho_v\Delta h_i)\theta_i}-\frac{\Theta_{p,a}\rho_a\Delta t}{(\rho_v\Delta h_i)R_i\theta_i}B_{T,i}-\frac{\lambda\rho_a\Delta t}{(\rho_v\Delta h_i)R_i'\theta_i}B_{q,i}+\frac{\lambda\rho_a\Delta t}{(\rho_v\Delta h_i)R_i'\theta_i}\beta_i \tag{S2.37}$$

So, to abbreviate (where E_i, F_i and G_i are known assumed constants for the level and timestep in question, (S2.37) can be written as:

$$iii)\quad \boxed{T_{L,i}^{t+1}=E_i q_{a,i-1}^{t+1}+F_i T_{a,i-1}^{t+1}+G_i} \tag{S2.38}$$

so we define the coefficients as:

$$E_i=\left(-\frac{\Delta t A_{q,i}\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}-\frac{\Delta t D_{T,i}\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)R_i\theta_i}\right)/$$
$$\left(1-\frac{\Delta t\alpha\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}-\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)\theta_i R_i}-\frac{\eta_2\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\Delta t\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}C_{q,i}+\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)\theta_i R_i}C_{T,i}\right) \tag{S2.39}$$

$$F_i=\left(-\frac{\Delta t A_{T,i}\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)R_i\theta_i}-\frac{\Delta t D_{q,i}\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}\right)/$$
$$\left(1-\frac{\Delta t\alpha\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}-\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)\theta_i R_i}-\frac{\eta_2\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\Delta t\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}C_{q,i}+\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)\theta_i R_i}C_{T,i}\right) \tag{S2.40}$$

$$G_i=\left(T_{L,i}^{t}+\frac{\eta_3\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\eta_1 R_{LW}^{down}\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\eta_4 R_{SW}^{down}\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\lambda\rho_a\Delta t\beta_i}{(\rho_v\Delta h_i)R_i'\theta_i}\right.$$
$$\left.-\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)R_i\theta_i}B_{T,i}-\frac{\Delta t\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}B_{q,i}\right)$$
$$/\left(1-\frac{\Delta t\alpha\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}-\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)\theta_i R_i}-\frac{\eta_2\Delta t}{(\rho_v\Delta h_i)\theta_i}+\frac{\Delta t\lambda\rho_a}{(\rho_v\Delta h_i)R_i'\theta_i}C_{q,i}+\frac{\Delta t\Theta_{p,a}\rho_a}{(\rho_v\Delta h_i)\theta_i R_i}C_{T,i}\right) \tag{S2.41}$$

S2.11 Solving latent and sensible heat flux equations between layers by induction

To prove by induction, we must express $T_{a,i}^{t+1}$ and $q_{a,i}^{t+1}$ in terms that are identical to (S2.32) **and** (S2.33) We first seek to eliminate $T_{a,i+1}^{t+1}$ from b) and c) We first substitute the assumed expressions for temperature and humidity in the layer above or, that is to say, (equations (S2.34) and (S2.35) here) We substitute for $T_{a,i+1}^{t+1}$ in b), to eliminate that term:

$$b) \quad \frac{T_{a,i}^{t+1} - T_{a,i}^{t}}{\Delta t} = k_i \frac{A_{T,i+1}T_{a,i}^{t+1} + B_{T,i+1} + C_{T,i+1}T_{L,i+1}^{t+1} + D_{T,i+1}q_{a,i}^{t+1}}{\Delta z_i \Delta h_i} - \frac{k_i T_{a,i}^{t+1}}{\Delta z_i \Delta h_i} - \frac{k_{i-1}T_{a,i}^{t+1}}{\Delta z_{i-1}\Delta h_i} + \frac{k_{i-1}T_{a,i-1}^{t+1}}{\Delta z_{i-1}\Delta h_i} + \frac{T_{L,i}^{t+1}}{\Delta h_i R_i} - \frac{T_{a,i}^{t+1}}{\Delta h_i R_i} \tag{S2.42}$$

$$b) \quad T_{a,i}^{t+1}\left(1 - \Delta t\left(A_{T,i+1}\frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i} - \frac{1}{\Delta h_i R_i}\right)\right) = T_{a,i}^{t} + \frac{B_{T,i+1}k_i \Delta t}{\Delta z_i \Delta h_i} + T_{a,i-1}^{t+1}\left(\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}\right)\Delta t + q_{a,i}^{t+1}\left(\frac{k_i D_{T,i+1}}{\Delta z_i \Delta h_i}\right)\Delta t + T_{L,i}^{t+1}\left(\frac{\Delta t}{\Delta h_i R_i}\right) + T_{L,i+1}^{t+1}\left(\frac{k_i C_{T,i+1}}{\Delta z_i \Delta h_i}\right) \tag{S2.43}$$

Similarly, we substitute for $q_{a,i+1}^{t+1}$ in c), in order to eliminate that term:

$$c) \quad \frac{q_{a,i}^{t+1} - q_{a,i}^{t}}{\Delta t} = k_i \frac{(A_{q,i+1}q_{a,i}^{t+1} + B_{q,i+1} + C_{q,i+1}T_{L,i+1}^{t+1} + D_{q,i+1}T_{a,i}^{t+1})}{\Delta z_i \Delta h_i} - \frac{k_i q_{a,i}^{t+1}}{\Delta z_i \Delta h_i} - \frac{k_{i-1}q_{a,i}^{t+1}}{\Delta z_{i-1}\Delta h_i} + \frac{k_{i-1}q_{a,i-1}^{t+1}}{\Delta z_{i-1}\Delta h_i} + \frac{\alpha_i T_{L,i}^{t+1} + \beta_i - q_{a,i}^{t+1}}{\Delta h_i R_i'} \tag{S2.44}$$

$$c) \quad q_{a,i}^{t+1}\left(1 - \Delta t\left(A_{q,i+1}\frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i} - \frac{1}{\Delta h_i R_i'}\right)\right) = q_{a,i}^{t} + \left(\frac{B_{q,i+1}k_i \Delta t}{\Delta z_i \Delta h_i} + \frac{\beta \Delta t}{\Delta h_i R_i'}\right) + q_{a,i-1}^{t+1}\left(\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}\Delta t\right) T_{a,i}^{t+1}\left(\frac{k_i D_{q,i+1}}{\Delta z_i \Delta h_i}\right)\Delta t + (T_{L,i+1}^{t+1})\left(\frac{k_i}{\Delta z_i \Delta h_i}\right)C_{q,i+1}\Delta t + T_{L,i}^{t+1}\left(\frac{\alpha_i}{\Delta h_i R_i'}\right)\Delta t \tag{S2.45}$$

Now, we substitute expression iii) for the leaf temperature in the layer above (S2.38). This step is in order to eliminate the term $T_{L,i+1}^{t+1}$ from both expressions:

$$b) \quad T_{a,i}^{t+1}\left(1-\Delta t\left(A_{T,i+1}\frac{k_i}{\Delta z_i \Delta h_i}-\frac{k_i}{\Delta z_i \Delta h_i}-\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}-\frac{1}{\Delta h_i R_i}\right)\right)=$$
$$T_{a,i}^{t}+\frac{B_{T,i+1}k_i\Delta t}{\Delta z_i\Delta h_i}+T_{a,i-1}^{t+1}\left(\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}\right)\Delta t+q_{a,i}^{t+1}\left(\frac{k_iD_{T,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t+T_{L,i}^{t+1}\left(\frac{\Delta t}{\Delta h_i R_i}\right)$$
$$+\left(E_{i+1}q_{a,i}^{t+1}+F_{i+1}T_{a,i}^{t+1}+G_{i+1}\right)\left(\frac{k_iC_{T,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t \quad \text{(S2.46)}$$

$$c) \quad q_{a,i}^{t+1}\left(1-\Delta t\left(A_{q,i+1}\frac{k_i}{\Delta z_i \Delta h_i}-\frac{k_i}{\Delta z_i \Delta h_i}-\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}-\frac{1}{\Delta h_i R_i'}\right)\right)=$$
$$q_{a,i}^{t}+\left(\frac{B_{q,i+1}k_i\Delta t}{\Delta z_i\Delta h_i}+\frac{\beta\Delta t}{\Delta h_i R_i'}\right)+q_{a,i-1}^{t+1}\left(\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}\Delta t\right)$$
$$T_{a,i}^{t+1}\left(\frac{k_iD_{q,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t+\left(E_{i+1}q_{a,i}^{t+1}+F_{i+1}T_{a,i}^{t+1}+G_{i+1}\right)\left(\frac{k_i}{\Delta z_i\Delta h_i}\right)C_{q,i+1}\Delta t$$
$$+T_{L,i}^{t+1}\left(\frac{\alpha}{\Delta h_i R_i'}\right)\Delta t \quad \text{(S2.47)}$$

We now abbreviate equation a) as:

$$b) \quad T_{a,i}^{t+1}X_1 = X_2 + X_3T_{a,i-1}^{t+1} + X_4q_{a,i}^{t+1} + X_5T_{L,i}^{t+1} \quad \text{(S2.48)}$$

and abbreviate equation b) as:

$$c) \quad q_{a,i}^{t+1}Y_1 = Y_2 + Y_3q_{a,i-1}^{t+1} + Y_4T_{a,i}^{t+1} + Y_5T_{L,i}^{t+1} \quad \text{(S2.49)}$$

where:

$$X_1 = 1-\Delta t\left(A_{T,i+1}\frac{k_i}{\Delta z_i \Delta h_i}-\frac{k_i}{\Delta z_i \Delta h_i}-\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}-\frac{1}{\Delta h_i R_i}\right)-F_{i+1}\left(\frac{k_iC_{T,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t \quad \text{(S2.50)}$$

$$X_2 = T_{a,i}^{t}+\frac{B_{T,i+1}k_i\Delta t}{\Delta z_i\Delta h_i}+G_{i+1}\left(\frac{k_iC_{T,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t \quad \text{(S2.51)}$$

$$X_3 = \left(\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}\right)\Delta t \quad \text{(S2.52)}$$

$$X_4 = \left(\frac{k_iD_{T,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t+E_{i+1}\left(\frac{k_iC_{T,i+1}}{\Delta z_i\Delta h_i}\right)\Delta t \quad \text{(S2.53)}$$

$$X_5 = \left(\frac{\Delta t}{\Delta h_i R_i}\right) \tag{S2.54}$$

$$Y_1 = 1 - \Delta t \left(A_{q,i+1}\frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i} - \frac{1}{\Delta h_i R'_i}\right) - E_{i+1}\left(\frac{k_i}{\Delta z_i \Delta h_i}\right) C_{q,i+1}\Delta t \tag{S2.55}$$

$$Y_2 = q^t_{a,i} + \left(\frac{B_{q,i+1}k_i\Delta t}{\Delta z_i \Delta h_i} + \frac{\beta_i \Delta t}{\Delta h_i R'_i}\right) + G_{i+1}\left(\frac{k_i}{\Delta z_i \Delta h_i}\right) C_{q,i+1}\Delta t \tag{S2.56}$$

$$Y_3 = \left(\frac{k_{i-1}}{\Delta z_{i-1}\Delta h_i}\Delta t\right) \tag{S2.57}$$

$$Y_4 = \left(\frac{k_i D_{q,i+1}}{\Delta z_i \Delta h_i}\right)\Delta t + F_{i+1}\left(\frac{k_i}{\Delta z_i \Delta h_i} C_{q,i+1}\right)\Delta t \tag{S2.58}$$

$$Y_5 = \left(\frac{\alpha_i}{\Delta h_i R'_i}\right)\Delta t \tag{S2.59}$$

We then cross-substitute for $q^{t+1}_{a,i}$ from c) to b), to eliminate that term:

$$b) \quad T^{t+1}_{a,i} X_1 = X_2 + X_3 T^{t+1}_{a,i-1} + X_4\left(\frac{Y_2}{Y_1} + \frac{Y_3}{Y_1} q^{t+1}_{a,i-1} + \frac{Y_4}{Y_1} T^{t+1}_{a,i} + \frac{Y_5}{Y_1} T^{t+1}_{L,i}\right) + X_5 T^{t+1}_{L,i} \tag{S2.60}$$

$$b) \quad T^{t+1}_{a,i}\left(X_1 - X_4\frac{Y_4}{Y_1}\right) = T^{t+1}_{a,i-1}X_3 + \left(X_2 + X_4\frac{Y_2}{Y_1}\right) + T^{t+1}_{L,i}\left(X_4\frac{Y_5}{Y_1} + X_5\right) + q^{t+1}_{a,i-1}\left(X_4\frac{Y_3}{Y_1}\right) \tag{S2.61}$$

similarly, we cross-substitute for $T^{t+1}_{a,i}$ from b) to a), to eliminate that term:

$$c) \quad q^{t+1}_{a,i} Y_1 = Y_2 + Y_3 q^{t+1}_{a,i-1} + Y_4\left(\frac{X_2}{X_1} + \frac{X_3}{X_1} T^{t+1}_{a,i-1} + \frac{X_4}{X_1} q^{t+1}_{a,i} + \frac{X_5}{X_1} T^{t+1}_{L,i}\right) + Y_5 T^{t+1}_{L,i} \tag{S2.62}$$

$$c) \quad q^{t+1}_{a,i}\left(Y_1 - Y_4\frac{X_4}{X_1}\right) = q^{t+1}_{a,i-1}Y_3 + \left(Y_2 + Y_4\frac{X_2}{X_1}\right) + T^{t+1}_{L,i}\left(Y_4\frac{X_5}{X_1} + Y_5\right) + T^{t+1}_{a,i-1}\left(Y_4\frac{X_3}{X_1}\right) \tag{S2.63}$$

So this demonstrates the expressions b) and c) can be described in terms of the respective original substitutions (S2.32) **and** (S2.33)**.** The respective coefficients from (S2.32) and (S2.33) may be described as:

$$A_{T,i} = \frac{X_3}{X_1 - X_4\left(\frac{Y_4}{Y_1}\right)} \tag{S2.64}$$

$$B_{T,i} = \frac{X_2 + X_4\left(\frac{Y_2}{Y_1}\right)}{X_1 - X_4\left(\frac{Y_4}{Y_1}\right)} \tag{S2.65}$$

$$C_{T,i} = \frac{\left(X_4\left(\frac{Y_5}{Y_1}\right) + X_5\right)}{X_1 - X_4\left(\frac{Y_4}{Y_1}\right)} \tag{S2.66}$$

$$D_{T,i} = \frac{X_4\left(\frac{Y_3}{Y_1}\right)}{X_1 - X_4\left(\frac{Y_4}{Y_1}\right)} \tag{S2.67}$$

and:

$$A_{q,i} = \frac{Y_3}{Y_1 - Y_4\left(\frac{X_4}{X_1}\right)} \tag{S2.68}$$

$$B_{q,i} = \frac{Y_2 + Y_4\left(\frac{X_2}{X_1}\right)}{Y_1 - Y_4\left(\frac{X_4}{X_1}\right)} \tag{S2.69}$$

$$C_{q,i} = \frac{\left(Y_4\left(\frac{X_5}{X_1}\right) + Y_5\right)}{Y_1 - Y_4\left(\frac{X_4}{X_1}\right)} \tag{S2.70}$$

$$D_{q,i} = \frac{Y_4\left(\frac{X_3}{X_1}\right)}{Y_1 - Y_4\left(\frac{X_4}{X_1}\right)} \tag{S2.71}$$

Now, all of the coefficients X_1, X_2, X_3, X_4, X_5, Y_1, Y_2, Y_3, Y_4 and Y_5 and, in turn, the coefficients $A_{T,i}$, $B_{T,i}$, $C_{T,i}$, $D_{T,i}$, $A_{q,i}$, $B_{q,i}$, $C_{q,i}$ and $D_{q,i}$ can be described in terms of the coefficients from the level above and the potentials (i.e. T and q) at the previous timestep.

So we have a set of coefficients that may be determined for each time-step, and we have the means to determine T_S (and q_S via the saturation assumption). We thus have a process to calculate the temperature and humidity profiles for each timestep by systematically calculating each of the coefficients from the top of the column (the 'downwards sweep') then calculating the 'initial value' (the surface temperature and humidity) and finally calculating each T_a, q_a and T_L by working up the column (the 'upwards sweep').

The term $T_{L,i+1}^{t+1}$ can also be described in terms of the variables at the level below by using equation iii) and its terms E_i, F_i and G_i. We can therefore describe the changes in the canopy between the present timestep 't' and the next timestep 't+1' by 'working down' the column from the interaction with the LMDZ atmospheric model to determine the coefficients A_T, B_T, C_T etc. and then 'working up' the column to determine the potentials T and q.

Table 1: *Input coefficients at the top layer of the model, where $A_{T,n}, B_{T,n}\ldots$ etc are the respective coefficients at the top of the surface model and $A_{T,atmos}$, $B_{T,atmos}$ are the coefficients at the lowest level of the atmospheric model.*

stand-alone model	**coupled model**
$A_{T,n} = 0$	$A_{T,n} = A_{T,atmos}$
$B_{T,n} = B_{T,input}$	$B_{T,n} = B_{T,atmos}$
$C_{T,n} = 0$	$C_{T,n} = 0$
$D_{T,n} = 0$	$D_{T,n} = 0$
$A_{q,n} = 0$	$A_{q,n} = A_{q,atmos}$
$B_{q,n} = B_{q,input}$	$B_{q,n} = B_{q,atmos}$
$C_{q,n} = 0$	$C_{q,n} = 0$
$D_{q,n} = 0$	$D_{q,n} = 0$

S3 The boundary conditions

S3.1 The upper boundary conditions

In stand-alone simulations, the top level variables $A_{T,n}$, $C_{T,n}$, $D_{T,n}$ and $A_{q,n}$, $C_{q,n}$, $D_{q,n}$, are set to zero and $B_{T,n}$ and $B_{q,n}$ set to the input temperature and specific humidity respectively for the relevant time step (as in Best et al., 2004) In coupled simulations, $A_{T,n}$, $B_{T,n}$ and $B_{q,n}$, $C_{q,n}$ are taken from the respective values at lowest level of the atmospheric model. Table 1 summarises the boundary conditions for both the coupled and un-coupled simulations.

S3.2 The lower boundary condition

We need to solve the lowest level transport equations seperately:

$$b) \quad \frac{T_{a,1}^{t+1}-T_{a,1}^{t}}{\Delta t} = k_1\frac{(T_{a,2}^{t+1}-T_{a,1}^{t+1})}{\Delta z_1\Delta h_1} - \left(\frac{1}{\rho_a\Theta_{p,a}}\right)\frac{\phi_H}{\Delta h_1} + \left(\frac{1}{\Delta h_1}\right)\frac{(T_{L,1}^{t+1}-T_{a,1}^{t+1})}{R_1} \quad \text{(S3.1)}$$

$$c) \quad \frac{q_{a,1}^{t+1}-q_{a,1}^{t}}{\Delta t} = k_1\frac{(q_{a,2}^{t+1}-q_{a,1}^{t+1})}{\Delta z_1\Delta h_1} - \left(\frac{1}{\rho_a\lambda}\right)\frac{\phi_{LE}}{\Delta h_1} + \left(\frac{1}{\Delta h_1}\right)\frac{(\alpha_i T_{L,1}^{t+1}+\beta_1-q_{a,1}^{t+1})}{R_1'} \quad \text{(S3.2)}$$

We substitute to the above to eliminate $T_{a,2}^{t+1}$ from b) and $q_{a,2}^{t+1}$ from c):

$$T_{a,1}^{t+1} = A_{T,1}\phi_H + B_{T,1} + C_{T,1}T_{L,1}^{t+1} + D_{T,1}\phi_{LE} \quad \text{(S3.3)}$$

and:

$$q_{a,1}^{t+1} = A_{q,1}\phi_{LE} + B_{q,1} + C_{q,1}T_{L,1}^{t+1} + D_{q,1}\phi_H \quad \text{(S3.4)}$$

Now for the leaf at level 1, just above the ground level:

$$\begin{aligned} a) \quad T_{L,1}^{t+1} - T_{L,1}^{t} = {} & \frac{\lambda\rho_a\Delta t\beta_1}{(\rho_v\Delta h_1)R_1'\theta_1} + \frac{\eta_4 R_{SW}^{down}\Delta t}{(\rho_v\Delta h_1)\theta_1} + \frac{\eta_1 R_{LW}^{down}\Delta t}{(\rho_v\Delta h_1)\theta_1} + \frac{\eta_3\Delta t}{(\rho_v\Delta h_1)\theta_1} \\ & + T_{L,1}^{t+1}\left(\Theta_{p,a}\rho_a\frac{\Delta t}{(\rho_v\Delta h_1)R_1\theta_1} + \lambda\rho_a\frac{\Delta t\alpha}{(\rho_v\Delta h_1)R_1'\theta_1} + \frac{\eta_2\Delta t}{(\rho_v\Delta h_1)\theta_1}\right) \\ & - T_{a,1}^{t+1}\Theta_{p,a}\rho_a\left(\frac{\Delta t}{R_1\theta_1(\rho_v\Delta h_1)}\right) - q_{a,1}^{t+1}\lambda\rho_a\left(\frac{\Delta t}{(\rho_v\Delta h_1)R_1'\theta_1}\right) \quad \text{(S3.5)} \end{aligned}$$

and substitute for $T_{a,1}^{t+1}$ and $q_{a,1}^{t+1}$:

$$\begin{aligned} a) \quad T_{L,1}^{t+1} - T_{L,1}^{t} = {} & \frac{\lambda\rho_a\Delta t\beta_i}{(\rho_v\Delta h_1)R_1'\theta_1} + \frac{eta_4 R_{SW}^{down}\Delta t}{(\rho_v\Delta h_1)\theta_1} + \frac{\eta_1 R_{LW}^{down}\Delta t}{(\rho_v\Delta h_1)} + \frac{\eta_3\Delta t}{(\rho_v\Delta h_1)\theta_1} \\ & + T_{L,1}^{t+1}\left(\Theta_{p,a}\rho_a\frac{\Delta t}{(\rho_v\Delta h_1)R_1\theta_1} + \lambda\rho_a\frac{\Delta t\alpha}{(\rho_v\Delta h_1)R_1'\theta_1} + \frac{\eta_2\Delta t}{(\rho_v\Delta h_1)\theta_1}\right) \\ & - \frac{\Theta_{p,a}\rho_a\Delta t}{R_1\theta_1(\rho_v\Delta h_1)}(A_{T,1}\phi_H + B_{T,1} + C_{T,1}T_{L,1}^{t+1} + D_{T,1}\phi_{LE}) \\ & \quad - \frac{\lambda\rho_a\Delta t}{R_1'\theta_1(\rho_v\Delta h_1)}(A_{q,1}\phi_{LE} + B_{q,1} + C_{q,1}T_{L,1}^{t+1} + D_{q,1}\phi_H) \quad \text{(S3.6)} \end{aligned}$$

In a similar approach to the previous section, this should be reduced to the form:

$$T_{L,1}^{t+1} = E_1\phi_{LE} + F_1\phi_H + G_1 \tag{S3.7}$$

and the expression re-arranged to isolate the factors E_1, F_1 and G_1:

$$\begin{aligned}
a) \quad & T_{L,1}^{t+1}\left(1 - \left(\frac{\Theta_{p,a}\rho_a\Delta t}{(\rho_v\Delta h_1)R_1\theta_1} + \frac{\lambda\rho_a\Delta t\alpha}{(\rho_v\Delta h_1)R_1'\theta_1} + \frac{\eta_2\Delta t}{(\rho_v\Delta h_1)\theta_1}\right) + \frac{C_{T,1}\Theta_{p,a}\rho_a\Delta t}{R_1\theta_1(\rho_v\Delta h_1)}\right. \\
&\left. + \frac{C_{q,1}\lambda\rho_a\Delta t}{(\rho_v\Delta h_1)R_1'\theta_1}\right) = T_{L,1}^t + \phi_{LE}\left(-\frac{\Theta_{p,a}\rho_a\Delta t}{R_1\theta_1(\rho_v\Delta h_1)}D_{T,1} - \frac{\lambda\rho_a\Delta t}{(\rho_v\Delta h_1)R_1'\theta_1}A_{q,1}\right) \\
&\qquad + \phi_H\left(-\frac{\Theta_{p,a}\rho_a\Delta t}{R_1\theta_1(\rho_v\Delta h_1)}A_{T,1} - \frac{\lambda rho_a\Delta t}{(\rho_v\Delta h_1)R_1'\theta_1}D_{q,1}\right) \\
&\qquad + \left(\frac{\lambda\rho_a\Delta t\beta_i}{(\rho_v\Delta h_1)R_1'\theta_1} + \frac{\eta_4 R_{SW}^{down}\Delta t}{(\rho_v\Delta h_1)\theta_1} + \frac{\eta_1 R_{LW}^{down}\Delta t}{(\rho_v\Delta h_1)\theta_1}\right. \\
&\qquad \left. + \frac{\eta_3\Delta t}{(\rho_v\Delta h_1)\theta_1} - \frac{B_{T,1}\Theta_{p,a}\rho_a\Delta t}{R_1\theta_1(\rho_v\Delta h_1)} - \frac{B_{q,1}\lambda\rho_a\Delta t}{(\rho_v\Delta h_1)R_1'\theta_1}\right)
\end{aligned} \tag{S3.8}$$

Now, substituting for $T_{a,2}^{t+1}$ in expression b):

$$\begin{aligned}
b) \quad & \frac{T_{a,1}^{t+1} - T_{a,1}^t}{\Delta t} = \\
& k_1\frac{(A_{T,2}T_{a,1}^{t+1} + B_{T,2} + C_{T,2}(E_2 q_{a,1}^{t+1} + F_2 T_{a,1}^{t+1} + G_2) + D_{T,2}q_{a,1}^{t+1} + T_{a,1}^{t+1})}{\Delta z_1\Delta h_1} \\
&\qquad - \left(\frac{1}{\rho_a\Theta_{p,a}}\right)\frac{\phi_H}{\Delta h_1} + \frac{T_{L,1}^{t+1} - T_{a,1}^{t+1}}{R_1\Delta h_1}
\end{aligned} \tag{S3.9}$$

$$\begin{aligned}
b) \quad & T_{a,1}^{t+1}\left(\frac{1}{\Delta t} - \frac{k_1 A_{T,2}}{\Delta z_1\Delta h_1} - \frac{k_1 C_{T,2}F_2}{\Delta z_1\Delta h_1} + \frac{k_1}{\Delta z_1\Delta h_1} + \frac{1}{\Delta h_1 R_1}\right) = \\
& \frac{T_{a,1}^t}{\Delta t} + q_{a,1}^{t+1}\left(\frac{k_1 C_{T,2}E_2 + k_1 D_{T,2}}{\Delta z_1\Delta h_1}\right) + \\
& T_{L,1}^{t+1}\left(\frac{1}{\Delta h_1 R_1}\right) + \left(\frac{k_1 B_{T,2}}{\Delta z_1\Delta h_1} + \frac{k_1 C_{T,2}G_2}{\Delta z_1\Delta h_1}\right) - \left(\frac{1}{\rho_a\Theta_{p,a}}\right)\phi_H\left(\frac{1}{\Delta h_1}\right)
\end{aligned} \tag{S3.10}$$

and for $q_{a,2}^{t+1}$ in expression c):

$$c) \quad \frac{q_{a,1}^{t+1} - q_{a,1}^{t}}{\Delta t} = k_1 \frac{(A_{q,2} q_{a,1}^{t+1} + B_{q,2} + C_{q,2}(T_{L,2}^{t+1}) + D_{q,2} T_{a,1}^{t+1} - q_{a,1}^{t+1})}{\Delta z_1 \Delta h_1} = - \left(\frac{1}{\rho_a \lambda}\right) \frac{\phi_{LE}}{\Delta h_1} + \frac{1}{\Delta h_1} \frac{\alpha T_{L,1}^{t+1} + \beta_1 - q_{a,1}^{t+1}}{R_1'} \tag{S3.11}$$

$$c) \quad \frac{q_{a,1}^{t+1} - q_{a,1}^{t}}{\Delta t} = k_1 \frac{(A_{q,2} q_{a,1}^{t+1} + B_{q,2} + C_{q,2}(E_2 q_{a,1}^{t+1} + F_2 T_{a,1}^{t+1} + G_2) + D_{q,2} T_{a,1}^{t+1} - q_{a,1}^{t+1})}{\Delta z_1 \Delta h_1} - \left(\frac{1}{\rho_a \lambda}\right) \frac{\phi_{LE}}{\Delta h_1} + \frac{1}{\Delta h_1} \frac{(\alpha T_{L,1}^{t+1} + \beta_1 - q_{a,1}^{t+1})}{R_1'} \tag{S3.12}$$

$$c) \quad q_{a,1}^{t+1}\left(\frac{1}{\Delta t} - \frac{k_1 A_{q,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{q,2} E_2}{\Delta z_1 \Delta h_1} + \frac{k_1}{\Delta z_1 \Delta h_1}\right) = \frac{q_{a,1}^{t}}{\Delta t} + T_{a,1}^{t+1}\left(\frac{C_{q,2}F_2}{\Delta z_1 \Delta h_1} + \frac{D_{q,2}}{\Delta z_1 \Delta h_1}\right) + T_{L,1}^{t+1}\left(\frac{\alpha}{\Delta h_1 R_1'}\right) + \left(\frac{k_1 B_{q,2}}{\Delta z_1 \Delta h_1} + \frac{C_{q,2} G_2}{\Delta z_1 \Delta h_1} + \frac{\beta_1}{\Delta h_1 R_1'}\right) - \phi_{LE}\left(\frac{1}{\Delta h_1}\right) \tag{S3.13}$$

We now isolate the terms in (S3.8):

$$a) \quad T_{L,1}^{t+1} = E_1 \phi_{LE} + F_1 \phi_H + G_1 \tag{S3.14}$$

so we have:

$$E_1 = \left(-\frac{\Theta_{p,a}\rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} D_{T,1} - \frac{\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} A_{q,1}\right) / \left(1 - \left(\frac{\Theta_{p,a}\rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1)\theta_1}\right) + \frac{C_{T,1}\Theta_{p,a}\rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} + \frac{C_{q,1}\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1}\right) \tag{S3.15}$$

$$F_1 = \left(-\frac{\Theta_{p,a}\rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} A_{T,1} - \frac{\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta} D_{q,1}\right) / \left(1 - \left(\frac{\Theta_{p,a}\rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_2 \Delta t}{(rho_v \Delta h_1)\theta_1}\right) + \frac{C_{T,1}\Theta_{p,a}\rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} + \frac{C_{q,1}\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1}\right) \tag{S3.16}$$

and:

$$G_1 = \left(T_{L,1}^t + \frac{\lambda \rho_a \Delta t \beta_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_4 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_1) \theta_1} + \frac{\eta_1 R_{LW}^{down} \Delta t}{(\rho_v \Delta h_1 \theta_1)} + \frac{\eta_3 \Delta t}{(\rho_v \Delta h_1) \theta_1} - \frac{B_{T,1} \Theta_{p,a} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} - \frac{B_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} \right) / \left(1 - \left(\frac{\Theta_{p,a} \rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} \right) + \frac{C_{T,1} \Theta_{p,a} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} + \frac{C_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} \right) \tag{S3.17}$$

We now seek to rearrange b) and c) into expressions of the form:

$$i) \quad T_{a,1}^{t+1} X_1 = X_2 + \phi_H X_3 + q_{a,1}^{t+1} X_4 + T_{L,1}^{t+1} X_5 \tag{S3.18}$$

and:

$$ii) \quad q_{a,1}^{t+1} Y_1 = Y_2 + \phi_{LE} Y_3 + T_{a,1}^{t+1} Y_4 + T_{L,1}^{t+1} Y_5 \tag{S3.19}$$

The same process as in the previous section means that we can assign $A_{T,1}$, $B_{T,1}$, $C_{T,1}$, $D_{T,1}$, $A_{q,1}$, $B_{q,1}$, $C_{q,1}$, $D_{q,1}$ exactly as previously (expressions (S2.64) to (S2.71)), and define X_1 to Y_5 as follows:

$$X_1 = 1 - \Delta t \left(\frac{k_1 A_{T,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{T,2} F_2}{\Delta z_1 \Delta h_1} + \frac{k_1}{\Delta z_1 \Delta h_1} + \frac{1}{\Delta h_1 R_1} \right) \tag{S3.20}$$

$$X_2 = T_{a,1}^t + \Delta t \left(\frac{k_1 B_{T,2}}{\Delta z_1 \Delta h_1} + \frac{k_1 C_{T,2} G_2}{\Delta z_1 \Delta h_1} \right) \tag{S3.21}$$

$$X_3 = -\Delta t \left(\frac{1}{\Delta h} \right) \left(\frac{1}{\rho_a \Theta_{p,a}} \right) \tag{S3.22}$$

$$X_4 = \Delta t \left(\frac{k_1 C_{T,2} E_2 + k_1 D_{T,2}}{\Delta z_1 \Delta h_1} \right) \tag{S3.23}$$

$$X_5 = \Delta t \left(\frac{1}{\Delta h_1 R_1} \right) \tag{S3.24}$$

$$Y_1 = 1 - \Delta t \left(\frac{k_1 A_{q,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{q,2} E_2}{\Delta z_1 \Delta h_1} + \frac{1}{\Delta h_1 R'} + \frac{k_1}{\Delta z_1 \Delta h_1} \right) \tag{S3.25}$$

$$Y_2 = q^t_{a,1} + \Delta t \left(\frac{k_1 B_{q,2}}{\Delta z_1 \Delta h_1} + \frac{k_1 C_{q,2} G_2}{\Delta z_1 \Delta h_1} + \frac{\beta_1}{\Delta h_1 R'_1} \right) \tag{S3.26}$$

$$Y_3 = -\Delta t \left(\frac{1}{\Delta h_1} \right) \left(\frac{1}{\rho_a \lambda} \right) \tag{S3.27}$$

$$Y_4 = \Delta t \left(\frac{k_1 C_{q,2} F_2}{\Delta z_1 \Delta h_1} + \frac{k_1 D_{q,2}}{\Delta z_1 \Delta h_1} \right) \tag{S3.28}$$

$$Y_5 = \Delta t \left(\frac{\alpha_1}{\Delta h_1 R'_1} \right) \tag{S3.29}$$

Now, for the lower boundary condition we consider the interaction between the lowest atmospheric level (level 1) and the infinitesimal surface layer (level S). Fluxes of the sensible and latent heat from this layer are given, respectively, by:

$$i) \quad \phi_H = -(\rho_a \Theta_{p,a}) k_S \frac{T^{t+1}_{a,1} - T^{t+1}_S}{\Delta z_S} \tag{S3.30}$$

$$ii) \quad \phi_{LE} = -(\rho_a \lambda) k_S \frac{q^{t+1}_{a,1} - q^{t+1}_S}{\Delta z_S} \tag{S3.31}$$

$$i) \phi_H = \frac{\rho_a \Theta_{p,a} k_s}{\Delta z_s} (A_{T,1} \phi_H + B_{T,1} + C_{T,1} T^{t+1}_{L,1} + D_{T,1} \phi_{LE} - T^{t+1}_S) \tag{S3.32}$$

$$ii) \phi_{LE} = \frac{(\rho_a \lambda) k_s}{\Delta z_s} (A_{q,1} \phi_{LE} + B_{q,1} + C_{q,1} T^{t+1}_{L,1} + D_{q,1} \phi_H - q^{t+1}_S) \tag{S3.33}$$

use a substitution:

$$T_{L,1}^{t+1} = E_1\phi_{LE} + F_1\phi_H + G_1 \tag{S3.34}$$

$$i) \quad \phi_H = -\frac{(\rho_a\Theta_{p,a})k_S}{\Delta z_S}(A_{T,1}\phi_H + B_{T,1} + C_{T,1}(E_1\phi_{LE} + F_1\phi_H + G_1) \\ + D_{T,1}\phi_{LE} - T_S^{t+1}) \tag{S3.35}$$

$$ii) \quad \phi_{LE} = -\frac{(\rho_a\lambda)k_S}{\Delta z_S}(A_{q,1}\phi_{LE} + B_{q,1} + C_{q,1}(E_1\phi_{LE} + F_1\phi_H + G_1) \\ + D_{q,1}\phi_H - q_S^{t+1}) \tag{S3.36}$$

$$i) \quad \phi_H(1 + \frac{(\rho_a\Theta_{p,a})k_S}{\Delta z_S}(A_{T,1} + C_{T,1}F_1) = -\frac{(\rho_a\Theta_{p,a})k_S}{\Delta z_S}(B_{T,1} + C_{T,1}G_1 - T_S^{t+1}) \\ - \frac{(\rho_a\Theta_{p,a})k_S}{\Delta z_S}(\phi_{LE}(C_{T,1}E_1 + D_{T,1})) \tag{S3.37}$$

$$ii) \quad \phi_{LE}(1 + \frac{(\rho_a\lambda)k_S}{\Delta z_S}(A_{q,1} + C_{q,1}E_1)) = -\frac{(\rho_a\lambda)k_S}{\Delta z_S}(B_{q,1} + C_{q,1}G_1 - q_S^{t+1}) \\ - \frac{(\rho_a\lambda)k_S}{\Delta z_S}(\phi_H(C_{q,1}F_1 + D_{q,1})) \tag{S3.38}$$

$$ii) \quad \phi_{LE}(1+\frac{(\rho_a\lambda)k_S}{\Delta z_S}(A_{q,1}+C_{q,1}E_1)) = -\frac{(\rho_a\lambda)k_S}{\Delta z_S}(B_{q,1}+C_{q,1}G_1-(\alpha_S T_S^{t+1}+\beta_S)) \\ - \frac{(\rho_a\lambda)k_S}{\Delta z_S}(\phi_H(C_{q,1}F_1 + D_{q,1})) \tag{S3.39}$$

and abbreviate to:

$$i) \quad \Omega_1\phi_H = \Omega_2 + \Omega_3 T_S^{t+1} + \Omega_4\phi_{LE} \tag{S3.40}$$

$$ii) \quad \Omega_5\phi_{LE} = \Omega_6 + \Omega_7 T_S^{t+1} + \Omega_8\phi_H \tag{S3.41}$$

where:

$$\Omega_1 = \left(1 + \frac{(\rho_a \Theta_{p,a})k_S}{\Delta z_S}(A_{T,1} + C_{T,1}F_1)\right) \tag{S3.42}$$

$$\Omega_2 = -\frac{(\rho_a \Theta_{p,a})k_S}{\Delta z_S}(B_{T,1} + C_{T,1}G_1) \tag{S3.43}$$

$$\Omega_3 = \frac{\rho_a \Theta_{p,a} k_S}{\Delta z_S} \tag{S3.44}$$

$$\Omega_4 = -\frac{(\rho_a \Theta_{p,a} k_S)}{\Delta z_S}(C_{T,1}E_1 + D_{T,1}) \tag{S3.45}$$

$$\Omega_5 = \left(1 + \frac{(\rho_a \lambda)k_S}{\Delta z_S}(A_{q,1} + C_{q,1}E_1)\right) \tag{S3.46}$$

$$\Omega_6 = -\frac{(\rho_a \lambda)k_S}{\Delta z_S}(B_{q,1} + C_{q,1}G_1 - \beta_S) \tag{S3.47}$$

$$\Omega_7 = \frac{(\rho_a \lambda)k_S}{\Delta z_S}\alpha_S \tag{S3.48}$$

$$\Omega_8 = -\frac{(\rho_a \lambda)k_S}{\Delta z_S}(C_{q,1}F_1 + D_{q,1}) \tag{S3.49}$$

$$\xi_1 = \frac{\Omega_2 + \frac{\Omega_4}{\Omega_5}\Omega_6}{\Omega_1 - \frac{\Omega_4}{\Omega_5}\Omega_8} \tag{S3.50}$$

$$\xi_2 = \frac{\Omega_3 + \frac{\Omega_4 \Omega_7}{\Omega_5}}{\Omega_1 - \frac{\Omega_4}{\Omega_5}\Omega_8} \tag{S3.51}$$

$$\xi_3 = \frac{\Omega_6 + \frac{\Omega_8 \Omega_2}{\Omega_1}}{\Omega_5 - \frac{\Omega 8 \Omega_4}{\Omega_1}} \tag{S3.52}$$

$$\xi_4 = \frac{\Omega_7 + \frac{\Omega_8 \Omega_3}{\Omega_1}}{\Omega_5 - \frac{\Omega_8 \Omega_4}{\Omega_1}} \tag{S3.53}$$

cross substitute:

$$i) \quad \Omega_1 \phi_H = \Omega_2 + \Omega_3 T_S^{t+1} + \frac{\Omega_4}{\Omega_5}(\Omega_6 + \Omega_7 T_S^{t+1} + \Omega_8 \Phi_H) \tag{S3.54}$$

$$i) \quad \Phi_H \left(\Omega_1 - \frac{\Omega_4}{\Omega_5} \Omega_8 \right) = \left(\Omega_2 + \frac{\Omega_4}{\Omega_5} \Omega_6 \right) + T_S^{t+1} \left(\Omega_3 + \frac{\Omega_4 \Omega_7}{\Omega_5} \right) \tag{S3.55}$$

and:

$$ii) \quad \Omega_5 \phi_{LE} = \Omega_6 + \Omega_7 T_S^{t+1} + \frac{\Omega_8}{\Omega_1}(\Omega_2 + \Omega_3 T_S^{t+1} + \Omega_4 \Phi_{LE}) \tag{S3.56}$$

$$ii) \quad \Phi_{LE} \left(\Omega_5 - \frac{\Omega_8}{\Omega_1} \Omega_4 \right) = \left(\Omega_6 + \frac{\Omega_8}{\Omega_1} \Omega_2 \right) + T_S^{t+1} \left(\Omega_7 + \frac{\Omega_8 \Omega_3}{\Omega_1} \right) \tag{S3.57}$$

rewrite:

$$i) \quad \phi_H = \xi_1 + \xi_2 T_S^{t+1} \tag{S3.58}$$

$$ii) \quad \phi_{LE} = \xi_3 + \xi_4 T_S^{t+1} \tag{S3.59}$$

$$T_S^{t+1} = T_S^t + \frac{\Delta t}{\theta_0}((R_{LW} + R_{SW} + \xi_1 + \xi_2 T_S^{t+1} + \xi_3 + \xi_4 T_S^{t+1}) - J_{soil}) \tag{S3.60}$$

$$T_S^{t+1} \left(1 - \xi_2 \frac{\Delta t}{\theta_0} - \xi_4 \frac{\Delta t}{\theta_0} \right) = T_S^t + \frac{\Delta t}{\theta_0}(R_{LW} + R_{SW} + \xi_1 + \xi_3 - J_{soil}) \tag{S3.61}$$

and so:

$$T_S^{t+1} = \frac{T_S^t + \frac{\Delta t}{\theta_0}(R_{LW} + R_{SW} + \xi_1 + \xi_3 - J_{soil})}{(1 - \xi_2 \frac{\Delta t}{\theta_0} - \xi_4 \frac{\Delta t}{\theta_0})} \tag{S3.62}$$

We therefore have an expression for the surface temperature T_S^{t+1}, in terms of the down-welling radiation that is incident on the surface (R_{LW} and R_{SW}), the heat capacity of the infinitesimal surface layer (θ_0), the vegetation layer directly above the surface (ξ_1, ξ_2, ξ_3 and ξ_4) and the heat from the soil system (J_{soil}).

The radiation that is received by the lowermost level is provided by the radiation scheme.

So to re-write the above equation including the factors $\eta_{1,S}$, $\eta_{2,S}$, $\eta_{3,S}$ and $\eta_{4,S}$:

$$T_S^{t+1} = \frac{T_S^t + \frac{\Delta t}{\theta_0}(\eta_{1,S} R_{LW}^{down} + \eta_{3,S} + \eta_{4,S} R_{SW}^{down} + \xi_1 + \xi_3) - J_{soil}}{(1 - \frac{\Delta t}{\theta_0}(\xi_2 + \xi_4 + \eta_{2,S}))} \tag{S3.63}$$

S4 Notation list

symbol	description
T^t, T^{t+1}	Temperature at the 'present' and 'next' timestep respectively (K)
q^t, q^{t+1}	Specific humidity at the 'present' and 'next' timestep (kg/kg)
T_i^L	Leaf temperature at level 'i' (K)
q_i^L	Leaf specific humidity at level 'i' (kg/kg)
T_i^a	Atmospheric temperature at level 'i' (K)
q_i^a	Atmospheric specific humidity at level 'i' (kg/kg)
ΔT	Interval between 'present' and 'next' timestep (s)
Δz_i	Difference in height between potential at level 'i' and level 'i+1' (m)
Δh_i	Thickness of level 'i' (m)
ϵ_i	Emissivity fraction at level 'i' (-)
ω_i	Leaf interception coefficient at level 'i' (-)
K_{LW}, K_{SW}	Canopy extinction coefficient for longwave and shortwave, respectively (-)
ρ_i^{alb}	Albedo of vegetation layer 'i' (-)
λ	Latent heat of vapourisation (J/kg)
ρ_v, ρ_a	Vegetation and atmospheric density, respectively (kg/m^3)
σ	Stefan-Boltzmann constant (5.67 x 10^{-8} $Wm^{-2}K^{-4}$)
θ_i	Leaf layer heat capacity at level 'i'(J/ (kg K))
$\Theta_{p,a}$	Specific heat capacity of air (J/(kg K))
R_i, R_i'	Stomatal resistance at level 'i' for sensible and latent heat flux, respectively (s/m)
LE_i, H_i	Latent heat and sensible heat flux at level 'i', respectively (W/m^2)
LE_{tot}, H_{tot}	Total latent heat and sensible heat flux at canopy top, respectively (W/m^2)
$R_{LW,i}, R_{SW,i}$	Long-wave and short wave radiation received by level 'i', respectively (W/m^2)
k_i	Diffusivity coefficient for level 'i' (m^2/s)
$A_{T,i}, B_{T,i}, C_{T,i}, D_{T,i}$	Components for substituted equation i)
$A_{q,i}, B_{q,i}, C_{q,i}, D_{q,i}$	Components for substituted equation ii)
E_i, F_i, G_i	Components for substituted equation iii)
θ_0	Heat capacity of the infinitesimal surface layer ($J/(Km^2)$)
J_{soil}	Heat flux from the sub-soil (W/m^2)
ϕ_H, ϕ_{LE}	Respectively sensible and latent heat flux from the infinitesimal surface layer (W/m^2)

Supplementary material

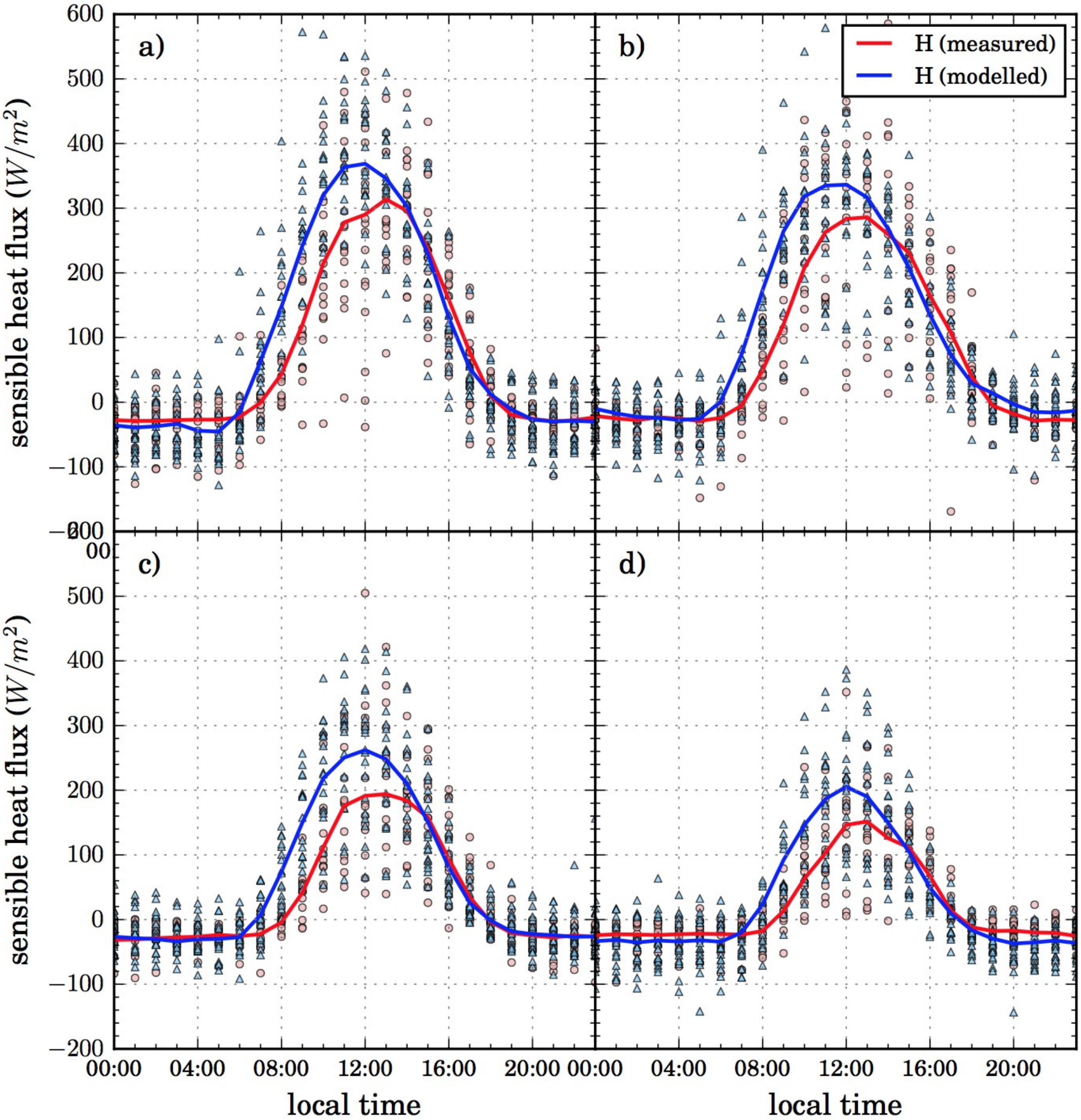

Figure S1 : The analysis of Figure 3 is repeated for the four seasons of the year (every 5th measurement is shown in the background in a lighter colour). Hourly average sensible heat flux (annual average) : a) spring ; b) summer ; c) autumn ; d) winter

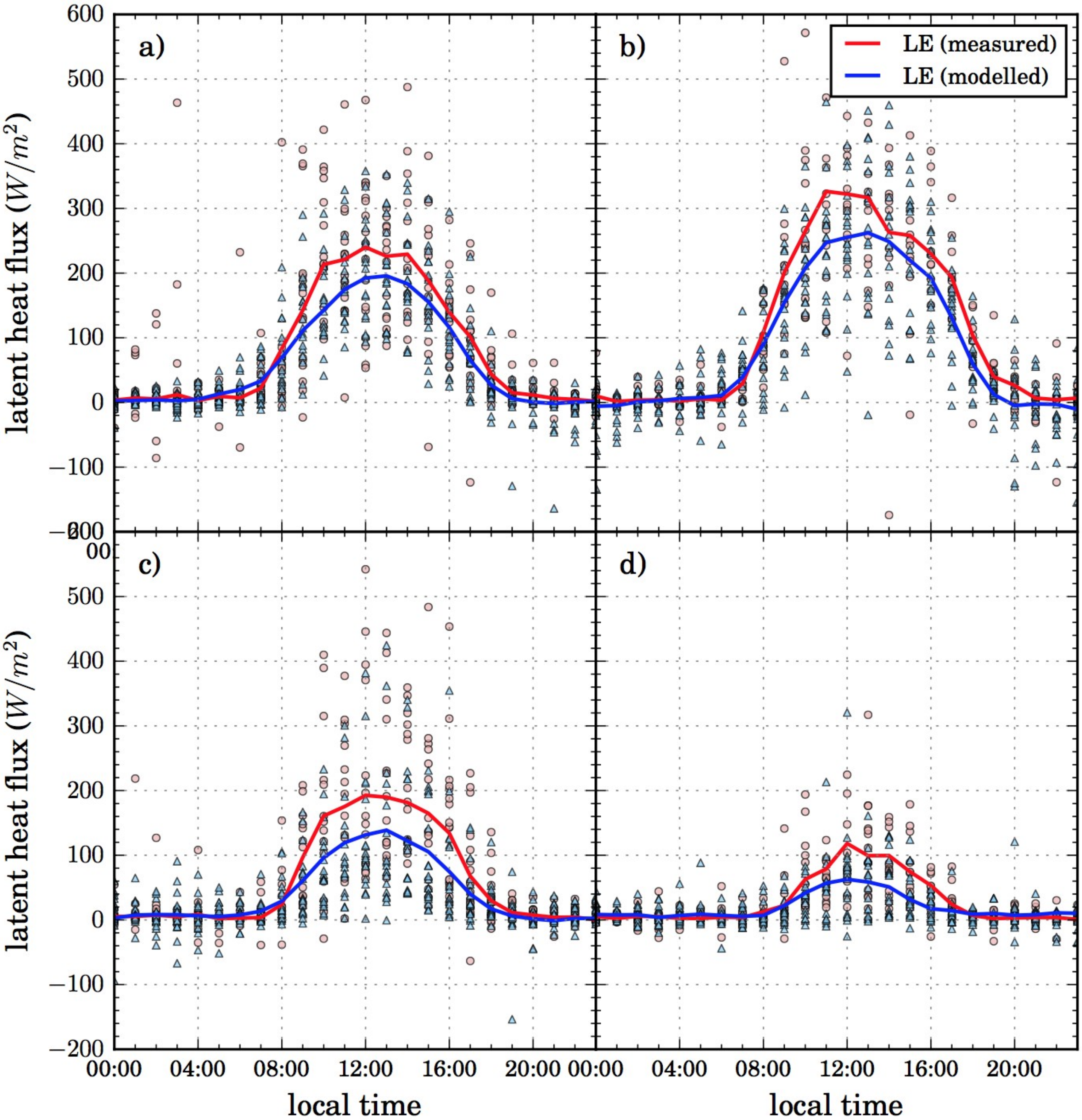

Figure S2: The analysis of Figure 3 repeated for the four seasons of the year (every 5th measurement is shown in the background in a lighter colour). hourly latent sensible heat flux (annual average) : a) spring ; b) summer ; c) autumn ; d) winter

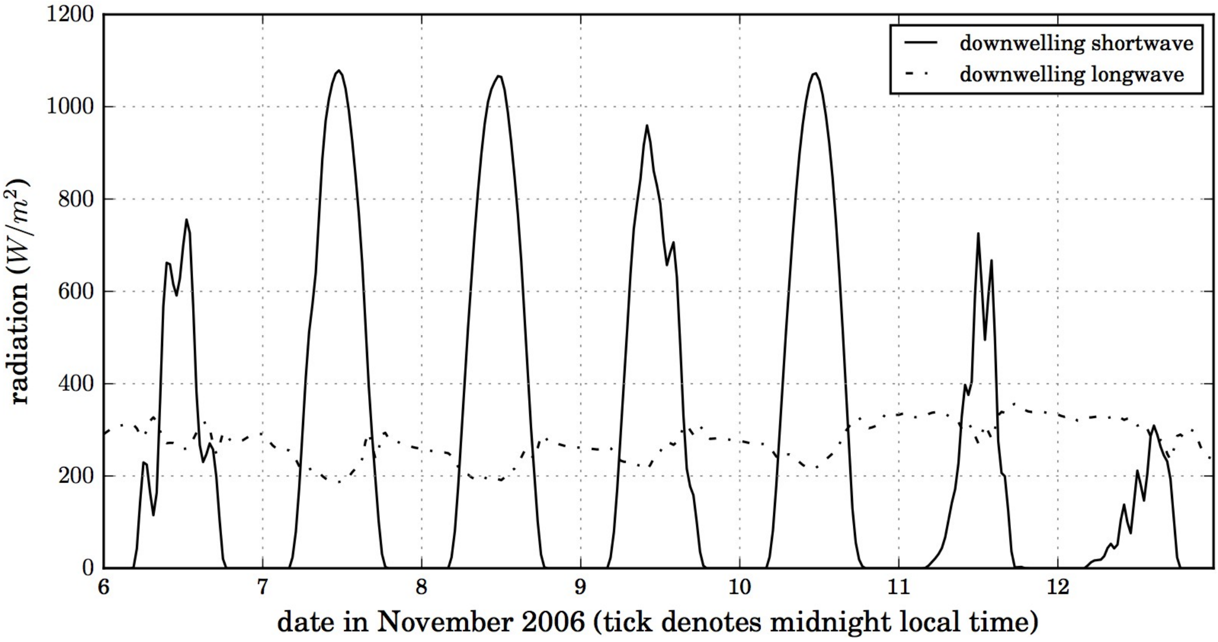

Figure S3 – Fluxnet derived values of downwelling shortwave and longwave radiation that was used to force the model (shown for the intensive period from 6th to 12th of November 2006.