Dear editor Lutz Gross,

Thank you for processing our manuscript. We have revised the manuscript according to the comments by two reviewers and here replied each comment bellow. The original comments are in plain text and the replies in italics. The main modifications are stated at last.

Information of our manuscript is as following:

Title: Non-singular spherical harmonic expressions of geomagnetic vector and gradient tensor fields in the local north-oriented reference frame

Author(s): J. Du et al.

MS No.: gmd-2014-215

MS Type: Technical/Development/Evaluation Paper

Referee #1 by Prof. Mehdi Eshagh (Received and published: 10 December 2014)

A. General comments

The paper deals with non-singular formulation of the elements of the vector and tensor of the Earth's magnetic field similar to the works done by Petrovskaya and Vershkov (2006) and Eshagh (2008, 2009). The main difference is related to the normalization factor as in the geomagnetism the semi-normalised associated Legendre functions (ALFs) are used, but in the gravity field studies the fully-normalised ones. The developments are very trivial, but can be useful. In addition, the authors provide the non-singular formulae for the third-order derivatives of the geomagnetic field. The paper is recommended for publication in Geosciences Model Development after a major revision. The following general and specific comments are provided for improving the paper.

B. Specific comments

1. The authors are asked to write some words about the differences between the works done by Petrovskaya and Vershkov (2006) and Eshagh (2008, 2009) and to explain why semi-normalised ALFs are used for the geomagnetic field.

>Jinsong Du et al.: Thank you. In geomagnetic field studies, the Schmidt semi-normalized associated Legendre functions (SSALFs) is usually used (e.g. Blakely, 1995; Langel and Hinze, 1998). As for the differences between the works done in gravity field studies by Petrovskaya and

Vershkov (2006) and Eshagh (2008, 2009), we have added the corresponding content in the end of section 2.1 in the revised manuscript, which are as following: It should be stated that our work differs from those presented by Petrovskaya and Vershkov (2006) and Eshagh (2009) in the LNORF and also the associated Legendre functions (ALFs). Nonetheless, the following mathematical derivations are carried out based on their studies in gravity field.

2. In the abstract, it is written higher-order derivatives, whilst the paper considers the third-order ones. It should be revised.

>Jinsong Du et al.: Thank you for pointing this out. We have changed the 'higher-order derivatives' to 'third-order derivatives'.

3. According to the reference system theory, the local north-oriented frame is defined as a frame whose z-axis is radially upward and the system is left handed. The equations that e.g. Eshagh (2009) has used are based on such a frame. Please explain why this frame is defined differently in the paper.

>Jinsong Du et al.: Thank you. For the geomagnetic fields modeling and their applications, it is usual to utilize a local topocentric coordinate system (please see the page 113 in the chapter '5 Sources of the Geomagnetic Field and the Modern Data That Enable Their Investigation' by Nils Olsen et al. (2010) in 'Handbook of Geomathematics' edited by W. Freeden et al.). In the local reference frame, the X axis points toward geographic North and the Y axis geographic East and the Z axis vertically down. This reference frame is an orthogonal right-handed coordinate system. We have added the corresponding reference to the revised manuscript in section 2.1.

4. The paper presents the mathematical derivations in 7 subsections, but the problem is that the reader cannot find the connection with these mathematical proofs and the traditional expressions. It is recommended that the authors start with the traditional expressions of the vector and tensor of the geomagnetic field as well as the third-order derivatives, and discuss about their importance and roles in geomagnetic studies, and in the mathematical derivations they refer to the traditional formulae so that the reader can see the connections between the new and old formulae. For example, see the Eshagh (2009) that you have referred to.

>Jinsong Du et al.: Thank you very much. According to your suggestion, we have adjusted this part and stated the connection with the studies by Petrovskaya and Vershkov (2006) and Eshagh (2009) in the revised manuscript. Based on these connections, our mathematical derivations are

clearer than those in the discussion paper.

5. The appendix repeats the things that have been already presented in the paper by Eshagh (2009). Please remove it! Those coefficients related to the third-order derivatives can simply be moved into the text.

>Jinsong Du et al.: Thank you. In fact, because of the differences in the local-north-oriented reference frame and also the normalized associated Legendre functions, some coefficients in the Appendix are different with those presented in the paper by Eshagh (2008, 2009). Therefore, we have added the coefficients into the text in the revised manuscript.

6. The purpose of the numerical investigation is not clear. If the goal is just to present the maps of the vector and tensor quantities based on the new formulae, then what will be the role of considering two geomagnetic models? One of them should be enough, otherwise the author should discuss about the discrepancies between the models. In addition, the maps of the third-order derivatives are missing, and this could be a good contribution, which the paper deals with improperly.

>Jinsong Du et al.: Thank you for your suggestion. The two models are different. The one is the core field, which is dominated by the spherical harmonic degrees/orders from 1 to 12~20. Another one is the lithospheric field, which is dominated by the spherical harmonic degrees/orders higher than ~16. Originally, we want to use these two models to test the correctness of the formulae in the full range of the spherical harmonic degrees/orders. In the revised manuscript, we have used only the GRIMM_L120 v0.0 (Lesur et al., 2013) with degrees and orders of 16~90 to illustrate the purpose. At the same time, a core field model with spherical harmonic degrees/orders 1~15 is also used to test and the results not shown here indicate the correctness of the formulae in the full range of the spherical harmonic degrees/orders, where the computational stability of the Legendre function with ultrahigh-order is not considered. Meanwhile, in the revised manuscript, we only show the results near the two poles. The third-order derivatives are also presented aiming to further interpretations of the lithospheric magnetic field models in the future.

C. Technical comments

1. All abbreviations should be defined properly in the introduction even if they are well known and they should be given some reference, e.g. ESA, GOCE, CHAMP, SAC-C, ST-5, Ørsted…

>Jinsong Du et al.: We have defined all abbreviations in the revised manuscript or added the

corresponding references.

2. The abbreviation 'SHA' has been defined but never used. Please remove it!

>*Jinsong Du et al.: Thank you for pointing out this abbreviation and we have removed it.*

3. In Section 2, above Eq. (1), it is written that '… at point P' whilst P will be introduced later as the ALF. Simply write any point with the geocentric distance r, co-latitude θ and longitude φ. The same holds for the text above Eq. (2a).

>*Jinsong Du et al.: We have added some corresponding descriptions about the* $P(r,\theta,\varphi)$ *when appearing first time in the text.*

4. Below Eq. (44), the abbreviation SH has not been defined already. Please write the full name!

>*Jinsong Du et al.: We have changed this abbreviation and used its full name.*

5. The sentence above '2-derivation of …' write: 'the Kronecker delta'.

>*Jinsong Du et al.: Thank you for pointing this out and we have corrected it.*

6. The article 'the' should not be used when an equation is referred by its number. For example, write: Eq. (1) and NOT 'the Eq. (1)'. The same holds for 'Lemma 3'.

>*Jinsong Du et al.: We have removed the corresponding expression 'the' in the revised manuscript and thank you.*

Referee #2 by Anonymous Referee #2 (Received and published: 8 April 2015)

This paper provides new expressions for the gradient, the double-gradient, and some elements of the triple-gradient tensors that are stable at the poles in the local-north frame. Calculations of the gradient and double-gradient are provided for two field models. Unless one is performing a global analysis that includes data at or very near the poles, then I see the impact of this paper as limited. However, the paper still provides a useful alternative to the standard gradient and double-gradient formulae and should be published, but with more emphasis on comparison with the standard formulae. Too much effort is spent talking about the usefulness of gradients. This is not a paper about convincing people to use gradients, and it is a paper about using new, better formulae than the standard ones.

General comments

1. Given that the expressions are stable at the poles, are there any other advantages in using them? I ask this because, as stated earlier, unless one is doing a global analysis that includes data at the

poles, can't you just rotate the underlying spherical coordinate system such that the pole is no longer in the area of interest, which means that you can use the standard expressions? Are the new expressions less computationally intensive? Do they require less storage?

>Jinsong Du et al.: Thank you very much. Our method has two main features. The one is the non-singularity at the poles. Another one is that there is no derivative of the Legendre function. Therefore, recursive calculation by the Clenshaw or Horner algorithms can be avoided. The computational efficiency can be improved and the storage is less required. Please note that we don't discuss the calculation of the Legendre function. Your suggested rotation is indeed correct and can be performed. However, compared with the rotation approach, our method doesn't need additional computation and thus reduce the complexity and also the computing time. According to this comment, we have added a sentence in the revised manuscript as following: A rotation of the coordinate system is always possible to avoid the polar singularity, but this solution is very ineffective for large data sets.

2. Even in the case where I want to compute the gradient and double-gradient at the poles, can't I rotate the coordinate system around the polar axis to eliminate the problems with 1/sin (theta)? If so, why use your new expressions?

>Jinsong Du et al.: Thank you. These questions are very similar with those in (1) above. We have emphasized the advantages of our method compared with the standard ones in the last paragraph of section 3 in the revised manuscript, which are as following: Compared with the traditional formulae in section 2.1, there are two advantages of our derived formulae in section 2.3. On the one hand, the traditional derivatives up to second-order are removed in the new formulae; therefore, the relatively complicated method by the Horner's recursive algorithm (Holmes and Featherstone, 2002b) can be avoided. On the other hand, the singular terms of $1/\sin\theta$ *and* $1/\sin^2\theta$ *are removed in the new formulae; consequently, the scale factor of e.g.* 10^{-280} *(Holmes and Featherstone, 2002a,b) is not required when the computing point approaches to the poles and the magnetic fields at the poles can also be calculated in the defined reference frame. In fact, there are differences between the results by our expressions and those by the Horner's recursive algorithm, for instance, if using the same model and the parameters as those in Fig. 1 and Fig. 2, the differences of the three components* B_x, B_y *and* B_z *are at a level of* $[-3\times10^{-11}\ nT : +3\times10^{-11}\ nT]$.

3. Tables 1 and 2 and Figures 1 and 2 are fairly useless given that you should be showing the

superiority of your new expressions over the standards. Therefore, you should have similar tables and figures for the standard expressions, being sure to show the polar neighborhoods in which the standard expressions begin to degrade. Furthermore, why have you not included polar projections in Figures 1 and 2 since this is the most important area for comparison? Also, you do not need to show two field models, just show either Figure 1 or 2.

>Jinsong Du et al.: Thank you for your valuable suggestion. Our original purpose of using two models is to test the validity for the full range of the degrees and orders. In the revised manuscript, we have used only the GRIMM_L120 v0.0 (Lesur et al., 2013) with degrees and orders of 16~90 to illustrate the purpose. At the same time, a core field model with spherical harmonic degrees/orders 1~15 is also used to test and the results not shown here indicate the correctness of the formulae in the full range of the spherical harmonic degrees/orders, where the computational stability of the Legendre function with ultrahigh-order is not considered. Meanwhile, in the revised manuscript, we only show the results near the two poles. The third-order derivatives are also presented aiming to further interpretations of the lithospheric magnetic field models in the future.

4. At the poles you (arbitrarily) define x_p and y_p to be aligned along some meridians and you show the smoothness of the functions across the poles when approached along these meridians in Figure 3. However, what happens if you approach the poles from an arbitrary meridian? Are the functions still smooth?

>Jinsong Du et al.: Thank you. As shown in Figure 3 in the revised manuscript, the magnetic V, B_z and B_{zz} components at the poles are independent of the direction of the $\mathbf{x}_P$ and $\mathbf{y}_P$ axes and thus smooth cross the poles. However, while changing with the direction of the $\mathbf{x}_P$ and $\mathbf{y}_P$ axes at the poles, the B_x, B_y, B_{xz}, B_{yz}, B_{xzz} and B_{yzz} components have the periods of 360° and the B_{xx}, B_{xy}, B_{yy}, B_{xxz}, B_{xyz} and *B_{yyz} components have the periods of 180°. These variations can be accurately described by sine or cosine function and the differences among these magnetic effects are magnitude, period and initial phase. Therefore, B_x, B_y, B_{xz}, B_{yz}, B_{xx}, B_{xy}, B_{yy}, B_{xzz}, B_{yzz} B_{xxz}, B_{xyz}* and *B_{yyz} components are not smooth cross the poles.*

Main modifications

We have revised the manuscript according to the comments by two reviewers. The small modifications on the grammars and English expressions are shown in the revised manuscript. The main modifications are as following:

1. We have defined all abbreviations in the revised manuscript or added the corresponding references, such as ESA, GOCE, CHAMP, SAC-C, ST-5, Ørsted.

2. We have added four references, which are Sabaka et al. (2015), Kotsiaros et al. (2015), Olsen et al. (2015) and Olsen et al. (2010).

3. According to the Reviewer 2, we have stated why we don't use the rotation approach in the lines 61~63 in the revised manuscript.

4. According to the Reviewer 1, we have stated the connection with the studies by Petrovskaya and Vershkov (2006) and Eshagh (2009) in the end of the section 2.1 in the revised manuscript.

5. According to the Reviewer 1, we have removed the Appendix A. The Appendix B has been removed in to the last paragraph of section 4.

6. According to the two reviewers, we have used only the GRIMM_L120 v0.0 (Lesur et al., 2013) with degrees and orders of 16~90 to illustrate the purpose. At the same time, a core field model with spherical harmonic degrees/orders 1~15 is also used to test and the results not shown here indicate the correctness of the formulae in the full range of the spherical harmonic degrees/orders, where the computational stability of the Legendre function with ultrahigh-order is not considered. Meanwhile, in the revised manuscript, we only show the results near the two poles. The third-order derivatives are also presented aiming to further interpretations of the lithospheric magnetic field models in the future.

7. According to the Reviewer 2, we have emphasized the advantages of our method compared with the standard ones in the last paragraph of section 3 in the revised manuscript.

8. Considering the contents, the discussion part has been removed in to section 3 in the revised manuscript.

Best regards,

Jinsong Du et al.

5 May 2015

Non-singular spherical harmonic expressions of geomagnetic vector and gradient tensor fields in the local north-oriented reference frame

By

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Non-singular spherical harmonic expressions of geomagnetic vector and gradient tensor fields in the local north-oriented reference frame

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Abstract

General expressions of magnetic vector (MV) and magnetic gradient tensor (MGT) in terms of the first- and second-order derivatives of spherical harmonics at different degrees/orders, are relatively complicated and singular at the poles. In this paper, we derived alternative non-singular expressions for the MV, the MGT and also the third-order partial derivatives of the magnetic potential field in local north-oriented reference frame. Using our newly derived formulae, the magnetic potential, vector and gradient tensor fields and also the third-order partial derivatives of the magnetic potential field at an altitude of 300 km are calculated based on a global lithospheric magnetic field model GRIMM_L120 (version 0.0) with spherical harmonic degrees 16~90. The corresponding results at the poles are discussed and the validity of the derived formulas is verified using the Laplace equation of the magnetic potential field.

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1 Introduction

Compared to the magnetic vector and scalar measurements, magnetic gradients lead to more robust models of the lithospheric magnetic field. The ongoing *Swarm* mission of the European

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Space Agency (ESA) provides measurements not only of the vector and scalar data but also an estimate of their east-west gradients (e.g. Olsen et al., 2004, 2015; Friis-Christensen et al., 2006). Kotsiaros and Olsen (2012, 2014) proposed to recover the lithospheric magnetic field through Magnetic Space Gradiometry in the same way that has been done for modeling the gravitational potential field from the satellite gravity gradient tensor measurements by the Gravity field and steady-state Ocean Circulation Explorer (GOCE). Purucker et al. (2005, 2007), Sabaka et al. (2015) and Kotsiaros et al. (2015) also reported efforts to model the lithospheric magnetic field using magnetic gradient information from the satellite constellation. Their results showed that by using gradients data, the modelled lithospheric magnetic anomaly field has enhanced shorter wavelength content and has a much higher quality compared to models built from vector field data. This is because the gradients data can remove the highly time-dependant contributions of the magnetosphere and ionosphere that are correlated between two side-by-side satellites.

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The order-2 magnetic gradient tensor consists of spatial derivatives highlighting certain structures of the magnetic field (e.g. Schmidt and Clark, 2000, 2006). It can be used to detect the hidden and small-scale magnetized sources (e.g. Pedersen and Rasmussen, 1990; Harrison and Southam, 1991) and to investigate the orientation of the lineated magnetic anomalies (e.g. Blakely and Simpson, 1986). Quantitative magnetic interpretation methods such as the analytic signal, edge detection, spatial derivatives, Euler deconvolution, and transforms, all set in Cartesian coordinate system (e.g. Blakely, 1995; Purucker and Whaler, 2007; Taylor et al., 2014) also require calculating the higher-order derivatives of the magnetic anomaly field and need to be extended to regional and global scales to handle the curvature of the Earth and other planets. Ravat et al. (2002) and Ravat (2011) utilized the analytic signal method and the total gradient to interpret the satellite-altitude magnetic anomaly data. Therefore, both the magnetic field modelling and also the geological interpretations require the calculation for the partial derivatives of the magnetic field, possibly at the poles for specific systems of coordinates. Spherical harmonic analysis, established originally by Gauss (1839), is generally used to model the global magnetic internal fields of the Earth and other terrestrial planets (e.g. Maus et al., 2008; Langlais et al., 2009; Thébault et al., 2010, Finlay et al., 2010; Lesur et al., 2013, Sabaka et al., 2013; Olsen et al., 2014). Series of spherical harmonic functions themselves made of Schmidt semi-normalized associated Legendre functions (SSALFs) (e.g. Blakely, 1995; Langel and Hinze, 1998), are fitted by least-squares to magnetic measurements, giving the spherical harmonic coefficients (i.e. the Gaussian coefficients) defining the model. Kotsiaros and Olsen (2012, 2014) presented the MV and the MGT using a spherical harmonic representation and, of course, their expressions are singular as they approach the poles. Even if there are satellite data gaps around the poles, it is advisable to use non-singular spherical harmonic expressions for the MV and the MGT in case airborne or shipborne magnetic data are utilized (e.g. Golynsky et al., 2013; Maus, 2010). A rotation of the coordinate system is always possible to avoid the polar singularity, but this solution is very ineffective for large data sets.

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In this paper, following Petrovskaya and Vershkov (2006) and Eshagh (2008, 2009) for the gravitational gradient tensor in the local north oriented, orbital reference and geocentric spherical frames, the non-singular expressions in terms of spherical harmonics for the MV, the MGT and the third-order derivatives of the magnetic potential field in the specially defined local-north-oriented reference frame (LNORF) are presented. In the next section, the traditional expressions of the MV and the MGT are first stated, then some necessary propositions are proved and at last new

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non-singular expressions are derived. In section 3, the new formulae are tested using the global lithospheric magnetic field model GRIMM_L120 (version 0.0) (Lesur et al., 2013) and compared with the results by traditional formulae. Finally, some conclusions are drawn and further applications are also discussed.

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2 Methodology

In this section, the traditional expressions of MV and MGT are presented, and their numerical problems are stated. Then based on some necessary mathematical derivations, new expressions are given.

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2.1 Traditional expressions

The scalar potential V of the Earth's magnetic field in a source-free region can be expanded in the truncated series of spherical harmonics at the point $P(r,\ \theta,\ \varphi)$ with the geocentric distance r, co-latitude θ and longitude φ (e.g. Backus et al., 1996):

$$V(r,\theta,\varphi) = a\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\tilde{P}_l^m(\cos\theta) \quad , \qquad (1)$$

where a=6371.2 km is the radius of the Earth's magnetic reference sphere; $\tilde{P}_l^m(\cos\theta)$ (or $\tilde{P}_l^m$ for simplification) is the SSALF of degree l and order m; L is the maximum spherical harmonic degree; g_l^m and h_l^m are the geomagnetic harmonic coefficients describing internal sources of the Earth.

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If considered in the LNORF {**x**,**y**,**z**} (e.g. Olsen et al., 2010), where **z**-axis points downward in geocentric radial direction, **x**-axis points to the north, and **y**-axis towards the east (that is, a right-handed system). At the poles, we define that the **x**-axis points to the meridian of 180° E (or

180° W) at north pole and of 0° at south pole, which will be discussed in section 3. Therefore, the three components of the MV can be expressed as:

$$B_x(r,\theta,\varphi) = -\frac{1}{r}\frac{\partial}{\partial(-\theta)}V(r,\theta,\varphi) = \sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+2}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\left[\frac{\partial}{\partial\theta}\tilde{P}_l^m(\cos\theta)\right], \tag{2a}$$

$$B_y(r,\theta,\varphi) = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}V(r,\theta,\varphi) = \sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+2} m\left(g_l^m \sin m\varphi - h_l^m \cos m\varphi\right)\left[\frac{1}{\sin\theta}\tilde{P}_l^m(\cos\theta)\right], \tag{2b}$$

$$B_z(r,\theta,\varphi) = -\frac{\partial}{\partial(-r)}V(r,\theta,\varphi) = -\sum_{l=1}^{L}\sum_{m=0}^{l}(l+1)\left(\frac{a}{r}\right)^{l+2}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\tilde{P}_l^m(\cos\theta). \tag{2c}$$

The MGT can be written as (e.g. Kotsiaros and Olsen, 2012)

$$\nabla\mathbf{B} = \begin{pmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{pmatrix} = \begin{pmatrix} \partial B_x/\partial x & \partial B_x/\partial y & \partial B_x/\partial z \\ \partial B_y/\partial x & \partial B_y/\partial y & \partial B_y/\partial z \\ \partial B_z/\partial x & \partial B_z/\partial y & \partial B_z/\partial z \end{pmatrix}, \tag{3}$$

where nine elements are expressed respectively as:

$$B_{xx} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right) \times\left[-\frac{\partial^2}{\partial\theta^2}\tilde{P}_l^m(\cos\theta) + (l+1)\tilde{P}_l^m(\cos\theta)\right], \tag{4a}$$

$$B_{xy} = B_{yx} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3} m\left(g_l^m \sin m\varphi - h_l^m \cos m\varphi\right) \times\left[-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\tilde{P}_l^m(\cos\theta) + \frac{\cos\theta}{\sin^2\theta}\tilde{P}_l^m(\cos\theta)\right], \tag{4b}$$

$$B_{xz} = B_{zx} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}(l+2)\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\left[\frac{\partial}{\partial\theta}\tilde{P}_l^m(\cos\theta)\right], \tag{4c}$$

$$B_{yy}=\frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m\cos m\varphi+h_l^m\sin m\varphi\right)$$
$$\times\left[(l+1)\widetilde{P}_l^m(\cos\theta)+\frac{m^2}{\sin^2\theta}\widetilde{P}_l^m(\cos\theta)-\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\widetilde{P}_l^m(\cos\theta)\right], \tag{4d}$$

$$B_{yz}=B_{zy}=\frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}(l+2)m\left(g_l^m\sin m\varphi-h_l^m\cos m\varphi\right)\left[\frac{1}{\sin\theta}\widetilde{P}_l^m(\cos\theta)\right], \tag{4e}$$

$$B_{zz}=-\frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}(l+1)(l+2)\left(g_l^m\cos m\varphi+h_l^m\sin m\varphi\right)\widetilde{P}_l^m(\cos\theta). \tag{4f}$$

The expressions for V, B_z and B_{zz} can be calculated stably even for very high *spherical harmonic* degrees and orders by using the Holmes and Featherstone (2002a) scheme. However, there exist the singular terms of 1/sinθ and 1/ sin$^2\theta$ in Eq. (2b), Eq. (4b), Eq. (4d) and Eq. (4e) when the computing point approaches to the poles. Besides, some expressions contain the terms of first- and second-order derivatives of SSALFs, such as Eq. (2a) and Eq. (4a) ~ (4d). Nevertheless, the derivatives up to second-order for very high degree and orders of SSALFs can be recursively calculated by the Horner algorithm (Holmes and Featherstone, 2002b). These algorithms are relatively complicated and thus we want to use alternative expressions to avoid the singular terms and also the partial derivatives of SSALFs. It should be stated that our work differs from those presented by Petrovskaya and Vershkov (2006) and Eshagh (2009) in the LNORF and also the associated Legendre functions (ALFs). Nonetheless, the following mathematical derivations are carried out based on their studies in gravity field.

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2.2 Mathematical derivations

To deal with the singular terms and first- and second-order derivatives of the SSALFs, some useful mathematical derivations are introduced and proved in the following.

1 - Derivation of $\partial\widetilde{P}_l^m/\partial\theta$:

Based on Eq. (Z.1.44) in Ilk (1983)

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$$\partial P_l^m / \partial\theta = 0.5\left[(l+m)(l-m+1)P_l^{m-1} - P_l^{m+1}\right], \qquad (5)$$

and the relation between the ALFs and the SSALFs as

$$\widetilde{P}_l^m = \sqrt{C_m (l-m)!/(l+m)!}\,P_l^m, \qquad (6)$$

thus the first-order derivative of the SSALFs can be deduced as:

$$\partial \widetilde{P}_l^m / \partial\theta = a_{l,m}\widetilde{P}_l^{m-1} + b_{l,m}\widetilde{P}_l^{m+1}, \qquad (7a)$$

$$a_{l,m} = 0.5\sqrt{l+m}\sqrt{l-m+1}\sqrt{C_m / C_{m-1}}, \qquad (7b)$$

$$b_{l,m} = -0.5\sqrt{l+m+1}\sqrt{l-m}\sqrt{C_m / C_{m+1}}, \qquad (7c)$$

where $C_m = 2 - \delta_{m,0} = \begin{cases} 1, m = 0 \\ 2, m \neq 0 \end{cases}$ and δ is the Kronecker delta.

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2 - Derivation of $\partial^2 \widetilde{P}_l^m / \partial\theta^2$:

According to Eq. (23) in Eshagh (2008) as

删除的内容：the

$$\begin{aligned}\partial^2 P_l^m / \partial\theta^2 = {} & 0.25(l+m)(l-m+1)(l+m-1)(l-m+2)P_l^{m-2} \\ & -0.25\left[(l+m)(l-m+1)+(l-m)(l+m+1)\right]P_l^m \\ & +0.25P_l^{m+2}\end{aligned}, \qquad (8)$$

the second-order derivative of the SSALFs can be written as:

$$\partial^2 \widetilde{P}_l^m / \partial\theta^2 = c_{l,m}\widetilde{P}_l^{m-2} + d_{l,m}\widetilde{P}_l^m + e_{l,m}\widetilde{P}_l^{m+2}, \qquad (9a)$$

$$c_{l,m} = 0.25\sqrt{l+m}\sqrt{l+m-1}\sqrt{l-m+2}\sqrt{l-m+1}\sqrt{C_m / C_{m-2}}, \qquad (9b)$$

$$d_{l,m} = -0.25\left[(l+m)(l-m+1)+(l-m)(l+m+1)\right], \qquad (9c)$$

$$e_{l,m} = 0.25\sqrt{l+m+2}\sqrt{l+m+1}\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+2}}. \qquad (9d)$$

3 - Derivation of $\widetilde{P}_l^m / \sin\theta$:

Using Eq. (Z.1.42) in Ilk (1983)

删除的内容：the

$$P_l^m / \sin\theta = 0.5\left[(l+m)(l+m-1)P_{l-1}^{m-1} + P_{l-1}^{m+1}\right]/m, \quad m \geq 1, \qquad (10)$$

and Eq. (6), we can obtain that

删除的内容：the

$$\widetilde{P}_l^m / \sin\theta = f_{l,m}\widetilde{P}_{l-1}^{m-1} + g_{l,m}\widetilde{P}_{l-1}^{m+1}, \quad m \geq 1, \tag{11a}$$

$$f_{l,m} = 0.5\sqrt{l+m}\sqrt{l+m-1}\sqrt{C_m / C_{m-1}} / m, \quad m \geq 1, \tag{11b}$$

$$g_{l,m} = 0.5\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+1}} / m, \quad m \geq 1. \tag{11c}$$

4 - Derivation of $\widetilde{P}_l^m / \sin^2\theta$:

Employing Eq. (31) in Eshagh (2008) as

删除的内容：the

$$\begin{aligned} P_l^m / \sin^2\theta = \{&(l+m)(l+m-1)(l-m+1)(l-m+2)/(m-1)P_l^{m-2} \\ &+ [(l+m)(l+m-1)/(m-1) + (l-m)(l-m-1)/(m+1)]P_l^m \\ &+ 1/(m+1)P_l^{m+2}\}/(4m) \end{aligned}, \quad m \geq 2, \tag{12}$$

and Eq. (6), we have

删除的内容：the

$$\widetilde{P}_l^m / \sin^2\theta = h_{l,m}\widetilde{P}_l^{m-2} + k_{l,m}\widetilde{P}_l^m + n_{l,m}\widetilde{P}_l^{m+2}, \quad m \geq 2, \tag{13a}$$

$$h_{l,m} = 0.25\sqrt{l+m}\sqrt{l+m-1}\sqrt{l-m+1}\sqrt{l-m+2}\sqrt{C_m / C_{m-2}} / [m(m-1)], \quad m \geq 2, \tag{13b}$$

$$k_{l,m} = 0.25[(l+m)(l+m-1)/(m-1) + (l-m)(l-m-1)/(m+1)] / m, \quad m \geq 2, \tag{13c}$$

$$n_{l,m} = 0.25\sqrt{l-m}\sqrt{l-m-1}\sqrt{l+m+2}\sqrt{l+m+1}\sqrt{C_m / C_{m+2}} / [m(m+1)], \quad m \geq 1. \tag{13d}$$

5 - Derivation of $\partial\widetilde{P}_l^m / (\sin\theta\partial\theta)$:

Using Eq. (36) in Eshagh (2008) as

删除的内容：the

$$\begin{aligned} \partial P_l^m / (\sin\theta\partial\theta) = 0.25\{&(l+m)(l+m-1)(l+m-2)(l-m+1)/(m-1)P_{l-1}^{m-2} \\ &+ [(l+m)(l-m+1)/(m-1) - (l+m+1)(l+m)/(m+1)]P_{l-1}^m \\ &- 1/(m+1)P_{l-1}^{m+2}\} \end{aligned}, \quad m \geq 2, \tag{14}$$

and Eq. (6), we can derive

删除的内容：the

$$\partial\widetilde{P}_l^m / (\sin\theta\partial\theta) = o_{l,m}\widetilde{P}_{l-1}^{m-2} + q_{l,m}\widetilde{P}_{l-1}^m + x_{l,m}\widetilde{P}_{l-1}^{m+2}, \quad m \geq 2, \tag{15a}$$

$$o_{l,m} = 0.25\sqrt{l+m}\sqrt{l+m-1}\sqrt{l+m-2}\sqrt{l-m+1}\sqrt{C_m / C_{m-2}} / (m-1), \quad m \geq 2, \tag{15b}$$

$$q_{l,m} = 0.25\sqrt{l-m}\sqrt{l+m}[(l-m+1)/(m-1) - (l+m+1)/(m+1)], \quad m \geq 2, \tag{15c}$$

$$x_{l,m} = -0.25\sqrt{(l+m+1)}\sqrt{l-m}\sqrt{l-m-1}\sqrt{l-m-2}\sqrt{C_m / C_{m+2}} / (m+1). \tag{15d}$$

6 - Derivation of $\partial\widetilde{P}_l^m / (\sin\theta\partial\theta) - \widetilde{P}_l^m \cos\theta / \sin^2\theta$:

According to Petrovskaya and Vershkov (2006) and Eshagh (2009), we can write

$$\begin{aligned}&\partial P_l^m/(\sin\theta\partial\theta)-P_l^m\cos\theta/\sin^2\theta\\&=0.5\left[(m-1)(l+m)(l-m+1)P_l^{m-1}/\sin\theta-(m+1)P_l^{m+1}/\sin\theta\right]/m,\quad m\ge 1,\end{aligned}\tag{16}$$

and using Eq. (36) in Eshagh (2008), we can obtain

删除的内容： the

$$P_l^{m-1}/\sin\theta=0.5\left[(l-m+2)(l-m+3)P_{l+1}^{m-2}+P_{l+1}^{m}\right]/(m-1),\quad m\ge 2,\tag{17a}$$

$$P_l^{m+1}/\sin\theta=0.5\left[(l-m)(l-m+1)P_{l+1}^{m}+P_{l+1}^{m+2}\right]/(m+1).\tag{17b}$$

Substituting Eq. (17) into the right hand side of Eq. (16) and after simplification, we can derive

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$$\begin{aligned}&\partial P_l^m/(\sin\theta\partial\theta)-P_l^m\cos\theta/\sin^2\theta\\&=0.25\left[(l+m)(l-m+1)(l-m+2)(l-m+3)P_{l+1}^{m-2}\right.\\&\left.+2m(l-m+1)P_{l+1}^{m}-P_{l+1}^{m+2}\right]/m,\quad m\ge 1.\end{aligned}\tag{18}$$

And combing Eq. (6), we obtain that

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$$\begin{aligned}&\partial \tilde{P}_l^m/(\sin\theta\partial\theta)-\tilde{P}_l^m\cos\theta/\sin^2\theta\\&=0.25\left[\sqrt{l+m}\sqrt{l-m+1}\sqrt{l-m+2}\sqrt{l-m+3}\sqrt{C_m/C_{m-2}}\tilde{P}_{l+1}^{m-2}\right.\\&+2m\sqrt{l-m+1}\sqrt{l+m+1}\tilde{P}_{l+1}^{m}\\&\left.-\sqrt{l+m+1}\sqrt{l+m+2}\sqrt{l+m+3}\sqrt{l-m}\sqrt{C_m/C_{m+2}}\tilde{P}_{l+1}^{m+2}\right]/m,\quad m\ge 1.\end{aligned}\tag{19}$$

7 - Derivation of $\left[(l+1)\sin^2\theta\tilde{P}_l^m+m^2\tilde{P}_l^m-\sin\theta\cos\theta\partial\tilde{P}_l^m/\partial\theta\right]/\sin^2\theta$:

Based on Lemma 3 in Eshagh (2009) as

删除的内容：the

$$\sin\theta\cos\theta\partial P_l^m/\partial\theta=mP_l^m+(l+1)\sin^2\theta P_l^m-\sin\theta P_{l+1}^{m+1},\tag{20}$$

we can derive

$$\begin{aligned}&\left[(l+1)\sin^2\theta P_l^m+m^2P_l^m-\sin\theta\cos\theta\partial P_l^m/\partial\theta\right]/\sin^2\theta\\&=m(m-1)P_l^m/\sin^2\theta+P_{l+1}^{m+1}/\sin\theta.\end{aligned}\tag{21}$$

According to Eq. (10), we can write

删除的内容： the

$$P_{l+1}^{m+1}/\sin\theta=0.5\left[(l+m+2)(l+m+1)P_l^m+P_l^{m+2}\right]/(m+1).\tag{22}$$

Inserting Eq. (12) and Eq. (22) into Eq. (21), and after some simplifications, we obtain that

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$$\left[(l+1)\sin^2\theta P_l^m + m^2 P_l^m - \sin\theta\cos\theta\partial P_l^m/\partial\theta\right]/\sin^2\theta$$
$$= 0.25(l+m)(l+m-1)(l-m+1)(l-m+2)P_l^{m-2}$$
$$+0.25[(l+m)(l+m-1)+(l-m)(l-m-1)(m-1)/(m+1)$$
$$+2(l+m+2)(l+m+1)/(m+1)]P_l^m + 0.25P_l^{m+2} \quad (23)$$

And combing with Eq. (6), we can derive

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$$\left[(l+1)\sin^2\theta \widetilde{P}_l^m + m^2 \widetilde{P}_l^m - \sin\theta\cos\theta\partial \widetilde{P}_l^m/\partial\theta\right]/\sin^2\theta$$
$$= 0.25\sqrt{l+m}\sqrt{l+m-1}\sqrt{l-m+1}\sqrt{l-m+2}\sqrt{C_m/C_{m-2}}\widetilde{P}_l^{m-2}$$
$$+0.25[(l+m)(l+m-1)+(l-m)(l-m-1)(m-1)/(m+1)$$
$$+2(l+m+2)(l+m+1)/(m+1)]\widetilde{P}_l^m$$
$$+0.25\sqrt{l+m+1}\sqrt{l+m+2}\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m/C_{m+2}}\widetilde{P}_l^{m+2} \quad (24)$$

2.3 New expressions

Inserting the corresponding mathematical derivations in the last section into Eq. (2) and Eq. (4) and after some simplifications, the new expressions for MV and MGT can be written as:

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删除的内容：the

$$B_x = \sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+2}\left(g_l^m\cos m\varphi + h_l^m\sin m\varphi\right)\left(a_{l,m}^x\widetilde{P}_l^{m-1} + b_{l,m}^x\widetilde{P}_l^{m+1}\right), \quad (25a)$$

$$B_y = \sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+2}\left(g_l^m\sin m\varphi - h_l^m\cos m\varphi\right)\left(a_{l,m}^y\widetilde{P}_{l-1}^{m-1} + b_{l,m}^y\widetilde{P}_{l-1}^{m+1}\right), \quad (25b)$$

$$B_z = \sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+2}\left(g_l^m\cos m\varphi + h_l^m\sin m\varphi\right)\left(a_{l,m}^z\widetilde{P}_l^{m}\right), \quad (25c)$$

$$B_{xx} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m\cos m\varphi + h_l^m\sin m\varphi\right)\left(a_{l,m}^{xx}\widetilde{P}_l^{m-2} + b_{l,m}^{xx}\widetilde{P}_l^{m} + c_{l,m}^{xx}\widetilde{P}_l^{m+2}\right), \quad (26a)$$

$$B_{xy} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m\sin m\varphi - h_l^m\cos m\varphi\right)\left(a_{l,m}^{xy}\widetilde{P}_{l+1}^{m-2} + b_{l,m}^{xy}\widetilde{P}_{l+1}^{m} + c_{l,m}^{xy}\widetilde{P}_{l+1}^{m+2}\right), \quad (26b)$$

$$B_{xz} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m\cos m\varphi + h_l^m\sin m\varphi\right)\left(a_{l,m}^{xz}\widetilde{P}_l^{m-1} + b_{l,m}^{xz}\widetilde{P}_l^{m+1}\right), \quad (26c)$$

$$B_{yy} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m\cos m\varphi + h_l^m\sin m\varphi\right)\left(a_{l,m}^{yy}\widetilde{P}_l^{m-2} + b_{l,m}^{yy}\widetilde{P}_l^{m} + c_{l,m}^{yy}\widetilde{P}_l^{m+2}\right), \quad (26d)$$

$$B_{yz} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m \sin m\varphi - h_l^m \cos m\varphi\right)\left(a_{l,m}^{yz}\tilde{P}_{l-1}^{m-1} + b_{l,m}^{yz}\tilde{P}_{l-1}^{m+1}\right), \quad (26e)$$

$$B_{zz} = \frac{1}{a}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+3}\left(g_l^m \cos m\lambda + h_l^m \sin m\varphi\right)a_{l,m}^{zz}\tilde{P}_l^m. \quad (26f)$$

where the corresponding coefficients of the SSALFs are given as following:

$$\begin{cases} a_{l,m}^{x} = 0.5\sqrt{l+m}\sqrt{l-m+1}\sqrt{C_m / C_{m-1}} \\ b_{l,m}^{x} = -0.5\sqrt{l+m+1}\sqrt{l-m}\sqrt{C_m / C_{m+1}} \end{cases} \quad (27a)$$

$$\begin{cases} a_{l,m}^{y} = 0.5\sqrt{l+m}\sqrt{l+m-1}\sqrt{C_m / C_{m-1}} \\ b_{l,m}^{y} = 0.5\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+1}} \end{cases}, \quad (27b)$$

$$a_{l,m}^{z} = -(l+1), \quad (27c)$$

$$\begin{cases} a_{l,m}^{xx} = -0.25\sqrt{l+m}\sqrt{l+m-1}\sqrt{l-m+2}\sqrt{l-m+1}\sqrt{C_m / C_{m-2}} \\ b_{l,m}^{xx} = 0.25[(l+m)(l-m+1)+(l-m)(l+m+1)]+(l+1) \\ c_{l,m}^{xx} = -0.25\sqrt{l+m+2}\sqrt{l+m+1}\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+2}} \end{cases} \quad (27d)$$

$$\begin{cases} a_{l,m}^{xy} = -0.25\sqrt{l+m}\sqrt{l-m+1}\sqrt{l-m+2}\sqrt{l-m+3}\sqrt{C_m / C_{m-2}} \\ b_{l,m}^{xy} = -0.5m\sqrt{l-m+1}\sqrt{l+m+1} \\ c_{l,m}^{xy} = 0.25\sqrt{l+m+1}\sqrt{l+m+2}\sqrt{l+m+3}\sqrt{l-m}\sqrt{C_m / C_{m+2}} \end{cases} \quad (27e)$$

$$\begin{cases} a_{l,m}^{xz} = 0.5(l+2)\sqrt{l+m}\sqrt{l-m+1}\sqrt{C_m / C_{m-1}} = (l+2)a_{l,m}^{x} \\ b_{l,m}^{xz} = -0.5(l+2)\sqrt{l+m+1}\sqrt{l-m}\sqrt{C_m / C_{m+1}} = (l+2)b_{l,m}^{x} \end{cases} \quad (27f)$$

$$\begin{cases} a_{l,m}^{yy} = 0.25\sqrt{l+m}\sqrt{l+m-1}\sqrt{l-m+1}\sqrt{l-m+2}\sqrt{C_m / C_{m-2}} \\ b_{l,m}^{yy} = 0.25[(l+m)(l+m-1)+(l-m)(l-m-1)(m-1)/(m+1) \\ \quad +2(l+m+2)(l+m+1)/(m+1)] \\ c_{l,m}^{yy} = 0.25\sqrt{l+m+1}\sqrt{l+m+2}\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+2}} \end{cases} \quad (27g)$$

$$\begin{cases} a_{l,m}^{yz} = 0.5(l+2)\sqrt{l+m}\sqrt{l+m-1}\sqrt{C_m / C_{m-1}} = (l+2)a_{l,m}^{y} \\ b_{l,m}^{yz} = 0.5(l+2)\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+1}} = (l+2)b_{l,m}^{y} \end{cases} \quad (27h)$$

$$a_{l,m}^{zz} = -(l+1)(l+2) = (l+2)a_{l,m}^{z}. \quad (27i)$$

Furthermore, some other higher-order partial derivatives and their transforms are usually used to image geologic boundaries in magnetic prospecting, such as the higher-order enhanced analytic signal (e.g. Hsu et al., 1996). Therefore, we also give the third-order partial derivatives of the magnetic potential field as:

$$
\begin{aligned}
B_{xxz} &= \frac{\partial B_{xx}}{\partial z} = \frac{\partial^2 B_x}{\partial x \partial z} = \frac{\partial^2 B_x}{\partial z \partial x} \\
&= \frac{1}{a^2}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+4}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\left(a_{l,m}^{xxz}\tilde{P}_l^{m-2} + b_{l,m}^{xxz}\tilde{P}_l^{m} + c_{l,m}^{xxz}\tilde{P}_l^{m+2}\right),
\end{aligned}
\tag{28a}
$$

$$
\begin{aligned}
B_{xyz} &= \frac{\partial B_{xy}}{\partial z} = \frac{\partial B_{yx}}{\partial z} = \frac{\partial^2 B_x}{\partial y \partial z} = \frac{\partial^2 B_x}{\partial z \partial y} = \frac{\partial^2 B_y}{\partial x \partial z} = \frac{\partial^2 B_y}{\partial z \partial x} \\
&= \frac{1}{a^2}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+4}\left(g_l^m \sin m\varphi - h_l^m \cos m\varphi\right)\left(a_{l,m}^{xyz}\tilde{P}_{l+1}^{m-2} + b_{l,m}^{xyz}\tilde{P}_{l+1}^{m} + c_{l,m}^{xyz}\tilde{P}_{l+1}^{m+2}\right),
\end{aligned}
\tag{28b}
$$

$$
\begin{aligned}
B_{xzz} &= \frac{\partial B_{xz}}{\partial z} = \frac{\partial B_{zx}}{\partial z} = \frac{\partial^2 B_x}{\partial z^2} = \frac{\partial^2 B_z}{\partial x \partial z} = \frac{\partial^2 B_z}{\partial z \partial x} \\
&= \frac{1}{a^2}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+4}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\left(a_{l,m}^{xzz}\tilde{P}_l^{m-1} + b_{l,m}^{xzz}\tilde{P}_l^{m+1}\right),
\end{aligned}
\tag{28c}
$$

$$
\begin{aligned}
B_{yyz} &= \frac{\partial B_{yy}}{\partial z} = \frac{\partial^2 B_y}{\partial y \partial z} = \frac{\partial^2 B_y}{\partial z \partial y} \\
&= \frac{1}{a^2}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+4}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)\left(a_{l,m}^{yyz}\tilde{P}_l^{m-2} + b_{l,m}^{yyz}\tilde{P}_l^{m} + c_{l,m}^{yyz}\tilde{P}_l^{m+2}\right),
\end{aligned}
\tag{28d}
$$

$$
\begin{aligned}
B_{yzz} &= \frac{\partial B_{yz}}{\partial z} = \frac{\partial B_{zy}}{\partial z} = \frac{\partial^2 B_y}{\partial z^2} = \frac{\partial^2 B_z}{\partial y \partial z} = \frac{\partial^2 B_z}{\partial z \partial y} \\
&= \frac{1}{a^2}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+4}\left(g_l^m \sin m\lambda - h_l^m \cos m\lambda\right)\left(a_{l,m}^{yzz}\tilde{P}_{l-1}^{m-1} + b_{l,m}^{yzz}\tilde{P}_{l-1}^{m+1}\right),
\end{aligned}
\tag{28e}
$$

$$
\begin{aligned}
B_{zzz} &= \frac{\partial^2 B_z}{\partial z^2} \\
&= \frac{1}{a^2}\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+4}\left(g_l^m \cos m\varphi + h_l^m \sin m\varphi\right)a_{l,m}^{zzz}\tilde{P}_l^{m},
\end{aligned}
\tag{28f}
$$

where the corresponding coefficients of the SSALFs are presented as:

$$
\begin{cases}
a_{l,m}^{xxz} = (l+3)a_{l,m}^{xx} \\
b_{l,m}^{xxz} = (l+3)b_{l,m}^{xx} \\
c_{l,m}^{xxz} = (l+3)c_{l,m}^{xx}
\end{cases}
\tag{29a}
$$

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删除的内容：27a

删除的内容：27b

删除的内容：27c

删除的内容：27d

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删除的内容：in Appendix A and can be computed once for all points.

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带格式的：英语(美国)

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$$\begin{cases} a_{l,m}^{xyz} = (l+3)a_{l,m}^{xy} \\ b_{l,m}^{xyz} = (l+3)b_{l,m}^{xy} \\ c_{l,m}^{xyz} = (l+3)c_{l,m}^{xy} \end{cases} , \qquad (29b)$$

$$\begin{cases} a_{l,m}^{xzz} = 0.5(l+2)(l+3)\sqrt{l+m}\sqrt{l-m+1}\sqrt{C_m / C_{m-1}} \\ \qquad = (l+2)(l+3)a_{l,m}^{x} = (l+3)a_{l,m}^{xz} \\ b_{l,m}^{xzz} = -0.5(l+2)(l+3)\sqrt{l+m+1}\sqrt{l-m}\sqrt{C_m / C_{m+1}} \\ \qquad = (l+2)(l+3)b_{l,m}^{x} = (l+3)b_{l,m}^{xz} \end{cases} , \qquad (29c)$$

$$\begin{cases} a_{l,m}^{yyz} = (l+3)a_{l,m}^{yy} \\ b_{l,m}^{yyz} = (l+3)b_{l,m}^{yy} \\ c_{l,m}^{yyz} = (l+3)c_{l,m}^{yy} \end{cases} , \qquad (29d)$$

$$\begin{cases} a_{l,m}^{yzz} = 0.5(l+2)(l+3)\sqrt{l+m}\sqrt{l+m-1}\sqrt{C_m / C_{m-1}} \\ \qquad = (l+2)(l+3)a_{l,m}^{y} = (l+3)a_{l,m}^{yz} \\ b_{l,m}^{yzz} = 0.5(l+2)(l+3)\sqrt{l-m}\sqrt{l-m-1}\sqrt{C_m / C_{m+1}} \\ \qquad = (l+2)(l+3)b_{l,m}^{y} = (l+3)b_{l,m}^{yz} \end{cases} , \qquad (29e)$$

$$a_{l,m}^{zzz} = -(l+1)(l+2)(l+3) = (l+3)a_{l,m}^{zz} = (l+2)(l+3)a_{l,m}^{z} . \qquad (29f)$$

In this way, we avoid computing recursively the SSALFs with singular terms, their first- and second-order derivatives as in the traditional formulae. The cost is only to calculate two additional degrees and orders for the SSALFs at most. It should be mentioned that, in this study, we use the conventional form of SSALF that if $m < 0$, then $\widetilde{P}_l^m = (-1)^{|m|}\widetilde{P}_l^{|m|}$ and if $m > l$, then $\widetilde{P}_l^m = 0$.

3 Numerical investigation and discussion

We test the derived expressions and the numerical implementation in C/C++, by calculating the magnetic potential, vector and its gradients and also the third-order partial derivatives of the magnetic potential field on a grid with 0.125°×0.125° cell size at the altitude of 300 km relative to the Earth's magnetic reference sphere using the lithospheric magnetic field model GRIMM_L120

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(version 0.0) defined by Lesur et al. (2013). The magnetic potential, MV, MGT and the third-order partial derivatives of the magnetic potential field in the two polar regions mapped by the lithospheric field model with spherical harmonic degrees/orders 16~90 are shown in Fig. 1 and Fig. 2, respectively. The corresponding statistics around the north and south poles are, respectively, presented in Table 1 and Table 2. A simple test is that the MGT meets the Laplace's equation of the potential field, that is, the trace of the MGT should be equal to zero. Our numerical results show that the amplitudes of $B_{xx}+B_{yy}+B_{zz}$ in the north and south polar regions are in the range of [-2.012×10^{-15} pT/m : +2.026×10^{-15} pT/m] (1 Tesla = 10^{3} mT=10^{9} nT=10^{12} pT=10^{18} aT), respectively. The relative error is almost equal the machine accuracy. Therefore, this feature proves the validity of our derived formulae. In addition, as shown in Fig. 1 and Fig. 2, it is obvious that the MGT and also the third-order partial derivatives of the magnetic potential field enhance the lineation and contacts at the satellite altitude. It also reveals some small-scale anomalies, which is very helpful for the further geological interpretation. A core field model with spherical harmonic degrees/orders 1~15 is also used to test and the results not shown here indicate the correctness of the formulae in the full range of the spherical harmonic degrees/orders, where the computational stability of the Legendre function with ultrahigh-order is not considered.

Furthermore, the computed magnetic fields are smooth near the poles and don't have the singularities but some components have the dependence on the direction of reference frame at the poles. As shown in Fig. 3, the magnetic potential V, B_z, B_{zz} and B_{zzz} components at the poles are independent of the direction of the $\mathbf{x}_P$ and $\mathbf{y}_P$ axes, while changing with the direction of the $\mathbf{x}_P$ and $\mathbf{y}_P$ axes at the poles, the B_x, B_y, B_{xz}, B_{yz}, B_{xzz} and B_{yzz} components have a period of 360° and the B_{xx}, B_{xy}, B_{yy}, B_{xxz}, B_{xyz} and B_{yyz} components have a period of 180°. These variations can be accurately

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described by a sine or cosine function relating to the horizontal rotation of the reference frame and the differences among these magnetic effects are magnitude, period and initial phase. Therefore, B_x, B_y, B_{xz}, B_{yz}, B_{xx}, B_{xy}, B_{yy}, B_{xzz}, B_{yzz} B_{xxz}, B_{xyz} and B_{yyz} components are not smooth at/cross the poles. Therefore, to determine the single value at the poles (Fig. 1 and Fig. 2) we specially define that the **x**-axis points to the meridian of 180° E (or 180° W) at north pole and of 0° at south pole, that is, the LNORF moving from Greenwich meridian to the poles.

Compared with the traditional formulae in section 2.1, there are two advantages of our derived formulae in section 2.3. On the one hand, the traditional derivatives up to second-order are removed in the new formulae; therefore, the relatively complicated method by the Horner′s recursive algorithm (Holmes and Featherstone, 2002b) can be avoided. On the other hand, the singular terms of 1/sinθ and 1/ sin$^2\theta$ are removed in the new formulae; consequently, the scale factor of e.g. 10^{-280} (Holmes and Featherstone, 2002a,b) is not required when the computing point approaches to the poles and the magnetic fields at the poles can also be calculated in the defined reference frame. In fact, there are differences between the results by our expressions and those by the Horner′s recursive algorithm, for instance, if using the same model and the parameters as those in Fig. 1 and Fig. 2, the differences of the three components B_x, B_y and B_z are at a level of [-3×10^{-11} nT : $+3\times10^{-11}$ nT].

4 Conclusions

We develop in this paper the new expressions for the MV, the MGT and the third-order partial derivatives of the magnetic potential field in terms of spherical harmonics. The traditional expressions have complicated forms involving first- and second-order derivatives of the SSALFs

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and are singular when approaching to the poles. Our newly derived formulae don't contain the first- and second-order derivatives of the SSALFs and remove the singularities at the poles.

However, our formulae are derived in the spherical LNORF with specific definition at the poles. For an application to the magnetic data of a satellite gradiometry mission, it is necessary to describe the MV and the MGT in the local orbital or other reference frame, where the new MV and MGT are the linear functions of the MV and the MGT in the LNORF with coefficients related to the satellite track azimuth (e.g. Petrovskaya and Vershkov, 2006) or other rotation angles. The other main purpose of this paper is in the future to contribute to the signal processing and the geophysical & geological interpretation of global lithospheric magnetic field model, especially near polar areas.

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Supplementary software implementation is performed by the programming language C/C++. The source code and input data presented in this paper can be obtained by contacting the lead author via email.

Acknowledgements. This study is supported by International Cooperation Projection in Science and Technology (No.: 2010DFA24580), Hubei Subsurface Multi-scale Imaging Key Laboratory (Institute of Geophysics & Geomatics, China University of Geosciences, Wuhan) (Grant No.: SMIL-2015-06) and State Key Laboratory of Geodesy and Earth's Dynamics (Institute of Geodesy and Geophysics, CAS) (Grant No.: SKLGED2015-5-5-EZ). Jinsong Du is sponsored by the China Scholarship Council (CSC). We would like to thank Prof. Mehdi Eshagh and another anonymous reviewer for their constructive suggestion. All projected figures are drawn using the Generic Mapping Tools (GMT) (Wessel and Smith, 1991).

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Numerical constants in the Eq. (25), the Eq. (26) and the Eq. (27) are expressed in following:

$$\begin{cases} a^x_{l,m} = 0.5\sqrt{l+m}\sqrt{l-m} \\ b^x_{l,m} = -0.5\sqrt{l+m+1}\sqrt{l} \end{cases}$$, (A1)

$$\begin{cases} a^y_{l,m} = 0.5\sqrt{l+m}\sqrt{l+m} \\ b^y_{l,m} = 0.5\sqrt{l-m}\sqrt{l-m} \end{cases}$$, (A2)

$$a^z_{l,m} = -(l+1),$$ (A3)

$$\begin{cases} a^{xx}_{l,m} = -0.25\sqrt{l+m}\sqrt{l+} \\ b^{xx}_{l,m} = 0.25[(l+m)(l-m \\ c^{xx}_{l,m} = -0.25\sqrt{l+m+2} \end{cases}$$, (A4)

$$\begin{cases} a^{xy}_{l,m} = -0.25\sqrt{l+m}\sqrt{l-} \\ b^{xy}_{l,m} = -0.5m\sqrt{l-m+1} \\ c^{xy}_{l,m} = 0.25\sqrt{l+m+1}\sqrt{l} \end{cases}$$, (A5)

$$\begin{cases} a^{xz}_{l,m} = 0.5(l+2)\sqrt{l+m} \\ b^{xz}_{l,m} = -0.5(l+2)\sqrt{l+m} \end{cases}$$, (A6)

$$\begin{cases} a^{yy}_{l,m} = 0.25\sqrt{l+m}\sqrt{l+} \\ b^{yy}_{l,m} = 0.25[(l+m)(l+m \\ \quad +2(l+m+2)(l+ \\ c^{yy}_{l,m} = 0.25\sqrt{l+m+1}\sqrt{l} \end{cases}$$

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Tables and figures

Table 1. Statistics of the magnetic potential, MV, MGT and third-order partial derivatives of the magnetic potential field around the north pole (0°≤θ≤30°) at the altitude of 300 km using the lithospheric magnetic field model GRIMM_L120 (version 0.0) (Lesur et al., 2013) for spherical harmonic degrees 16~90.

Magnetic effects	Minimum	Maximum	Mean	Standard deviation
V [mT×m]	-5.1554771	+4.7867519	+0.0828017	±1.7377648
B_x [nT]	-14.7389250	+17.6917740	-0.0890689	±4.9797007
B_y [nT]	-15.1297000	+13.6053000	+0.0010738	±4.8239313
B_z [nT]	-19.8715270	+25.3666030	-0.1988485	±6.7066701
B_{xx} [pT/m]	-0.1054684	+0.0621351	+0.0001872	±0.0215871
B_{xy} [pT/m]	-0.0410371	+0.0491030	+0.0000003	±0.0115018
B_{xz} [pT/m]	-0.0929498	+0.1082861	+0.0006867	±0.0247522
B_{yy} [pT/m]	-0.0726248	+0.0505990	-0.0004789	±0.0186580
B_{yz} [pT/m]	-0.0868184	+0.0826627	+0.0000058	±0.0228174
B_{zz} [pT/m]	-0.1015986	+0.1511038	+0.0002917	±0.0336965
$B_{xx}+B_{yy}+B_{zz}$ [pT/m]	-2.012×10^{-15}	$+2.026\times10^{-15}$	$+8.085\times10^{-19}$	$\pm5.101\times10^{-16}$
B_{xxz} [aT/m^2]	-0.7589853	+0.4794999	+0.0002436	±0.1537058
B_{xyz} [aT/m^2]	-0.2628265	+0.3734132	-0.0000004	±0.0734794
B_{xzz} [aT/m^2]	-0.7067652	+0.8470055	+0.0140820	±0.1752880
B_{yyz} [aT/m^2]	-0.5259662	+0.4076568	-0.0134321	±0.1370902
B_{yzz} [aT/m^2]	-0.6058631	+0.6396412	+0.0000341	±0.1448002
B_{zzz} [aT/m^2]	-0.7609268	+1.1697371	+0.0131885	±0.2421663

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Table 2. Statistics of the magnetic potential, MV, MGT and third-order partial derivatives of the magnetic potential field around the south pole (150°≤θ≤180°) at the altitude of 300 km using the lithospheric magnetic field model GRIMM_L120 (version 0.0) (Lesur et al., 2013) for spherical harmonic degrees 16~90.

Magnetic effects	Minimum	Maximum	Mean	Standard deviation
V [mT×m]	-3.3267455	+4.6543369	+0.0801853	±1.2427083
B_x [nT]	-11.440070	+15.9109730	+0.3451248	±3.5403285
B_y [nT]	-9.1169009	+15.0436160	-0.0001605	±3.1560093
B_z [nT]	-22.202857	+14.5020010	-0.3022955	±4.7971494
B_{xx} [pT/m]	-0.0579914	+0.0704617	+0.0000845	±0.0166266
B_{xy} [pT/m]	-0.0364002	+0.0308075	-0.0000006	±0.0074702
B_{xz} [pT/m]	-0.0741850	+0.0831062	+0.0019925	±0.0187492
B_{yy} [pT/m]	-0.0569493	+0.0706456	+0.0019055	±0.0143289
B_{yz} [pT/m]	-0.0599346	+0.0897167	-0.0000012	±0.0154623
B_{zz} [pT/m]	-0.1367168	+0.0735795	-0.0019900	±0.0258066
$B_{xx}+B_{yy}+B_{zz}$ [pT/m]	-1.027×10^{-15}	$+2.012\times10^{-15}$	$+1.113\times10^{-18}$	$\pm5.059\times10^{-16}$
B_{xxz} [aT/m^2]	-0.4605216	+0.5307263	+0.0011232	±0.1328515
B_{xyz} [aT/m^2]	-0.2840344	+0.2947601	-0.0000015	±0.0526629
B_{xzz} [aT/m^2]	-0.5686811	+0.5634376	0.0181792	±0.1497829
B_{yyz} [aT/m^2]	-0.4262850	+0.5819095	+0.0186968	±0.1169641
B_{yzz} [aT/m^2]	-0.6194116	+0.6520948	-0.0000118	±0.1085051
B_{zzz} [aT/m^2]	-1.0199774	+0.5863084	-0.0198200	±0.2084566

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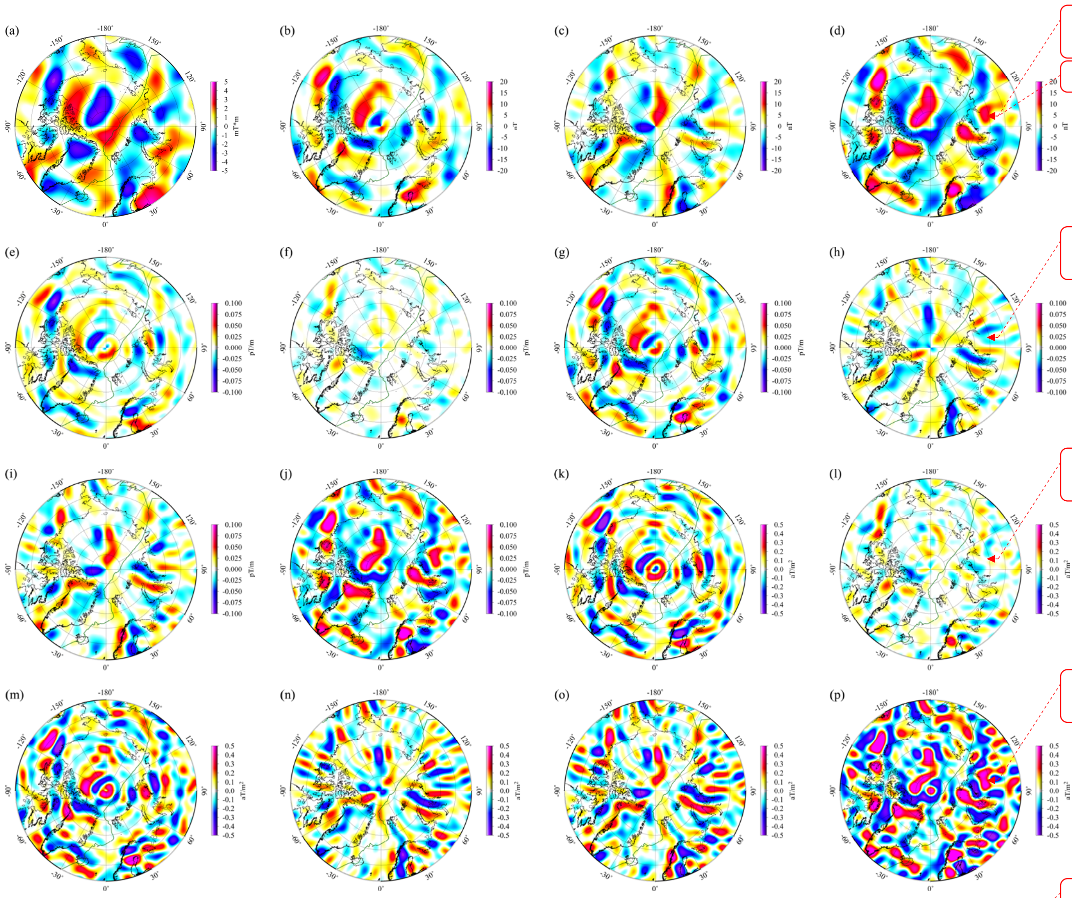

Figure 1. Lithospheric magnetic potential, vector and its gradients fields and third-order partial derivatives of the magnetic potential field around the north pole (0°≤θ≤30°) at the altitude of 300 km as defined by the lithospheric magnetic field model GRIMM_L120 (version 0.0) (Lesur et al., 2013) for spherical harmonic degrees 16~90. (a) is magnetic potential (V), (b) (c) and (d) are three components (B_x, B_y and B_z) of magnetic vector, (e), (f), (g), (h), (i) and (j) are six elements (B_{xx}, B_{xy}, B_{xz}, B_{yy}, B_{yz} and B_{zz}) of magnetic gradient tensor, (k), (l), (m), (n), (o) and (p) are six elements (B_{xxz}, B_{xyz}, B_{xzz}, B_{yyz}, B_{yzz} and B_{zzz}) of third-order partial derivatives of the magnetic potential field, respectively. The dark green lines are the plate boundaries by Bird (2003). All maps are shown by Polar Stereographic projections.

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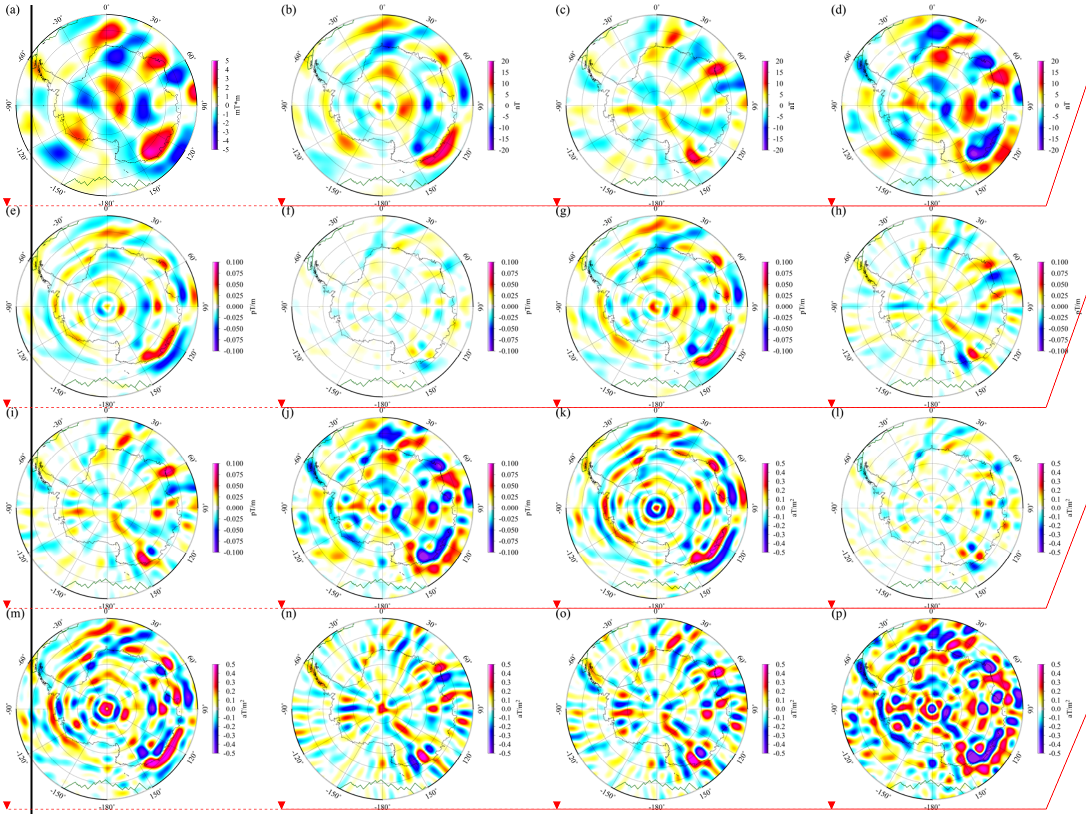
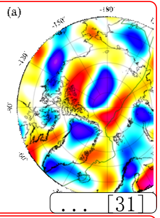


Figure 2. Lithospheric magnetic potential, vector and its gradients fields and third-order partial derivatives of the magnetic potential field around the south pole (150°≤θ≤180°) at the altitude of 300 km as defined by the lithospheric magnetic field model GRIMM_L120 (version 0.0) (Lesur et al., 2013) for spherical harmonic degrees 16~90. (a) is magnetic potential (V), (b) (c) and (d) are three components (B_x, B_y and B_z) of magnetic vector, (e), (f), (g), (h), (i) and (j) are six elements (B_{xx}, B_{xy}, B_{xz}, B_{yy}, B_{yz} and B_{zz}) of magnetic gradient tensor, (k), (l), (m), (n), (o) and (p) are six elements (B_{xxz}, B_{xyz}, B_{xzz}, B_{yyz}, B_{yzz} and B_{zzz}) of third-order partial derivatives of the magnetic potential field, respectively. The dark green lines are the plate boundaries by Bird (2003). All maps are shown by Polar Stereographic projections.



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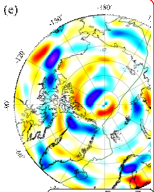

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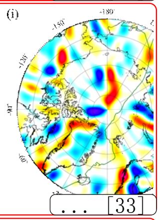

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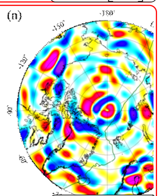

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删除的内容: Global …of the main field at the altitude of 300 km as defined by the main magnetic field model IGRF11 (Finlay et al., 2011) at the epoch of 2005.0.0 for spherical harmonic degrees 1~13.…on a Hammer projection centered at 90° E. [35]

删除的内容: main magnetic field model IGRF11 (Finlay et al., 2011) at the epoch of 2005.0.0 for spherical harmonic degrees 1~13 [36]

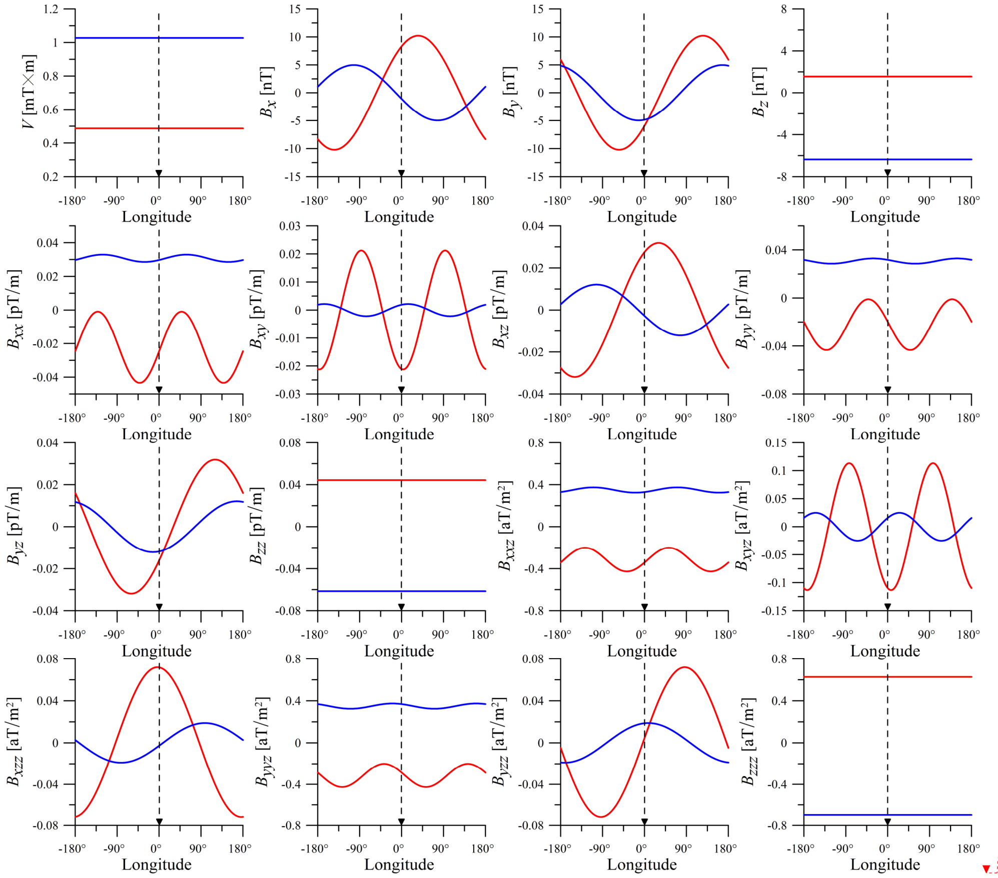
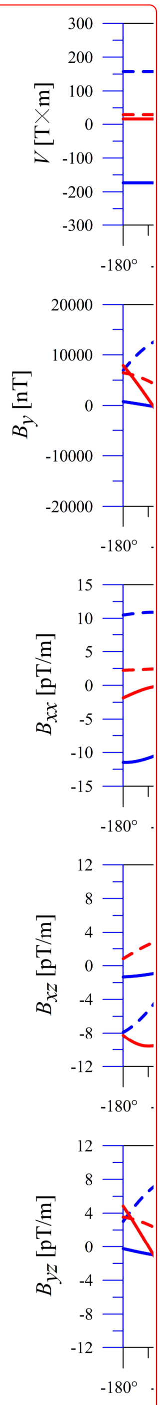


Figure 3. Limit values of magnetic potential (V), vector (B_x, B_y and B_z) and its gradients (B_{xx}, B_{xy}, B_{xz}, B_{yy}, B_{yz} and B_{zz}) and third-order partial derivatives of the magnetic potential field (B_{xxz}, B_{xyz}, B_{xzz}, B_{yyz}, B_{yzz} and B_{zzz}) at the poles when the local reference frames vary from different meridians (the direction of $\mathbf{x}_P$ axe changing from different meridian to the poles). Red and blue lines indicate the magnetic effects at north-pole and at south-pole, respectively. The reference frame is specially defined that the $\mathbf{x}_P$-axis points to the meridian of 180° E (or 180° W) at north pole and 0f 0° at south pole and the $\mathbf{y}_P$-axis points to the meridian of 90° E at two poles. The values at two poles showed by black dashed arrows are used to plot the maps in Fig. 1 and Fig. 2.



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