Replies to Executive Editor Comments:

*We thank the editor for the review and the coordination of the discussion around the manuscript, which has considerably clarified and improved the paper, making more clear its methodology, purpose and conclusions. Below we address the specific comments of the Editor regarding the availability of the models used:*

1. "– The paper must be accompanied by the code, or means of accessing the code, for the purpose of peer-review."

*See reply to comment no. 2 below.*

2. "– All papers must include a section at the end of the paper entitled "Code availability".

*The code for particle tracer ptr3D V2.0 is based on the standard equations of motion as described, e.g., in equation 1.1 of Roederer [1970]; however it has been parameterized in terms of spatial and temporal resolution for use in the particular study (i.e., maximum and minimum spatial limits of particle tracing, epoch & solar wind conditions, and time-steps of particle trajectory integration); it has thus been tuned to work accurately and efficiently for the particles under investigation, and hence at this stage it is not a generic code that can be provided for use as a generic particle tracer; it is envisioned that its next stage (ptr3D V3.0) that is being developed will be sufficiently tested to be able to stand on its own as a general particle tracer code, that can be used for any range of particle energies, time periods or regions.*

*The following text has been added in section 7:*

*"ptr3D V2.0 is a particle tracing code developed by the authors based on the equations of charged particle motion under the Lorenz force, as described in detail in the respective chapter of this paper, and its results can be verified by any other particle tracer. In its current version (V2.0) it has been tuned to work accurately and efficiently within the region, times and energies of the particles under investigation, and hence at this stage it is not a generic code that can be provided for use as a generic particle tracer; it is envisioned that its next version (ptr3D V3.0) will be released as a general particle tracer code, that can be used for any range of particle energies, times or regions."*

3. "– All papers must include a model name and version number (or other unique identifier) in the title. ".

*The particle tracer code name and version is ptr3D V2.0. This has been added in the text in the title, as well as has been corrected in the text, in lines: 19, 105, 107, 114, 121, 139, 144, 151, 154, 161, 166, 182, 192, 224, 238, 278, 283*

Replies to Referee #1

*We thank the Referee for a thorough examination and review of the manuscript, which has led to considerable improvements in the paper. In the improved version, the results and conclusions of the study are presented more clearly; furthermore, the revised version includes better definitions of the variables and the methods used for their calculation, as well as corrections of definitions and grammar. Below we address the specific comments of the Referee, pointing to the locations in the paper where corrections were made, and listing the specific changes:*

1. As mentioned above, the main issue preventing this manuscript from being ready for publication is the lack of discussion of the results of the study. What should we conclude from the various discrepancies between the models? How should scientists utilize the region maps illustrating constancy of the invariants? What physical mechanisms lead to breaking of the invariants in the regions illustrated? What should we use if we are interested in sorting data from those regions? There is a complete lack of discussion that is somewhat surprising and must be rectified for this to be considered a complete paper.

*We agree with Referee #1 that the results have not adequately been discussed in the paper; this comment has also been made by Referee #2. We have completely revised the final section of the Conclusions of the paper, and the following extensive discussion of the results of the study has been added in section 6, line 248 :*

*"In the discussions of particle transport, energization and loss in the Earth's radiation belts, a major question concerns the relative contribution between wave-particle interactions vs. radial diffusion, which is generally best discussed in terms of phase-space-density, calculated at constant adiabatic invariants. From the discussions herein, it is evident that caution should be exercised when considering the second and third adiabatic invariants to remain constant across all L-shells and local times within the radiation belts as well as for all particle energies and all geomagnetic conditions. In particular, in regions where the results from the various models diverge from the results from the particle tracer, which most closely follows the calculations of the invariants, we can conclude that the models should be used with caution, the lack of confidence in them being analogous to the magnitude of this divergence. In this paper it has been demonstrated that under extreme curvature of the magnetospheric magnetic field, particles of high energy and low pitch-angles cannot be considered to remain adiabatic in terms of their second and third invariants.*

*The physical mechanism that leads to breaking of the invariants in the regions illustrated does not involve temporal variations in the magnetic field of time scales shorter than the associated time-scales of the second and third invariants, i.e. the bounce period and drift period, as the fields used in the simulations above are all static. Instead, the breaking of the invariants in the above is associated with deviations of the magnetic field from a dipole configuration: in the definition of the invariants, in order for the second adiabatic invariant to remain constant it is required that the magnetic field between two mirroring points does not change much in one bounce period as the particle's guiding center drifts across field lines. Similarly, in order for the third adiabatic invariant to remain constant, it is required that the* *magnetic flux through the guiding center orbit of a particle around the earth should remain constant. However during active geomagnetic conditions the curvature of the field lines in the night side of the earth in combination with the large gyro-radii of large-energy particles leads to deviations from these conditions that need to be taken into account.*

*The present paper by no means aims to serve as a guide-line of the adiabaticity of particles at all energies, pitch angles and geomagnetic conditions; instead, it aims to raise awareness and caution in using general-purpose models and tools, such as IRBEM, LANL* and SPENVIS to calculate the values of the adiabatic invariants in regions and cases where they are not well defined."*

2. The definition and description of the invariants, particularly L∗, definitely needs some fleshing out. How is L∗ defined in terms of integrals of motion? How is it calculated in the various models? In particular, a brief description of how the authors implemented "the method described by Roederer" is needed.

*The following has been added in the manuscript in line 51*

*"…integrated along the trajectory of the particle for the entire drift shell."*

*and below in line 55 :*

*"A practical way to calculate Φ is to find the intersection C of a series of drift-shell field lines with the Earth's surface and to numerically compute Φ over the cap delineated by C, using the following equation:*

$$\Phi \simeq - - \frac{k_0}{R_E} \int_0^{2\pi} cos^2[\lambda_e(\phi)] d\phi$$

*where, λ e (φ) is the dipole latitude of the intersection C at a given longitude φ [Roederer, 1970]."*

***Specific Comments / Technical Corrections***

1) Abstract Line 2: "invariants respectively," → "invariants, respectively,".

*Corrected*

2) Abstract Line 6: "roughly" is a weasel word. Omit or be more precise, whichever is appropriate.

*"Equivalent" alone conveys the approximate nature of the relation between the two. "roughly" has been deleted*

3) Abstract Line 15: Omit "related".

*Corrected*

4) Abstract Line 15: “source code” → “software”.

*Corrected*

5) Abstract Line 19: Omit “geocentric distance”.

*Corrected*

6) Abstract Line 20: I would say you more than attempt to map, you do actually map.

*Deleted, also for other instances in the text*

7) Abstract Line 22: “proton” → “protons”.

*Corrected*

8) Abstract Line 24: See Item 5.

*Corrected*

9) Introduction Line 9: Omit “For particles in magnetic fields, and”, and capitalize “for”.

*Corrected*

10) Introduction Line 11: Omit “of the three types of motion mentioned above.”

*Corrected*

11) Equation 1: What is _? What is _? What is B(s)? These should be defined.

*Definitions have been added in lines 42 - 45 of the revised manuscript*

12) Equation 2: What is $\_{0}$ ? Is there an expression for $\Phi$?

*Definitions have been added in lines 47 and eq. 3 of the revised manuscript*

13) Title, Section 2.1: Is it LANLstar or LANL*? Be consistent.

*It’s now LANL* throughout the paper*

14) LANLstar Line 15: TS05 is just empirical. I’m not sure what “semi-empirical” means.

*Changed to “empirical”*

15) LANLstar Line 23: What is SpacePy?

*The following has been added: “a Python-based tool library for space science,”*

16) LANLstar Line 25: Why is LANLmax relevant for this discussion?

*The sentence on the description of LANLmax is not relevant for the discussion of LANLstar, and has been removed from the manuscript.*

17) IRBEM-lib Line 12: How is L* calculated?

*The following text has been added in the manuscript, on lines 91 and footnote 3 in the same page :*

*"Roederer's shell parameter L* is then deduced directly from the value of the third invariant using eq. 2"*

*Also modified text in line 104:*

*"The third invariant Φ is evaluated in UNILIB using Roederer's numerical method as for the case of IRBEM-lib"*

18) SPENVIS Line 23: "as given above"; where was it given?

*The following clarification has been added: "as described in the introduction"*

19) 3-D Tracer Line 7—8: "so as to facilitate" à "for"

*Corrected*

20) 3-D Tracer Line 13: TS05 only needs to be cited when first introduced.

*Deleted citation*

21) Calculations of I Line 4: what initial distances, exactly? How were they distributed between 4 and 8 ?

*The following clarification has been added: "in steps of 1 RE"*

Replies to Referee #2

*We thank the Referee for the review of the manuscript and for pointing out gaps in the continuity and clarity of the paper. In the revised version the motivation and the conclusions of the study are presented more clearly; furthermore, some of the definitions that were missing are now include and also the methodology of some of the calculations is now more complete. Below we address the specific comments of the Referee, pointing to the locations in the paper where corrections were made, and listing the specific changes:*

1. In the introduction section, the authors simply described the adiabatic invariants. For a technical research paper, what is the motivation of your work? What science questions you intend to address so that you carried out this work? More background information is needed.

*The following text has been added after the first paragraph of the abstract:*

*"The purpose of this work is to investigate where in the near-Earth magnetosphere we can safely calculate I and L* using tools with widespread use in the field of space physics, for various magnetospheric conditions and particle initial conditions."*

In addition, there is no definition of those parameters in equations (1) and (2).

*The definitions of those parameters have been added in lines 42-45 and 48 as follows:*

*" **I** is expressed in distance units (km or $R_E$) and so it gives an intuitive approximation of the length of the particle trajectory along a field line between the two mirror points. In place of the Third Adiabatic Invariant it is convenient to use L* or Roederer's L (Roederer, 1970). L* is defined as:*

$$L^* = -\frac{2\pi k_0}{\Phi R_E}$$

*where $\mathbf{k_0}$ is the Earth's dipole moment, $\mathbf{R_E}$ is the radius of the Earth (6370 km) and Φ is the Third Adiabatic Invariant and is defined as*

$$\Phi = \int_S B ds$$

*integrated along the trajectory of the particle for the entire drift shell. **L*** physically approximates the distance from the center of the Earth to the equatorial point of a given field line (in $\mathbf{R_E}$) if we assume a dipolar magnetic field for the Earth. **L*** is also an invariant, since it's inversely proportional to Φ (Roederer, 1970)."*

2. While the authors described how L* is calculated in most of these programs/models/methods, they omitted this in the IRBEM-lib.

*The following text has been added in the manuscript, on line 91 and also on footnote 3 in the same page:*

*"Roederer's shell parameter L* is then deduced directly from the value of the third invariant using eq. 2"*

*We also modified the manuscript in line 104 as follows:*

*"The third invariant Φ is evaluated in UNILIB using Roederer's numerical method as for the case of IRBEM-lib" (See also reply to comment 17 of ref. 1)*

3. Since most of the comparisons in this work are on L* and I, why is L*max (which is perhaps only available in LANL*) introduced or even calculated?

*The sentence on the description of LANLmax is not relevant for the discussion of LANLstar, and has been removed from the manuscript. (See also reply to comment 16 of ref. 1)*

4. Please elaborate the purpose of the work on "mapping regions of constant" in section 5.

*The following text has been added in the manuscript on line 190:*

*The purpose of this section is to demonstrate at which magnetic longitude the adiabaticity of I is broken, for different particle starting conditions as well as for different geomagnetic conditions. To this extent, in figure 11 we map the areas where I and therefore L* cannot be assumed to remain constant throughout a particle drift shell. With this map we demonstrate in a graphic representation the magnetic longitudes and distances from the Earth where I ceases to be adiabatic. In these areas the general-purpose models and tools described above, such as IRBEM, LANL* and SPENVIS, cannot be safely used to calculate the values of the adiabatic invariant I and therefore L*.*

5. What can we learn from these comparative results? Did the authors simply want to report these results or convey some implication? For example, if we were to calculate I or L*, which model or regions should we choose? The "Conclusion" section is rather a "Summary" section.

*We agree with Referee #2 that the results have not adequately been discussed in the paper, and the conclusions are not clearly summarized; a similar comment has also been made by Referee #1. We have completely revised the final section of the Conclusions of the paper, and the following extensive discussion of the results of the study has been added in section 6, line 248:*

*"In the discussions of particle transport, energization and loss in the Earth's radiation belts, a major question concerns the relative contribution between wave-particle interactions vs. radial diffusion, which is generally best discussed in terms of phase-space-density, calculated at constant adiabatic invariants. From the discussions herein, it is evident that caution should be exercised when considering the second and third adiabatic invariants to remain constant across all L-shells and local times within the radiation belts as well as for all particle energies and all geomagnetic conditions. In particular, in regions where the results from the various models diverge from the results from the particle tracer, which most closely follows the calculations of the invariants, we can conclude that the models should be used with caution, the lack of confidence in them being analogous to the magnitude of this divergence. In this paper it has been demonstrated that under extreme curvature of the magnetospheric*

*magnetic field, particles of high energy and low pitch-angles cannot be considered to remain adiabatic in terms of their second and third invariants.*

*The physical mechanism that leads to breaking of the invariants in the regions illustrated does not involve temporal variations in the magnetic field of time scales shorter than the associated time-scales of the second and third invariants, i.e. the bounce period and drift period, as the fields used in the simulations above are all static. Instead, the breaking of the invariants in the above is associated with deviations of the magnetic field from a dipole configuration: in the definition of the invariants, in order for the second adiabatic invariant to remain constant it is required that the magnetic field between two mirroring points does not change much in one bounce period as the particle's guiding center drifts across field lines. Similarly, in order for the third adiabatic invariant to remain constant, it is required that the magnetic flux through the guiding center orbit of a particle around the earth should remain constant. However during active geomagnetic conditions the curvature of the field lines in the night side of the earth in combination with the large gyro-radii of large-energy particles leads to deviations from these conditions that need to be taken into account.*

*The present paper by no means aims to serve as a guide-line of the adiabaticity of particles at all energies, pitch angles and geomagnetic conditions; instead, it aims to raise awareness and caution in using general-purpose models and tools, such as IRBEM, LANL* and SPENVIS to calculate the values of the adiabatic invariants in regions and cases where they are not well defined."*