Response to reviewers and changes made to the revised manuscript

October 7, 2014

1 Response to Reviewers

We would like to thank reviewers for their comments. Below we proposed several improvements based on these comments. We also incorporated them into a new version of the paper.

1.1 Review #1

- 1. p. 4086: Our text was poorly worded. We did not mean to imply these operators were equal, but instead we replace $\nu\Delta^2$ with a new operator, $(\nabla \cdot \tau \nabla)\Delta$ with a symmetric tensor τ . In case $\tau = \nu I$ where I is an identity, both operators are equal.
- 2. We address the comment about how equations (2) and (3) are derived from (1) above. Instead of a weak form for $Q_t = -\nu\Delta^2$, in (2) and (3) we formulate a weak form for $Q_t = -(\nabla \cdot \tau \nabla)\Delta Q$. As noted above, this later form is more general and is equal to the original formulation when $\tau = \nu I$.
- 3. Section 5.3: In this paper we wanted to focus only on the spatial error, and in all cases used timesteps where the time truncation error was negligible as compared to the spatial error. While running simulations for convergence we made sure that temporal errors did not dominate. For test case 2 (TC2), we obtain 4th order convergence when using timesteps near the CFL limit. For the test case 5 (TC5) we reduced time steps so that time truncation errors are of 4th order. For example, for the resolution with spatial scales Δx the time was Δt . For the refined grid with spatial scales $\Delta x/2$ we ran a simulation with $\Delta t/4$.

2 Review #2

- 1. This review suggested that we compare tensor and scalar hyperviscosities on the same grid for the test case 5. We supply one more plot in Figure 10 (panel (d)). We notice that the numerical noise at the transition region present in the simulation (b) with scalar HV is practically eliminated when tensor HV is used in (d). If the simulation in (c) for a better quality grid (low-connectivity grid) is considered optimal, then one can conclude that tensor hyperviscosity simulation provides a very-close-to-optimal result even if a low quality grid is used.
- 2. page 4086, comment on Δ^2 : We used the identity $\Delta = \nabla \cdot \nabla$.
- 3. page 4086, eqns. 2 and 3, the same as what Reviewer #1 pointed out: We explained this in the reply to Reviewer #1.
- 4. Figures 6 and 10: We enlarged the figures to make fonts bigger.
- 5. About convergence slope in Figure 12, TC5. We agree with the reviewer. Our speculation that this is due to the use of a reference solution is incorrect. It is more likely due to the fact that the mountain has a cusp in its shape, limiting the convergence to 1st order in the max norm and second order in the l2 norm.

3 Changes to the revised copy

As was suggested by the reviewers, we clarify our definition of a new hyperviscosity operator, add a simulation to compare tensor hyperviscosity on a low- and high-quality grids, explain choices for time stepping in convergence studies, and correct the explanation on the convergence rate in test case 5. We also address typos and typesetting issues. The full list of changes is below.

- 1. Page 2, line 100: The subscript t is explained.
- 2. Page 2, lines 102 ff: We clarified our definition of the hyperviscosity term as was suggested by both reviewers. We also added Eqn. (2) and expanded the derivation of the weak form in Eqns. 3 and 4.
- 3. In Figure 10 one more panel (panel (d), tensor hyperviscosity simulation and a highly distorted grid) was added. In Section 5.2, analysis of the new simulation was added. Table 2 has a new entry corresponding to this simulation.
- 4. In Section 5.3 we no longer state that certain convergence rates in TC5 are due to uncertainties in the reference solution.
- 5. In Section 5.3 we add a paragraph on time truncation errors. We also explain why these errors do not contaminate spatial convergence rates.
- 6. Page 6: Changed "... less noise the refined region" to "... less noise in the refined region".
- 7. Page 6: Changed "... reference solution from computed on a uniform high resolution grid" to "... reference solution that was computed on ..."
- 8. Page 6: Changed "... the improved resolution of these standing features leads to slightly less error that is obtained by..." to "... the improved resolution of these standing features leads to slightly less error than is obtained by ...".
- 9. Page 2: Changed 'flow-depended' to 'flow-dependent'. Changed 'valance' to 'valence'.
- 10. Page 4: Changed "...family have..." to "...family has...".
- 11. Figure 5, caption: Changed "... a closeups..." to "... closeups...".
- 12. Page 5: Changed "...halo and is ..." to "... halo is...", changed "...according to formula..." to "...according to the formula...".
- 13. Figure 6: Added units to captions.
- 14. Page 6: Changed "...then panel..." to "...than panel...", changed "...and to tensor..." to "...and tensor...".
- 15. Figure 10: Changed 10^6 to 10^{-6} .
- 16. Added a thank-you comment in the acknowledgement.
- 17. Figure 4: Changed "...Example of x8 Refined ..." to "...Example of ×8 refined..."
- 18. In Appendix, two matrices were substituted using symbols a_{11} , a_{12} , etc. due to the narrow column width.
- 19. For consistency, notations x2, x4, etc. were substituted by $\times 2$, $\times 4$... throughout the text.