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An improved coupling model for water flow, sediment transport and bed evolution (CASFE v.1)

S. He^{1,2}, W. Liu², X. Li^{1,2}, and C. Ouyang^{1,2}

¹Key laboratory of Mountain Hazards and Earth Surface Process, Chinese Academy of Science, Chengdu, China

²Institute of Mountain Hazards and Environment (IMHE), Chinese Academy of Sciences, Chengdu, China

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Correspondence to: X. Li (lixinpo@imde.ac.cn)

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Abstract

This paper presents a new coupling model to simulate water flow, sediment transport and bed evolution based on the shallow water assumption, depth-average integration as well as the morphological evolution (Chinese Academy of Sciences flow-erosion model, CASFE). The model takes account of the effects of rainfall, entrainment and deposition on the motion of water–sediment mixtures. Limitations and drawbacks of other authors' models are pointed out according to the comparison among these models. The finite volume method has been adopted to solve the one-dimensional dam-break problem considering an erodible bed. Numerical results indicate that the model can adequately describe the complex dynamic problems. Simulation results demonstrate that the entrainment and deposition significantly affects the flow dynamics and morphological evolution.

1 Introduction

In recent years, the catastrophic flood events in worldwide are increasingly frequent and its dynamics mechanism has attracted much interest as how to predict it by numerical method. It is generally accepted that reliable methods for predicting the water flow, sediment transport and bed evolution can play important role in development of control strategy for environmental protection. The mathematical models are considered to be one of the best ways for understanding the dynamics of water flow and sediment transport. In the past decades, numerical solutions, including finite element method, finite volume method and finite difference method, are mainly focused on the shallow water equations related with mass movement over a fixed bed. Nevertheless, in order to exhibit the geomorphological evolution, it is necessary to couple the shallow water equations describing the flow continuity and the momentum conservation equation, the sediment continuity equation and the bathymetry evolution equation. Thus, increasing attention is being paid to methods of solving the complete coupled

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equations. A discontinuous Galerkin finite element method (FEM) was implemented to represent the morphodynamic procedure by accounting for the sediment transport and bed evolution (Tassi et al., 2008). In addition, Cao (Cao et al., 2004) presents a one-dimensional dam-break model considering erodible sediment beds. The model used a weighted average flux method and the HLLC approximate Riemann solver with high-order accuracy promoted by SUPERBEE limiter. Many scholars (Fraccarollo et al., 2002; Benkhaldoun et al., 2011; Murillo and García-Navarro, 2010; Simpson and Castellort, 2006) used finite volume method (FVM) to solve the coupled model of water flow, sediment transport and bed evolution.

In this paper, a new mathematical model has been presented. This model enables predicting the change of water surface morphology and bed morphology in the time and space through water flow and sediment transport dynamics. The model presented in this study includes the shallow water equations for surface flow, conservation of sediment concentration, substrate erosion and deposition as well as morphodynamics models. In recent years, researches on models coupled sediment transport and water flow without interruption have been provided by Fagherazzi and Sun (2003), Cao et al. (2004, 2006), Simpson et al. (2006), Hu et al. (2009), Schippa et al. (2009), Benkhaldoun et al. (2011), and Li et al. (2011). The model here differs from these in that it takes account of the effect of rainfall, entrainment and deposition on momentum equation. The equations of the model are hyperbolic and conform to conservation laws. So far, the numerical schemes developed for solving hydrodynamics and morphodynamics problem are matured and the finite volume methods has been adopted for numerical simulation in this study.

This paper is organized as follows. The governing equations for couple model of water flow, sediment transport and bed evolution are presented in Sect. 2. The boundary kinematic condition is formulated in Sect. 3. The depth-integrated equations and the full model equations are derived in Sects. 4 and 5. The comparisons between the present model and the previous models are given in Sect. 6. The numerical results

and examples are displayed in Sect. 7. At last, Sect. 8 summarizes the concluding remarks.

2 Model descriptions

2.1 Governing equations

5 The water–sediment mixture is assumed to be an incompressible material with variable density throughout the entire body. Then the mass and momentum conservation laws can be written as

$$\left. \begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{U} &= 0 \\ \rho(\partial_t \mathbf{U} + \mathbf{U} \cdot \nabla \mathbf{U}) &= \rho \mathbf{g} - \nabla \cdot \mathbf{T} \end{aligned} \right\} \quad (1)$$

10 where ∇ is the gradient operator, t is time, ρ is the constant density of water–sediment mixture, \mathbf{U} is the velocity fields, \mathbf{T} is the Cauchy stress tensor, and $\mathbf{g} = (0, 0, g)$ is the gravitational acceleration. The analysis employs a Cartesian coordinate system in which x points in the down-slope direction, y points in the cross-slope direction, and z points upward, normal to the slope (see Fig. 1). Equation (1) can be written explicitly on components

$$\left. \begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} &= 0 \\ \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} &= - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) \\ \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} &= - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right) \\ \frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} &= \rho g - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right) \end{aligned} \right\} \quad (2)$$

where u , v , and w denote Cartesian velocity components in the x , y , and z directions. The model assumes that the water–sediment mixture consists of incompressible solid

grains of mass density ρ_s occupying the volume fraction c mixed with water of mass density ρ_w occupying the volume fraction $1 - c$, such that the water–sediment mixture bulk density is

$$\rho = \rho_w(1 - c) + \rho_s c \quad (3)$$

In the process of the water–sediment mixture movement, the water density and solid phase density are constant. However, affected by entrainment and deposition, the solid phase volume fraction changes with time, and so the density of the water–sediment mixture changes with time accordingly.

$$\frac{\partial \rho}{\partial t} = (\rho_s - \rho_w) \frac{\partial c}{\partial t} \quad (4)$$

The density of the saturated bed can be expressed as

$$\rho_o = \rho_w \rho + \rho_s (1 - \rho) \quad (5)$$

where ρ is bed sediment porosity.

2.2 Boundary kinematic condition

The conservation laws Eqs. (1) and (2) are subject to kinematic boundary conditions at the free surface, $z_t(\mathbf{x}, t) = 0$, and at the base, $z_b(\mathbf{x}, t) = 0$, of the water–sediment mixture (Gray, 2001)

$$z_t(\mathbf{x}, t) = 0 : \frac{\partial z_t}{\partial t} = w_1(z_t) - u_1(z_t) \frac{\partial z_t}{\partial x} - v_1(z_t) \frac{\partial z_t}{\partial y} - R \quad (6)$$

$$z_b(\mathbf{x}, t) = 0 : \frac{\partial z_b}{\partial t} = w_1(z_b) - u_1(z_b) \frac{\partial z_b}{\partial x} - v_1(z_b) \frac{\partial z_b}{\partial y} - \frac{E - D}{1 - \rho} \quad (7)$$

where the superscripts “t” and “b” indicate that a variable is evaluated at the surface and base, respectively, R is the rainfall rate, E, D represent the substrate entrainment and deposition fluxes at the bottom respectively, ρ is bed sediment porosity.

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The difference between the free surface and the base defines the water–sediment mixture thickness

$$h = z_t - z_b \quad (8)$$

5 The free surface of the debris-flows is traction free, that is

$$z_t(\mathbf{x}, t) = 0 : \mathbf{p}_t \mathbf{n}_t = 0 \quad (9)$$

2.3 Depth-integrated equations

10 A key step in further simplifying the equations of motion involves depth averaging to eliminate explicit dependence on z which is the coordinate normal to the bed (Savage and Hutter, 1989; Pitman and Le, 2005).

The depth-averaged solid phase volume fraction, velocities and stress components are defined by

$$\bar{c} = \frac{1}{h} \int_{z_b}^{z_t} c dz, \quad \bar{u} = \frac{1}{h} \int_{z_b}^{z_t} u dz, \quad \bar{v} = \frac{1}{h} \int_{z_b}^{z_t} v dz, \quad \bar{\tau}_{ij} = \frac{1}{h} \int_{z_1}^{z_2} \tau_{ij} dz \quad (10)$$

15 We start by integrating the mass balance equation of the water-sediment mixture in the z direction. Using Leibniz' formula to interchange the differentiation and integration operators, we obtain:

$$\int_{z_b}^{z_t} \left(\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} \right) dz = \frac{\partial \bar{\rho} h}{\partial t} + \frac{\partial \bar{\rho} h \bar{u}}{\partial x} + \frac{\partial \bar{\rho} h \bar{v}}{\partial y} + \rho(z_t) R - \rho(z_b) \frac{E - D}{1 - \rho} = 0 \quad (11)$$

20 Assuming that $\rho(z_t) = \bar{\rho}$ and $\rho(z_b) = \rho_0$, Eq. (11) reduces to

$$\frac{\partial h}{\partial t} + \frac{\partial h \bar{u}}{\partial x} + \frac{\partial h \bar{v}}{\partial y} = \frac{E - D}{1 - \rho} - R \quad (12)$$

The mass balance equation of the solid phase can be written

$$\frac{\partial c\rho_s}{\partial t} + \frac{\partial c\rho_s u}{\partial x} + \frac{\partial c\rho_s v}{\partial y} + \frac{\partial c\rho_s w}{\partial z} = 0 \quad (13)$$

The density of solid phase is constant, Eq. (15) reduce to

$$\frac{\partial c}{\partial t} + \frac{\partial cu}{\partial x} + \frac{\partial cv}{\partial y} + \frac{\partial cw}{\partial z} = 0 \quad (14)$$

Integrating the mass balance equation of the solid phase the z direction. Using Leibniz' formula to interchange the differentiation and integration operators, we obtain

$$\int_{z_b}^{z_t} \left(\frac{\partial c}{\partial t} + \frac{\partial cu}{\partial x} + \frac{\partial cv}{\partial y} + \frac{\partial cw}{\partial z} \right) dz = \frac{\partial \bar{c}h}{\partial t} + \frac{\partial \bar{c}h\bar{u}}{\partial x} + \frac{\partial \bar{c}h\bar{v}}{\partial y} + c(z_t)R - c(z_b) \frac{E-D}{1-\rho} = 0 \quad (15)$$

Assuming that $c(z_t) = \bar{c}$ and $c(z_b) = 1 - \rho$, Eq. (15) reduce to

$$\frac{\partial \bar{c}h}{\partial t} + \frac{\partial \bar{c}h\bar{u}}{\partial x} + \frac{\partial \bar{c}h\bar{v}}{\partial y} = E - D - \bar{c}R \quad (16)$$

The left-hand side of the x momentum Eq. (2) can be written as

$$LHS = \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z}$$

Depth averaging and using boundary conditions yields

$$\int_{z_b}^{z_t} LHS dz = \int_{z_b}^{z_t} \left[\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} \right] dz$$

$$\int_{z_b}^{z_t} LHS dz = \bar{\rho} \frac{\partial(h\bar{u})}{\partial t} + \frac{\partial(h\bar{u}^2)}{\partial x} + \frac{\partial(h\bar{u}\bar{v})}{\partial y} + \bar{\rho} u(z_t) R - \rho_0 u(z_b) \frac{E-D}{1-p} + \frac{\bar{u}(E-D)(\rho_0 - \rho)}{1-p} \quad (17)$$

5 Now, depth averaging the right-hand side of the x momentum Eq. (2) yields

$$\int_{z_b}^{z_t} (RHS) dz = -\frac{\partial(h\bar{\tau}_{xx})}{\partial x} - \frac{\partial(h\bar{\tau}_{xy})}{\partial y} - \tau_{xx}(z_b) \frac{\partial z_b}{\partial x} - \tau_{xy}(z_b) \frac{\partial z_b}{\partial y} - \tau_{xz}(z_b)$$

For the water–sediment mixture, shallow water equations is valid (such an assumption can be considered for low concentrations).

$$10 \quad \bar{\tau}_{xx} = \bar{\tau}_{yy} = \bar{\tau}_{zz} = \frac{1}{2} \rho g h \text{ and } \tau_{xy} = \tau_{yx} = 0$$

$$\int_{z_b}^{z_t} (RHS) dz = -\frac{\partial}{\partial x} \left(\frac{1}{2} \bar{\rho} g h^2 \right) - \bar{\rho} g h \frac{\partial z_b}{\partial x} - g h S_{fx}$$

$$\int_{z_b}^{z_t} (RHS) dz = -\bar{\rho} g h \frac{\partial h}{\partial x} - \frac{1}{2} (\rho_s - \rho_w) g h^2 \frac{\partial c}{\partial x} - \bar{\rho} g h \frac{\partial z_b}{\partial x} - g h S_{fx} \quad (18)$$

$$\frac{\partial(h\bar{u})}{\partial t} + \frac{\partial}{\partial x} \left(h\bar{u}^2 + \frac{1}{2} g h^2 \right) + \frac{\partial(h\bar{u}\bar{v})}{\partial y} = u(z_b) \frac{(E-D)\rho_0}{(1-p)\bar{\rho}} - u(z_t) R - \frac{\bar{u}(E-D)(\rho_0 - \rho)}{(1-p)\bar{\rho}} - \frac{(\rho_s - \rho_w) g h^2}{2\bar{\rho}} \frac{\partial c}{\partial x} - g h \left(\frac{\partial z_b}{\partial x} + S_{fx} \right) \quad (19)$$

The depth-integrated equations for the y momentum component are precisely analogous to those derived above for the x component. Thus, interchanging x and y as well as u and v in the preceding section yields the y component equation

$$\frac{\partial(h\bar{v})}{\partial t} + \frac{\partial(h\bar{u}\bar{v})}{\partial x} + \frac{\partial}{\partial y} \left(h\bar{v}^2 + \frac{1}{2}gh^2 \right) = v(z_b) \frac{(E-D)\rho_0}{(1-\rho)\bar{\rho}} - v(z_t)R - \frac{\bar{u}(E-D)(\rho_0-\rho)}{(1-\rho)\bar{\rho}} - \frac{(\rho_s-\rho_w)gh^2}{2\bar{\rho}} \frac{\partial \bar{c}}{\partial y} - gh \left(\frac{\partial z_b}{\partial y} + S_{fy} \right) \quad (20)$$

The relations between the friction slope and the substrate exchange fluxes are needed to close the governing equations at the boundary. According to the different conditions, different friction slope equations can be selected and the Darcy–Weisbach equation is considered to approximate the overland flow in our model:

$$S_{fx} = \frac{f\bar{u}\sqrt{\bar{u}^2 + \bar{v}^2}}{8gh}, \quad S_{fy} = \frac{f\bar{v}\sqrt{\bar{u}^2 + \bar{v}^2}}{8gh} \quad (21)$$

where f is the Darcy–Weisbach friction factor depends on the Reynolds number. In this paper we consider f as a constant. D and E represent the sediment deposition and entrainment fluxes across the surface between the flow and the bed. ρ is the bed sediment porosity. According to the previous works (Benkhaldoun et al., 2011; Cao et al., 2004), D and E can be expressed as

$$D = \omega_0(1 - C_\alpha)^m C_\alpha \quad (22)$$

$$E = \begin{cases} \phi(\theta - \theta_c) \sqrt{u^2 + v^2} h^{-1} d^{-0.2} & \text{if } \theta \geq \theta_c \\ 0 & \text{else} \end{cases} \quad (23)$$

where ω_0 is a parameter represent the velocity when a single particle settle in tranquil water, which is expressed as

$$\omega_0 = \sqrt{(13.95 \frac{v}{d})^2 + 1.09sgd} - 13.95 \frac{v}{d} \quad (24)$$

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$C_\alpha = c\alpha_c$ is the near-bed volumetric sediment concentration and α_c can be obtained by $\alpha_c = \min(2, (1 - \rho)/c)$, ν is the water kinematic viscosity, d is the grain diameter, m is an exponent, ϕ is a coefficient about the erosion forces, θ_c is a critical value of Shields parameter determining the initiation of sediment motion, $\theta = u^*/(sgd)$ is Shields parameter, $u^* = \sqrt{ghS_f}$ is the friction velocity, $s = \rho_s/\rho_w - 1$ is the submerged specific gravity of sediment.

To show the change of bed-load, the conservation law of the sediment is described by the Exner equation (Murillo et al., 2010; Li et al., 2011)

$$\frac{\partial z_b}{\partial t} = -\frac{E - D}{1 - \rho} \quad (25)$$

2.4 The full model equations

The complete sets of model equations are rewritten here. To simplify notation, hats and over-bars are dropped.

$$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} &= \frac{E-D}{1-\rho} - R \\ \frac{\partial ch}{\partial t} + \frac{\partial chu}{\partial x} + \frac{\partial chv}{\partial y} &= E - D - cR \\ \frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) + \frac{\partial(huv)}{\partial y} &= u(z_b) \frac{(E-D)\rho_0}{(1-\rho)\rho} - u(z_t)R - \frac{u(E-D)(\rho_0-\rho)}{(1-\rho)\rho} \\ &\quad - \frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - gh \left(\frac{\partial z_b}{\partial x} + S_{tx} \right) \\ \frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial}{\partial y} \left(hv^2 + \frac{1}{2}gh^2 \right) &= v(z_b) \frac{(E-D)\rho_0}{(1-\rho)\rho} - v(z_t)R - \frac{v(E-D)(\rho_0-\rho)}{(1-\rho)\rho} \\ &\quad - \frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial y} - gh \left(\frac{\partial z_b}{\partial y} + S_{ty} \right) \end{aligned} \right\} \quad (26)$$

$$\frac{\partial z_b}{\partial t} = -\frac{E-D}{1-\rho}$$

The model Eq. (26) constitutes a system of five equations in the variables $h(x, y, t)$, $u(x, y, t)$, $v(x, y, t)$, $c(x, y, t)$, $z_b(x, y, t)$. Together with initial and boundary conditions, the system constitutes a well-posed set of equations describing a water flow, sediment transport and bed evolution system.

2.5 Discussions

In this paper, based on the shallow water equations, coupled the water flow, sediment transport and bed evolution, to constitute a system model Eq. (26). Equation (26) differs from the conservation equations used in many previous models of the shallow water flow, sediment transport and bed evolution as summarized in Table 1.

3 Computational scheme

In this study, the numerical method chosen to solve Eq. (26) is a finite volume approach which is well suited for transient problems involving erosion and deposition of sediments, and is briefly outlined below. Rainfall is currently not considered in this numerical test, and so Eq. (26) can be reduced as follows

$$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} &= \frac{E-D}{1-p} \\ \frac{\partial ch}{\partial t} + \frac{\partial chu}{\partial x} &= E - D \\ \frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) &= u(z_b) \frac{(E-D)\rho_0}{(1-p)\rho} - u(z_t)R - \\ &\frac{u(E-D)(\rho_0-\rho)}{(1-p)\rho} - \frac{(\rho_s-\rho_w)gh^2}{2\bar{\rho}} \frac{\partial c}{\partial x} - gh \left(\frac{\partial z_b}{\partial x} + S_{tx} \right) \\ \frac{\partial z_b}{\partial t} &= -\frac{E-D}{1-p} \end{aligned} \right\} \quad (27)$$

In order to provide a reasonably compact presentation of the numerical scheme, it is useful to cast Eq. (27) in a more abstract form in vector format as follows

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = S \quad (28)$$

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in which,

$$U = \begin{pmatrix} h \\ hu \\ hc \end{pmatrix}, \quad F(U) = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huc \end{pmatrix},$$

$$S = \begin{pmatrix} u(z_b) \frac{(E-D)\rho_0}{(1-\rho)\rho} - gh \left(\frac{\partial z}{\partial x} + S_f \right) - \frac{(\rho_s - \rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - \frac{(\rho_0 - \rho)(E-D)u}{\rho(1-\rho)} \end{pmatrix}.$$

- 5 The finite volume method is based on the integral form of the conservation Eq. (28). The discretization of the integral form ensures that the basic quantities, mass and momentum will also be conserved across a discontinuity. Fractional step method is used to solve the shallow water equation (SWE) numerically.

Step 1: Solve the homogeneous SWE

$$10 \quad \frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0$$

- A number of approximate Riemann solvers have been constructed to solve the Riemann problem in an efficient manner and Roe's approximation is one of them. In this paper the 1-D Riemann problem at the cell interface is solved using Roe's approximation. In Roe's approximation, the nonlinear problem is linearized at the cell interface. At the cell interface we have a discontinuity with state U_L on the left side and state U_R on the right side. Equation (28) is linearized as follows

$$15 \quad \frac{\partial U}{\partial t} + \tilde{J} \frac{\partial U}{\partial x} = 0 \quad (29)$$

where \tilde{J} is the Roe-linearized normal Jacobian matrix of $F(U)$.

$$\tilde{J} = \begin{bmatrix} 0 & 1 & 0 \\ \tilde{m}^2 - \tilde{u}^2 & 2u & 0 \\ -\tilde{u}\tilde{c} & \tilde{c} & \tilde{u} \end{bmatrix} \quad (30)$$

where $\tilde{h} = \sqrt{h_R h_L}$, $\tilde{u} = \frac{\sqrt{h_R} u_L}{\sqrt{h_R} + \sqrt{h_L}}$, $\tilde{c} = \frac{\sqrt{h_R} c_R + \sqrt{h_L} c_L}{\sqrt{h_R} + \sqrt{h_L}}$, $\tilde{m} = \sqrt{\frac{1}{2}g(h_R + h_L)}$.

Thus, the eigenvalues and the eigenvectors and the weighting factors have the same form as that given by Eqs. (31)–(33)

$$\lambda_1 = \tilde{u} - \tilde{m}, \quad \lambda_2 = \tilde{u}, \quad \lambda_3 = \tilde{u} + \tilde{m} \quad (31)$$

$$\gamma_1 = \begin{bmatrix} 1 \\ \tilde{u} - \tilde{m} \\ \tilde{c} \end{bmatrix}, \quad \gamma_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \gamma_3 = \begin{bmatrix} 1 \\ \tilde{u} + \tilde{m} \\ \tilde{c} \end{bmatrix} \quad (32)$$

$$\alpha_1 = \frac{(h_R - h_L)}{2} - \frac{\tilde{h}(h_R - h_L)}{2\tilde{m}}; \quad \alpha_2 = \tilde{h}(c_R - c_L); \quad \alpha_3 = \frac{(h_R - h_L)}{2} + \frac{\tilde{h}(h_R - h_L)}{2\tilde{m}} \quad (33)$$

Once the cell interface problem has been linearized, it is a simple matter to compute the normal flux F_j .

$$F_{i+\frac{1}{2}} = \frac{1}{2}(F_i + F_{i+1}) - \frac{1}{2} \sum_{j=1}^2 \alpha_j |\lambda_j| \gamma_j \quad (34)$$

So that, the expression for the numerical scheme is

$$U_i^* = U_i^n + \frac{dt}{dx} (F_{i-\frac{1}{2}} - F_{i+\frac{1}{2}}) \quad (35)$$

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After we solve the homogeneous SWE, the conservation law of the sediment can be calculated as the first step.

$$(Z_b)_i^* = (Z_b)_i^n - \frac{E_i^n - D_i^n}{1 - \rho} \quad (36)$$

5 Step 2: Solve the source term

$$\frac{\partial U}{\partial t} = S \quad (37)$$

To reduce numerical instabilities, we adopt the semi-implicit method and the equations are discretized as below:

$$10 \frac{U_i^{n+1} - U_i^*}{\Delta t} = S_i^* \quad (38)$$

where U_i^* , S_i^* are solutions of Step 1.

With the U_i^{n+1} , we can calculate the conservation law of the sediment as the second step.

$$15 (Z_b)_i^{n+1} = (Z_b)_i^* - \frac{E_i^n - D_i^n}{1 - \rho} \quad (39)$$

where E_i^{n+1} , D_i^{n+1} can be solved with U_i^{n+1} .

The stability criterion adopted here is expressed by

$$20 \Delta t \leq \min \left(\frac{\text{CFL} \times \Delta x}{\max(|u| + m)} \right) \quad (40)$$

where $\text{CFL} (\leq 1)$ is Courant number, $m = \sqrt{gh}$, Δx is the distance between the centroid of the cell.

4 Numerical tests

In order to verify the stability of the algorithm and reliability of the numerical results, a classical test case was performed. In this paper, we use the 1-D dam-break problem with an erodible sediment flat bottom. Such dam break problem has been adopted to verify shock-capturing capacity and used extensively by many authors after an analytical solution was proposed by Stoker (1957). Figure 2 shows a schematic diagram of the problem, in this problem we set that the channel length $x = 200$ m and the dam is located at the middle of it ($x = 100$ m). The water depths on both sides of dam include upstream and downstream are $h_1 = 10$ m and $h_2 = 2$ m respectively. The channel bed is composed of non-cohesive uniform sediment and the initial height $z = 0$ m. The computational parameters are listed in Table 2.

As discussed above, the previous models which used non-sliding boundary condition and neglected the entrainment and deposition effect on the momentum equation underestimate the dam-break flow erosion ability and mobility. In reality, due to water erosion, the bottom boundary velocity is not zero (Iverson, 2012, 2013; Le and Pitman, 2009), and can be expressed as: $0 \leq u(z_b) \leq \bar{u}$. So comparisons of the model proposed in this paper ($u(z_b) = \bar{u}$) and that proposed by CAO ($u(z_b) = 0$) are done with the problem of one dimension dam-break over an erodible bed. Figure 3 shows the water depth variations along the channel for a dam-break flow over an erodible bed at time $t = 4$ s, 6 s and 8 s, using the current model and Cao et al. (2004)'s model. As can be expected, the erosion depth, volume fraction of solid in the model proposed in this paper is larger than that in Cao's model, which indicates the erosion can strength the flow and exhibit more erosion ability.

To study the influence of velocity in the bed on the erosion, the three cases with different velocity in bed ($0.25\bar{u}$, $0.5\bar{u}$, $0.75\bar{u}$) are preformed, and the results is shown in Fig. 4. As shown in the figure, the larger the velocity in the bed, the greater the mobility and erosion ability of the flow occurrence. Figure 5 shows the relationship between the maximum erosion depth and the velocity in the bed at different times ($t = 4, 6,$ and

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8 s). It can be seen that the relationship between the maximum erosion depth and the velocity in the bed is almost linear.

5 Conclusions

This paper presents a new coupled model of water flow, sediment transport and bed evolution based on the shallow water assumption, depth-average integration as well as the morphological evolution. Compared to the previous models, the present model takes account of the effects of rainfall, entrainment and deposition on the motion of the water–sediment mixture. In this model we perform numerical simulation by the finite volume method and the Roe’s approximate Riemann solver has been adopted. The results has been simulated are compared with the model of Cao et al. (2004) . Potential model to verify the veracity of our model and the feasibility of applying our model to some events such as channel development caused by rainfall, sediment transport caused by tsunami and/or floods.

One of the most important advantages that the formulation presented in this paper offers over many existing models is that the influence of the entrainment and deposition on momentum equation are considered. The one-dimensional dam-break problem is performed to validate the model. Comparisons are done with the model by Cao et al. (2004), and the influence of the velocity in bed on the erosion is analyzed. Through the analysis of numerical solutions simulated according to the actual scales and some model functions, we sum some conclusions as following:

- (1) The entrainment and deposition can enhance the mobility and erosion ability of water flow.
- (2) The influence of the velocity in the bed on the erosion is important, with increasing of the velocity in bed, the depth of erosion, the volume fraction of solid, and the flow depth increase significantly.

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(3) The relationship between the maximum erosion depth and the velocity in the bed is almost a linear one.

6 Code availability

A Matlab based function is provided to illustrate the presented model. This simple function is programmed only for the examples in the paper. The code has been uploaded with this submission and can also be obtained by contact Wei Liu (sponlol@163.com).

Supplementary material related to this article is available online at
[http://www.geosci-model-dev-discuss.net/7/2429/2014/
gmdd-7-2429-2014-supplement.zip](http://www.geosci-model-dev-discuss.net/7/2429/2014/gmdd-7-2429-2014-supplement.zip).

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References

- Benkhaldoun, F., Sahmim, S., and Seaid, M.: A two-dimensional finite volume morphodynamic model on unstructured triangular grids, *Int. J. Numer. Meth. Fl.*, 63, 1296–1327, doi:10.1002/fld.2129, 2011.
- Benkhaldoun, F., Sari, S., and Seaid, M.: A flux-limiter method for dam-break flows over erodible sediment beds, *Applied Mathematical Modelling*, 36, 4847–4861, doi:10.1016/j.apm.2011.11.088, 2012.

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- Cao, Z., Pender, G., Wallis, S., and Carling, P.: Computational dam-break hydraulics over erodible sediment bed, *J. Hydraul. Eng.*, 130, 689–703, doi:10.1061/(ASCE)0733-9429(2004)130:7(689), 2004.
- Cao, Z., Gareth, P., and Paul, C.: Shallow water hydrodynamic models for hyperconcentrated sediment-laden floods over erodible bed, *Adv. Water Resour.*, 29, 546–557, doi:10.1016/j.advwatres.2005.06.011, 2006.
- Fagherazzi, S. and Sun, T.: Numerical simulations of transportational cyclic steps, *Comput. Geosci.*, 29, 1143–1154, doi:10.1016/S0098-3004(03)00133-X, 2003.
- Fraccarollo, L. and Capart, H.: Riemann wave description of erosional dam-break flows, *J. Fluid Mech.*, 461, 183–228, doi:10.1017/S0022112002008455, 2002.
- Gray, J. M. N. T.: Granular flow in partially filled slowly rotating drums, *J. Fluid Mech.*, 441, 1–29, doi:10.1017/S0022112001004736, 2001.
- Hu, P. and Cao, Z.: Fully coupled mathematical modeling of turbidity currents over erodible bed, *Adv. Water Resour.*, 32, 1–15, doi:10.1016/j.advwatres.2008.07.018, 2009.
- Iverson, R. M.: Elementary theory of bed–sediment entrainment by debris flows and avalanches, *J. Geophys. Res.*, 117, F03006, doi:10.1029/2011JF002189, 2012.
- Iverson, R. M. and Ouyang, C.: Bed-sediment entrainment by rapidly evolving flows at Earth's surface: 1 review and reformulation of depth-integrated theory, *Rev. Geophys.*, submitted, 2013
- Li, S. and Duffy, C. J.: Fully coupled approach to modeling shallow water flow, sediment transport, and bed evolution in rivers, *Water Resour. Res.*, 47, W03508, doi:10.1029/2010WR009751, 2011.
- Le, L. and Pitman, E. B.: A model for granular flows over an erodible surface, *SIAM J. Appl. Math.*, 70, 1407–1427, doi:10.1137/060677501, 2009.
- Murillo, J. and García-Navarr, P.: An Exner-based coupled model for two-dimensional transient flow over erodible bed, *J. Comput. Phys.*, 229, 8704–873, doi:10.1016/j.jcp.2010.08.006, 2010.
- Pitman, E. B. and Le, L.: A two-fluid model for avalanche and debris flows, *Philos. T. Roy. Soc. A*, 363, 1573–1602, doi:10.1098/rsta.2005.1596, 2005.
- Savage, S. B. and Hutter, K.: The motion of a finite mass of granular material down a rough incline, *J. Fluid Mech.*, 199, 177–21, doi:10.1017/S0022112089000340, 1989.
- Schippa, L. and Pavan, S.: Bed evolution numerical model for rapidly varying flow in natural streams, *Comput. Geosci.*, 35, 390–40, doi:10.1016/j.cageo.2008.08.004, 2009.

Simpson, G. and Castelltort, S.: Coupled model of surface water flow, sediment transport and morphological evolution, *Comput. Geosci.*, 32, 1600–1614, doi:10.1016/j.cageo.2006.02.020, 2006.

5 Stocker, J. J.: *Water Waves, the Mathematical Theory with Applications*, Wiley Interscience, New York, 1957.

Tassi, P. A., Rhebergen, S., Vionnet, C. A., and Bokhove, O.: A discontinuous Galerkin finite element model for river bed evolution under shallow flows, *Comput. Method. Appl. M.*, 197, 2930–2947, doi:10.1016/j.cma.2008.01.023, 2008.

10 Xia, J., Lin, B., Falconer, R. A., and Wang, G.: Modelling dam-break flows over mobile beds using a 2-D coupled approach, *Adv. Water Resour.*, 33, 171–18, doi:10.1016/j.advwatres.2009.11.004, 2010.

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Table 1. Comparisons of other authors' conservation equation with Eq. (26) in this work.

Authors	Conservation equation	Limitation
Cao et al. (2004, 2006)	$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} &= \frac{E-D}{1-p} \\ \frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) &= gh \left(-\frac{\partial z}{\partial x} - S_f \right) \\ -\frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - \frac{(\rho_0-\rho)(E-D)u}{\rho(1-p)} & \\ \frac{\partial(hc)}{\partial t} + \frac{\partial(huc)}{\partial x} &= E - D \\ \frac{\partial z}{\partial t} &= \frac{D-E}{1-p} \end{aligned} \right\}$	<p>(1) One-dimension model (2) Rainfall is not considered (3) The effects of rainfall and entrainment deposition, $u(z_b) \frac{(E-D)\rho_0}{(1-p)\rho} - u(z_1)R$ are not considered in the momentum equation.</p>
Benkhaloud (2011)	$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} &= \frac{E-D}{1-p} \\ \frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) + \frac{\partial(huv)}{\partial y} &= gh \left(-\frac{\partial z}{\partial x} - S_f^x \right) \\ -\frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - \frac{(\rho_0-\rho)(E-D)u}{\rho(1-p)} & \\ \frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial}{\partial y} \left(hv^2 + \frac{1}{2}gh^2 \right) &= gh \left(-\frac{\partial z}{\partial y} - S_f^y \right) \\ -\frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial y} - \frac{(\rho_0-\rho)(E-D)v}{\rho(1-p)} & \\ \frac{\partial(hc)}{\partial t} + \frac{\partial(huc)}{\partial x} + \frac{\partial(hvc)}{\partial y} &= E - D \\ \frac{\partial z}{\partial t} + \frac{A_s}{1-p} \frac{\partial}{\partial x} \left[u \left(u^2 + v_2 \right) \right] + \frac{A_s}{1-p} \frac{\partial}{\partial y} \left[v \left(u^2 + v_2 \right) \right] &= \frac{D-E}{1-p} \end{aligned} \right\}$	<p>(1) Rainfall is not considered (2) The effects of rainfall and entrainment deposition in x direction, $u(z_b) \frac{(E-D)\rho_0}{(1-p)\rho} - u(z_1)R$ and y direction, $v(z_b) \frac{(E-D)\rho_0}{(1-p)\rho} - v(z_1)R$ are not considered in the momentum equations.</p>
Simpson (2006)	$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} &= \alpha + \frac{E-D}{1-p} \\ \frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) + \frac{\partial(huv)}{\partial y} &= gh \left(-\frac{\partial z}{\partial x} - S_f^x \right) \\ + \mathcal{E} \left[\frac{\partial^2(hu)}{\partial^2 x} + \frac{\partial^2(hu)}{\partial^2 y} \right] - \frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - \frac{(\rho_0-\rho)(E-D)u}{\rho(1-p)} & \\ \frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial}{\partial y} \left(hv^2 + \frac{1}{2}gh^2 \right) &= gh \left(-\frac{\partial z}{\partial y} - S_f^y \right) \\ + \mathcal{E} \left[\frac{\partial^2(hv)}{\partial^2 x} + \frac{\partial^2(hv)}{\partial^2 y} \right] - \frac{(\rho_s-\rho_w)gh^2}{2\rho} \frac{\partial c}{\partial y} - \frac{(\rho_0-\rho)(E-D)v}{\rho(1-p)} & \\ \frac{\partial(hc)}{\partial t} + \frac{\partial(huc)}{\partial x} + \frac{\partial(hvc)}{\partial y} &= E - D \\ \frac{\partial z}{\partial t} &= \frac{D-E}{1-p} \end{aligned} \right\}$	<p>The effects of entrainment and deposition in x direction, $u(z_b) \frac{(E-D)\rho_0}{(1-p)\rho} - u(z_1)R$ and y direction, $v(z_b) \frac{(E-D)\rho_0}{(1-p)\rho} - v(z_1)R$ are not considered in the momentum equations.</p>



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**Table 2.** Computational parameters of dam-break over erodible bed.

Coefficients	n	d (mm)	ρ_s (kgm^{-3})	ρ_w (kgm^{-3})	m	ν	ρ	ϕ	θ_c
Values	0.03	8	2840	1000	2	1.2×10^{-6}	0.4	0.015	0.045

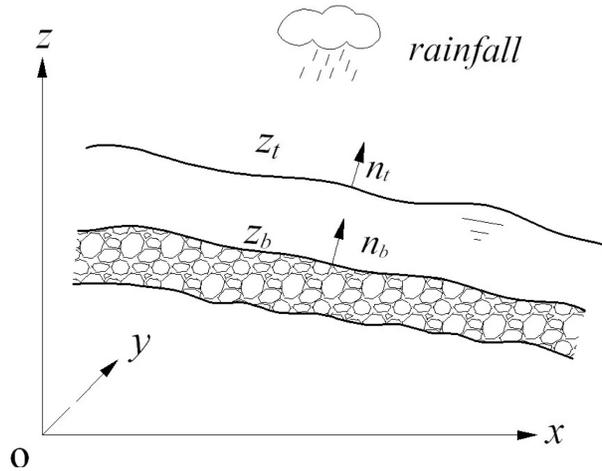


Fig. 1. Schematic depictions of a moving sediment-water layer that can exchange mass and momentum with a static bed layer under rainfall.

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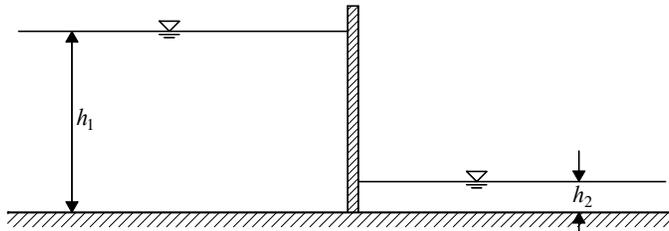


Fig. 2. Schematic diagram of a dam-break flow over erodible bed.

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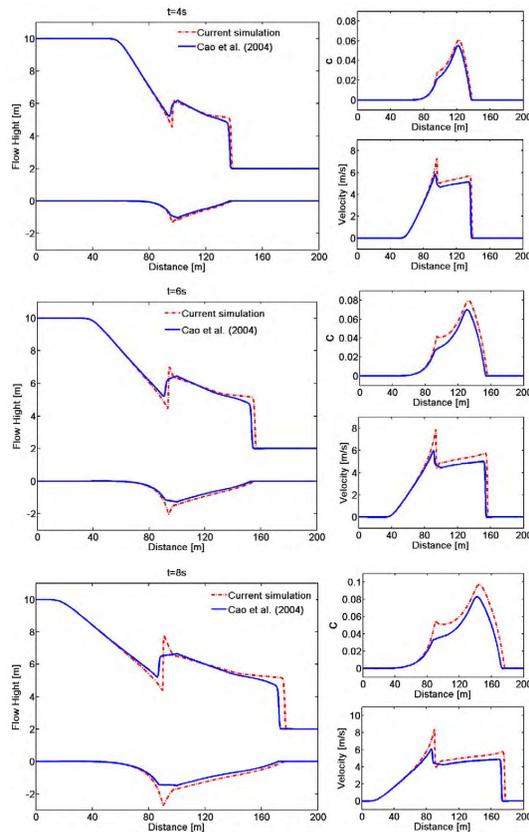


Fig. 3. The profiles of free surface, erodible bed, volume fraction of solid C and flow velocity at time $t = 4, 6$, and 8 s, respectively. Comparisons of current model results with that of Cao et al. (2004).

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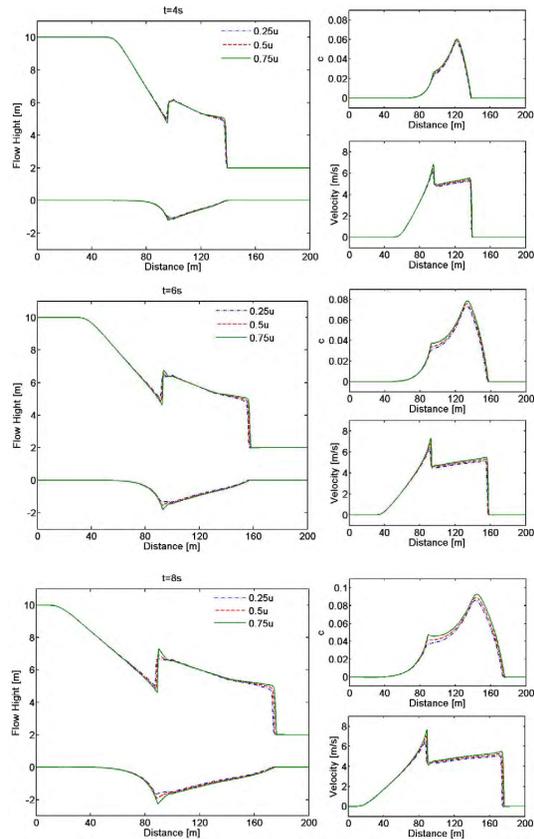


Fig. 4. Comparison of results of 1-D dam-break over an erodible bed with different velocity in the bed.

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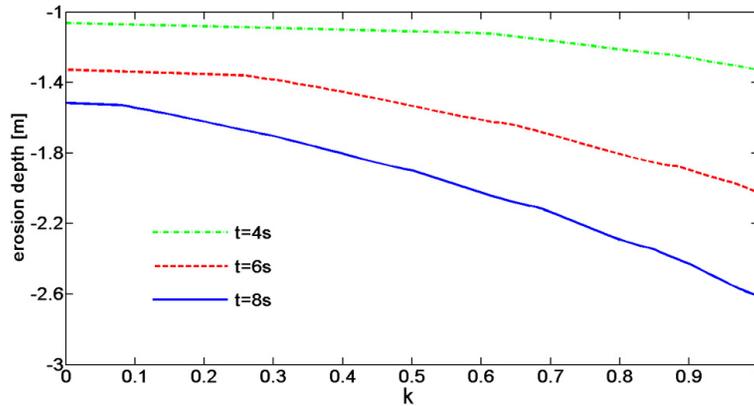


Fig. 5. The relationship between the maximum depth of erosion and the velocity in the bed.

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