Responses to Anonymous Referee #1 for gmd-2014-47: "On the sensitivity of 3-D thermal convection codes to numerical discretization: a model intercomparison" by P.-A. Arrial et al.

Anonymous Referee #1

The authors have provided a comparison between two numerical techniques, demonstrating differences in stability behaviour with respect to different initial conditions and Rayleigh number. These are interesting & important results highlighting how dynamics are influenced by choice of discretisation.
While touched upon in section 4, I felt that more could have been said about how resolution influenced simulation results relative to Rayleigh number. Perhaps a spherical harmonic power spectrum would be revealing, especially for more convective simulations where discretisation choice will more strongly impart on dynamics. For the dodecahedron initial condition, it does appear that the simulations tend towards different stable solutions regardless of resolution. This is perhaps owing to different stability behaviour for various modes (initially appearing as noise). It might be interesting to see if the CitcomS simulation can be persuaded towards the tetrahedron solution by seeding the simulation with some small amount of the required mode.

Many possible avenues for further investigation, though from my perspective the significance of different order FEM elements and alternate meshes would be interesting, as would comparison with other community codes to see if some behaviour consensus was observed.

Some very minor corrections:

Section 6, Line 16: As mentioned earlier in your report, CitcomS is a finite element code, not finite volume.
Figure 5: label within images states \delta=0.08 (citcom) & \delta = 0.09 (RBF), while dialog states that both have \delta=0.09
Figure 12: The labelling listed in the dialog seems inconsistent with the images.

Response:

Thank you very much for your helpful comments and corrections. We are going to answer these comments and provide some revisions for the paper.
Your comments focus on the dodecahedral pattern and the reasons that contribute to its destabilization. We think that there is no definitive answer to figure out which parameter influences the destabilization and the final convection pattern. However, we agree with you that the most probabilistic factor is the location of numerical noise or error that perturbs the position of the plumes and lead to different final convection patterns. We can provide more information and description of the results and how the destabilization occurs to highlight this hypothesis, especially for CitcomS. We propose to add the following paragraph on page 2044 after line 9.

For CitcomS, the mesh discretization shows a symmetrical effect. The shell is initially divided into 12 caps. Each cap is diametrically opposite to another one (Zhong et al., 2000). Thus during the transition, we can observe that destabilization occurs in symmetrical pairs with respect to the caps. As a result it is reasonable to presume that mesh discretization and the cap divisions influence the distribution of numerical errors and favor even modes. In these conditions, CitcomS won't reproduce the tetrahedral or the five-cell pattern observed with the RBF method, without adding an additional initial perturbation representing these odd modes.

You also suggested persuading CitcomS to converge to the tetrahedral pattern by adding an additional tetrahedral perturbation. We agree with you that a modification of the initial condition could modify the final convection pattern counterbalancing the numerical noise. However, one cannot conclude what is the best final pattern of convection nor is there a preference for one or the other. We could also try to impose an additional cubic perturbation to the RBF models or even a five-cell perturbation, and see what happens. There are many possibilities to investigate. The original goal of this paper was to compare the results with the same initial conditions, this is why we are not going to present other models for this part of the paper.
However, we also agree with your last comment and we encourage other codes to try to reproduce these results and see if a consensus can be reached. We added the following sentence to the page 2048 line 27:
We hope that this paper will stimulate further investigation on how the type and order of numerical discretization affects pattern formation in the context on benchmarking community codes.

Minor corrections
1) Section 6 line 16: Indeed, CitcomS is a finite element code. We apply this correction in the text.

2) Figure 5: The labels in the image are correct and we modified the caption accordinglyto it:
Fig5. Time trace of (a) the outer Nusselt number and (b) the RMS Velocity <u>*for Ra=70000*</u>

and delta=0.08 with CitcomS and \delta=0.09 with RBF. Both methods converge to an unsteady oscillating axisymmetric pattern dominated by the \ell = 2 mode (see Fig. 4).

3) Figure 12: In the caption the references to the final convection pattern need to be shifted from (b-d) to (c-e). The new caption is:

Fig. 12. Time traces of the evolution of the average temperature as a function of the parameter at Ra = 7000 for (a) the RBF-PS model and (b) CitcomS. (c–e) show the final convection patterns for each of these models with (c)\gamma =0, (d) \gamma=0.5, and (e)\gamma=1.0.

Other corrections:

- Page 2044 line 10: the section 4 title (Stability at higher order of symmetry) is not at its right place and must move to page 2043 line 13. The statement and description of the dodecahedral initial condition begin at this line.
- Page 2043 line 23: *an dodecahedral* becomes *a dodecahedral.*

Responses to Anonymous Referee #2 for gmd-2014-47: "On the sensitivity of 3-D thermal convection codes to numerical discretization: a model intercomparison" by P.-A. Arrial et al.

Anonymous Referee #2

General comments:

1) The authors presented an interesting piece of work in this paper. Using two different numerical codes/methods, CitcomS and RBF, the authors systematically explore the sensitivities of solutions of 3D spherical thermal convection to initial perturbations for relatively small Ra. This work reminds us the complexity of non-linear thermal convection in spherical geometry even at relatively small Ra. It also shows the importance and need of good benchmark studies, as mantle dynamics community has built more numerical codes and tools – the cases presented here can be used in future studies for benchmarks.

2) Some of interesting results from this study can be summarized as follows. It appears that results from RBF and CitcomS agree reasonably well in general, but for certain Ra and initial conditions, convection converges to different state. The study presents two such examples: A) low order symmetry case of cubic initial condition and B) high order symmetry case of dodecahedral initial condition. For cubic condition, while both RBF and CitcomS gave the same convective patterns for a large range of perturbation amplitude, the transitional states that occur for a small range of perturbation amplitude are different for these two methods. For dodecahedral initial condition, the first stationary and steady state dodecahedral pattern is nearly the same for these two methods, the second stationary state differs between the two methods and is also dependent on the initial perturbation amplitude.

Specific comments:

1) Can the authors add some comments on any possible or significant differences between CitcomS and RBF for RMS velocity, <T> and Nu in Figure 10 BEFORE the transition? Figure 8 seems to suggest that the RMS velocity is nearly the same between the two methods before the transition starts. Can we say the same for the difference cases in Figure 10?

2) It would be useful for the authors to make some general comments on how readers may evaluate convection results in general, even if they may be rather speculative. Should we take the current results as indication that convective patterns, RMS velocity, and averaged temperature and heat flux are not reliable and code-dependent? or we only need to be concerned for models with certain parameters and states (e.g., relatively small Ra, isoviscous convection, : : :)? My reading of the paper is that CitcomS and RBF for isoviscous thermal convection in spherical geometry mostly agree with each. However, it would be helpful for the authors to be more specific about this topic. I suspect that RMS does not work for variable viscosity (e.g., temperature-dependent viscosity) yet. Would variable viscosity convection show similar complexity or transitional states, depending

on initial state or Ra? How about Ra reaching Earth-like?

Technical corrections:

1) Page 2038, line 17, " : : : in 3-D spherical geometry (Moresi and Solomatov, 1995, Zhong et al., 2000; Tan et al. 2006)." Moresi and Solomatov (1995) was for 2-D Cartesian model Citcom, and CitcomS was first published in Zhong et al. (2000). Perhaps, it is better to move "Moresi and Solomatov, 1995" to the next line as the reference to Citcom.

2) Figure 5. It was stated in the figure caption that delta=0.09 for both methods, but in figure legends, delta=0.08 for CitcomS and 0.09 for RBF. Clarify them.

3) It seems that Fig. 10 was referenced before Fig. 9 in page 2044. Reorder them?

4) Figure 12 caption includes two b). Also, in line 23 of page 2046, figure 12e was incorrectly referenced for 5-cell pattern, but the 5-cell pattern is in figure 12d.

5) Page 2047, line 13, add "to" before "highlight".

6) Page 2047, line 16, replace "finite volume" with "finite element".

Response:

Thank you very much for your helpful comments and corrections. We answer to your comments in order:

Specific comments:

1) This is a very interesting comment. Indeed, we did not extensively describe and compare the dodecahedral stationary pattern between methods before the transition. As you observed in figure 8, the RMS velocity matches perfectly at a Rayleigh number of 7000. The same comparison and observation can be done with other Rayleigh numbers and parameters (Nusselt numbers, average temperature). We suggest adding the following text and table in page 2044 line 14:

In all cases of the Ra in Figure 9, the dodecahedral convection pattern is initially observed and stationary. This pattern is identical in both methods, whether one considers its geometry, the convergence of RMS velocity, average temperature or Nusselt Numbers before the transition (Tab. 1).

2) We do think that the current results are reliable. Results show a really good match for the cubic and five-cell steady states and the stationary dodecahedral pattern. We point out that when models are close to the transition, the destabilization can be influenced by the the discretization scheme of the method. As a reply to the first referee, we added a supplementary description of the destabilization of the dodecahedral pattern for CitcomS (see response to referee #1). We agree that the two methods mostly agree at some point and we modified the conclusion to make precise this point: page 2048, line 17

As a general observation, both methods show a good match on the cubic and five cell steady state patterns, and even for the stationary dodecahedral pattern before the transition. However, we hope that the above in depth computational study strongly illustrates how numerical discretization can impact both the resulting patterns of convection as well as the transitional states that occur.

At this time, RBF method has not incorporated variable viscosity. We did not explore the sensitivity of these models to temperature dependent-viscosity. We know that this kind of viscosity is probably going to modify the geometry of the final convection patterns and could possibly improve the stability of the dodecahedral pattern, for example. Reese et al, 1999 (*Phys. Earth Planet. Int.*) used a dodecahedral initial condition with a temperature dependent viscosity and investigated various viscosity ratios. However, the authors did not state the length of time integration. The results suggest that this pattern can reach a steady state with a variable viscosity. However, as the RBF method showed, the dodecahedral pattern can look steady but is actually destabilized to a lower order of symmetry. It would be really interesting to investigate the behavior of the models with variable viscosity.

For Rayleigh number higher than about 2(10^5), models become strongly time dependent, entering a turbulent regime, where symmetry is completely lost. Thus, at such high Ra, it is impossible to highlight the influence of numerical discretization on the resulting patterns of convection.

Technical corrections:

1) Indeed, the reference of (Moresi and Solomatov, 1995) would be better as a Citcom reference. We move this reference to the next line.

Page 2038, line 18: *Developed from the software Citcom (Moresi and Solomatov, 1995; Moresi et al., 1996), a code structured for 3D Cartesian geometry, CitcomS employs an Uzawa algorithm to solve the momentum equation coupled with the incompressibility constraints (Ramage and Wathen, 1994).*

2) The labels in the image are correct and we modify the caption according to it.

Fig5. Time trace of (a) the outer Nusselt number and (b) the RMS Velocity for Ra=70000 and delta=0.08 with CitcomS and \delta=0.09 with RBF. Both methods converge to an unsteady oscillating axisymmetric pattern dominated by the \ell = 2 mode (see Fig. 4).

3) The figure order is modified to follow the citation order. Thus the figure with Nusselt number and RMS velocity time traces becomes figure 9 and the figure with the final convection patterns for the dodecahedral test case becomes figure 10.

4) In the caption the references to the final convection pattern need to be shifted from (b-d) to (c-e). The new caption is:

Fig. 12. Time traces of the evolution of the average temperature as a function of the parameter at Ra = 7000 for (a) the RBF-PS model and (b) CitcomS. (c–e) show the final convection patterns for each of these models with (c)\gamma =0, (d) \gamma=0.5, and (e)\gamma=1.0.

5) We added “to” before “highlight” on line 13, page 2047

6) We changed “finite volume“ to “finite element” on line 16, page 2047.