Author's response

We are grateful to the reviewer for his/her careful reading of the manuscript and his/her thoughtful comments. We have followed all suggestions in all cases, and provide below a point-by-point response to the reviews. These are corrected in revised manuscript. This feedback has helped us improve and clarify the manuscript. Response to each referee is attached in the interactive discussion webpage, and here we add them with overall change.

We have provided the example of developed code as a supplement, which can be found in the interactive discussion webpage. The example tangent linear (TL) code in figure 2 and corresponding text in section 2.2 are revised following Fortran syntax. The validity and effectiveness of increasing step size for TLM is discussed and these results are included in the revised manuscript. Also, the code availability is stated in the revised manuscript. In current manuscript, discussion on the automatic differentiation tools is omitted for future study of comparison to hand-made codes of ADM.

Reponses to Anonymous Referee #1

(1) As far as I know HOMME has two basic formulations which are based on SE and DG methods, however, both rely on same grid system and parallel communications. But the treatment of flux at the element edges and numerical viscosity are based on different ideas. On page 1179, lines 10-15, authors mention that they employ DG formulation for developing a TLM. However, hyper-viscosity filter is based on SE method not the part of DG discretization. Please clarify how you apply hyper-viscosity. Is your TLM formulation based on SE or DG?

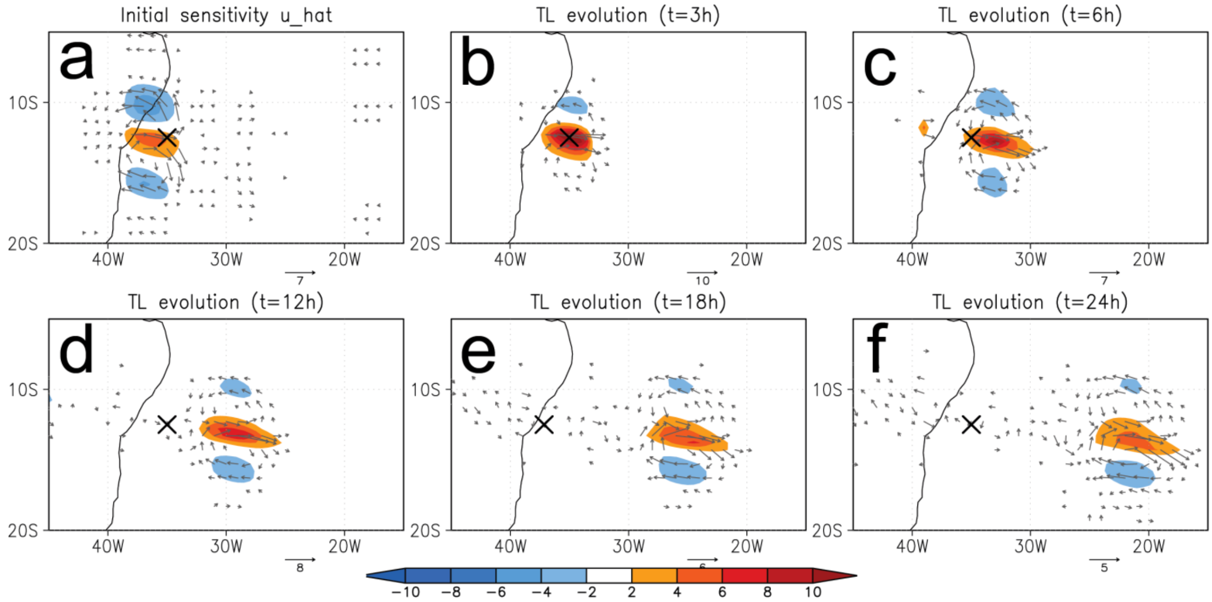
> **Authors' response**: Thanks for pointing out the crucial issue. As you mentioned, HOMME has two dynamic formula, the spectral element (SE; or continuous galerkin, CG) and discontinuous galerkin (DG). We have developed the tangent linear model for SE method. So, we omit the word of *DG* in the sentence, in order to avoid the confusion.
>
> Thus "… and DG method of the prognostic variables." is removed in section 2.1. And "For this study, the conservative 3 dimensional DG model is configured…" is change as "For this study, the conservative 3 dimensional CG model is configured…" in section 2.1.

(2) *On page 1184, lines 10-15: There are several basic operators such as div, curl, grad etc. involved in RHS estimation and hyper-viscosity computations. It will be interesting provide an example about your "hands-on" linearization for one of these operators at least, say "divergence_sphere", how does the liner version appear in terms of mathematical expressions? How do you handle the metric terms?*

> **Authors' response**: Thanks for your interest of how we coded the individual subroutines of HOMME. The subroutine of "*divergence_sphere*" is a linear routine, thus the corresponding TL code is not much interesting. Instead, the original nonlinear modules and developed tangent linear (TL) routines for "*divergence_sphere*" and "*preq_vertadv*" are attached in supplement. For example,"*preq_vertadv*" has a nonlinear part that is represent vertical advection terms. And, there are many "linearized" terms in its TL code. When developing TL codes, the linearizations are done with respect to the model state variables (i.e., prognostic variables) and their dependent variables. Thus, the metric terms are kept constant for TL code. These supplementary codes can be found in the interactive discussion webpage.

(3) *Did you ever test the TLM formulation in a simplified framework such as the spherical SW system, which is available in HOMME? If so, please provide some computational examples with one of the SW test cases.*

Authors' response: Yes. we have developed the TLM for global spectral element shallow water (SW) system before, and also developed the representer-based variational data assimilation system including the adjoint model (ADM) and simple background error covariance model for SW system. The draft for this development is recently finished and submitted for referred journal "Tellus A" with title of "*Representer-based variational data assimilation in a spectral element shallow water model on the cubed-sphere grid*". It can be provided, if the referee request. Below we attached Figure 3 of the submitted draft, which shows the spreads of observation information with linear dynamic models (i.e., ADM and TLM). The adjoint sensitivity at initial time (Fig. a) related to the assumed single zonal wind observation spreads out with TLM integration. Note that the subsequent evolutions (Fig. b-f) of geopotential height fields and wind vector fields are dynamically related with a geostrophic balance.



Caption for figure3. Spreads of observation information with the color shadings. (a) The structure of the initial sensitivity (time t =0) obtained by integrating ADM backward in time, and the zonal wind observation (×) is initially located at (35^{o}W, 12.5^{o}S) at 3 hour. Then, TLM is integrated with the initial sensitivity as the initial condition at time (b) 3, (c) 6, (d) 12, (e) 18, and (f) 24 hours, respectively. Arrows represent the wind vectors.

Minor Point: Abstract Line 5, "spectral element method"

Authors' response: Thank you. We were not able to find this typo in our version. This is corrected in final revised manuscript.

Reponses to Anonymous Referee #2

** In section 2.1: does "advanced time stepping" mean "adaptive time stepping"? You also mention that HOMME uses adaptive mesh refinement. Can you discuss the impact of these techniques on the accuracy of the linearization? Did you try running the TLM of an adaptive simulation? I don't think adaptivity was used in combination with the TLM, but I think this should be explicitly clarified.*

> **Authors' response**: As the reviewer point out, adaptive mesh refinement and time stepping are not used for TLM. To make it clearer, the following text is added and now reading in section the 2nd paragraph in section 2.1:
>
> "… Note that although the HOMME uses adaptive time stepping and adaptive mesh refinement, its TLM does not include such functions. ….

** Figure 2 is confusing: annotating the input and output variables with (I) and (O) isn't sensible fortran. (For example, gfortran errors with "Unexpected junk in formal argument list".) Fortran has a built-in mechanism for indicating intent, with the intent(in), intent(out) and intent(inout) attributes. I suggest that figure 2 be changed to use these instead, and to drop the unnecessarily novel (I)/(O) notation.*

> **Authors' response**: As the reviewer suggested, figure 2 is changed and followed the Fortran syntax, and the corresponding text is also changed in the 2nd paragraph in section 2.2.

```
Subroutine NL( a, b, tens )
real, intent(in) :: a, b
real, intent(out) :: tens
real :: tmp

tmp  = 3.0d0 * sin(a)
tens = tmp * b**2
End subroutine NL
```

```
Subroutine TL( a, b, tl_a, tl_b, tl_tens )
real, intent(in) :: a, b, tl_a, tl_b
real, intent(out) :: tl_tens
real :: tmp, tl_tmp

tl_tmp  = 3.0d0 * cos(a) * tl_a
tmp     = 3.0d0 * sin(a)
tl_tens = tl_tmp * b**2 + tmp * 2.0d0 * b * tl_b
End subroutine TL
```

** Section 2.4: "the high computational cost is extremely burdensome". This paper would be greatly improved with timing results — computational efficiency is crucial in 4DVAR applications of TLMs. Given that HOMME is explicit, the cost of (TLM + NLM) should be approximately twice that of the NLM, without any tricks (changing timesteps, or interpolating the original trajectory in time). Furthermore, the value of the tricks should be quantified by relating the error in the Taylor remainder (does the value of the LHS of* *equation (2) stay close to 1) against the computational savings (in seconds).*

Authors' response: Thanks for the excellent suggestions. We quantified total Wallclock run time and the mean relative error, defined, for different time steps, as

$$\frac{\left\| \mathbf{X}_{TL\_\Delta t} - \mathbf{X}_{NLD} \right\|}{\left\| \mathbf{X}_{NLD} \right\|}, \frac{\left\| \mathbf{X}_{TL\_2\Delta t} - \mathbf{X}_{NLD} \right\|}{\left\| \mathbf{X}_{NLD} \right\|}, \frac{\left\| \mathbf{X}_{TL\_3\Delta t} - \mathbf{X}_{NLD} \right\|}{\left\| \mathbf{X}_{NLD} \right\|}, \frac{\left\| \mathbf{X}_{TL\_4\Delta t} - \mathbf{X}_{NLD} \right\|}{\left\| \mathbf{X}_{NLD} \right\|}$$

where $\mathbf{X}_{TL}$ is a TLM field at T=5 hour, $\mathbf{X}_{NLD}$ is the corresponding difference fields between the two nonlinear model forecasts at 5 hour, and || || is a spatial averaged norm.

The following table gives these values for the mean relative error at time T=5 hour:

Variable	**1*Δt**	**2*Δt**	**3*Δt**	**4*Δt**
u	0.0124556	0.0128355	0.0135081	0.163502
v	0.0128028	0.0120578	0.0115803	0.13647
t	0.00696689	0.00650514	0.00596657	0.104771
ps	0.00697304	0.00639369	0.00547336	0.0750567

Also, we evaluated the total wallclock run time (second) for two days simulation and showed below:

Total Wallclock time for 5 hour forecast		
1*Δt	55.490	100%
2*Δt	31.249	56%
3*Δt	20.204	36%
4*Δt	18.080	33%

We added the above results in the manuscripts, and the 2nd paragraph in section 3.3 is now reading:

"…. This can be confirmed quantitatively by considering the relative mean error, defined, for any quantity $\boldsymbol{X}$ at the time T=5 hr, as

$$\| \boldsymbol{X}_{\mathrm{TLM}} - \boldsymbol{X}_{\mathrm{NLD}} \| / \| \boldsymbol{X}_{\mathrm{NLD}} \|, \tag{3}$$

where $\boldsymbol{X}_{\mathrm{TLM}}$ is a TLM field at T=5 hour, $\boldsymbol{X}_{\mathrm{NLD}}$ is the corresponding difference fields between t he two nonlinear model forecasts at 5 hour, and || || is a spatial averaged norm. Table 1 gives these values for the mean of the stat variable $\boldsymbol{X}$ at time T=5 hr. And the total wallclock time i s decreased, as the time step size is increased such that when Δt=150s is set to be 100%, 2Δt becomes 56%, 3Δt is 36%, and 4Δt for 33%...."

** The section on TAPENADE (note the misspelling in the manuscript) is quite strange. It's simply wrong that TAPENADE doesn't handle branches (see, e.g., figure 7 of 10.1145/2450153.2450158 for an example of TAPENADE applied to code with a branch). I strongly suggest that you consult with the developers of TAPENADE and make a fair comparison with AD tools (in particular in terms of computational efficiency). I expect that the hand-coded TLM will be quite a bit more efficient; quantifying this would improve the scientific value of the manuscript greatly, as it would give future developers of TLMs of other models in this* *area an idea of the tradeoffs.*

Authors' response: We agree with the reviewer in that many evaluations are necessary for fair comparison of TAPENADE-generated and hand-coded TLM. We leave this issue for the next study of ADM development and omit this paragraph in current version.

** The English used in the manuscript could be improved (for example, many articles are missing). I suggest having a native anglophone editor review the text and correct the minor grammatical errors.*

Authors' response: Done as suggested.

** The GMD manuscript types state that 'All papers must include a section at the end of the paper entitled "Code availability".' The code availability is discussed, but it's not in its own section as the journal requires. I suggest you review the discussion of the manuscript types and reformat appropriately.*

Authors' response: "Code availability" is now added in section 6, and reads:

"All codes in the current version of TLM are available upon the request. Any potential user interested in those modules should contact B.-J. Jung, and any feedback on them is welcome. Note that one may need help to use the TLM model optimally, but we do not have the resources to support the model in an open way. Since ADM is currently being developed based on the current version of TLM, all codes of ADM are also presumably available upon the request."

** Will the linearized version (and presumably, the future adjoint version) of HOMME be distributed with HOMME? I didn't see this mentioned in the text.*

Authors' response: This is discussed in Code availability.