

Interactive comment on “Non-orthogonal version of the arbitrary polygonal C-grid and a new diamond grid” by H. Weller

Anonymous Referee #2

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The manuscript is about low-order finite volume / finite difference discretizations of the shallow-water equations on a rotating sphere. It makes two main original contributions :

- a novel quadrilateral spherical mesh is proposed, with interesting geometrical properties
- certain vector reconstruction operators (H and perp) are proposed which take liberties with the symmetry or antisymmetry properties necessary for the exact discrete conservation of energy ; the consequences of taking these liberties is analyzed on a series of standard test cases.

In addition, the lack of convergence of the perp operator devised by Thuburn et al. C2431

(2010) and used in a number of ensuing publications is demonstrated. This lack of convergence had previously been overlooked since the perp operator does apparently not compromise the convergence of the whole shallow-water scheme it is included in. This is an interesting point since it highlights the importance of carefully assessing the properties not only of a whole scheme, but also of its individual building blocks, and at the same time provides an example where the whole scheme behaves (apparently) better than one of its building blocks.

The above calls for an analysis of the accuracy of the asymmetric and symmetric H operators in isolation. This would help interpreting the numerical results obtained for the whole shallow-water scheme, especially the differences observed when only the operator H differs between two experiments.

Ideally one would also like to have some indications of the truncation error of both H for nearly-orthogonal quadrilateral meshes, for instance a planar mesh made of identical parallelograms. This would shed light on the role of the off-diagonal terms of H .

Overall the paper is interesting and well-written. I recommend publication provided it is improved as suggested above and hereafter.

Other remarks :

- 3.2.1 The test flow (solid-body rotation) is solenoidal, it may be useful to do a similar convergence study (Fig. 5) with an irrotational flow, e.g. $u_{lat} = \cos(\phi)$, $u_{lon} = 0$.
- It should be mentioned that the symmetric H does not become diagonal when used on an orthogonal triangular/hexagonal C-grid but does become diagonal when used with an orthogonal Cartesian C-grid.
- Is (14-15) Perot's reconstruction ?

- Section 4 : I understand the linearized (unnumbered) equations are integrated over one time step using (21-22) and the eigenvalues μ of this operator are computed, stability being indicated by $|\mu| \leq 1$. It may be useful to express the same information in terms of $\log |\mu|$ or $\log |\mu|/dt$ especially for $|\mu|$. In fact since the time stepping is slightly dissipative it would be nice to see directly the (real part of) eigenvalues of the linearized operator without the time integration (e.g. eigenvalues of L where $(du/dt, dh/dt) = L.(u, h)$). Although in practice what matters is the stability of the whole spatial + temporal scheme it would be interesting to know whether L really has only eigenvalues on the left of the imaginary axis (despite the lack of energy conservation) or maybe a few have a slightly positive real part.
- What happens to this linear stability analysis if the TRiSK perp is replaced by the consistent perp (Eq. 16) ?
- Section 6 : Since much of the discussion is about the relative merits of symmetric vs asymmetric H it would be useful to have a direct assessment of their accuracy, e.g. something similar to Fig. 5 with H instead of perp. Error patterns would be interesting as well since, as mentioned before, the symmetric and asymmetric H should yield very similar errors in the center of a cubed sphere face due to the mesh being nearly Cartesian and orthogonal there.
- Fig.8 - 'normalized energy change' : Please specify how energy change is normalized ? I would suggest normalizing by initial available energy, i.e. kinetic+potential minus the potential energy of the flow at rest with the same mass (averaged height) (see e.g. Ringler et al. (2011) Eqs 16-18) Same remark for enstrophy. Also is kinetic energy defined by (10) ? With a symmetric H the kinetic contribution to the conserved energy is $\sum U_e V_e$ (Thuburn & Cotter, 2012, eq. 2.27).
- Lack of second-order convergence with the symmetric H on the HR grids seem
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consistent with its non-diagonal character. Conversely second-order convergence on the the cubed and diamond grids suggests a superconvergence of the symmetric and asymmetric H (formally first-order accurate only).

Ringler, T. D., Jacobsen, D., Gunzburger, M., Ju, L., Duda, M., & Skamarock, W. (2011). Exploring a Multiresolution Modeling Approach within the Shallow-Water Equations. *Monthly Weather Review*, 139(11), 3348–3368. doi:10.1175/MWR-D-10-05049.1