Geosci. Model Dev. Discuss., 6, C1552–C1554, 2013
www.geosci-model-dev-discuss.net/6/C1552/2013/

Geoscientific Model Development Discussions Open Access

Interactive comment on "Accuracy of the zeroth and second order shallow ice approximation – numerical and theoretical results" *by* J. Ahlkrona et al.

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Received and published: 27 September 2013

This paper examines in some detail the accuracy of the familiar (zero order) shallow ice approximation (SIA) and a far more complicated (but apparently higher order) expansion, the second order shallow ice approximation (SOSIA). As I understand it the SOSIA might be expected to offer two advantages over the SIA : its error with respect to the solution to the Stokes equations is intended to decay more rapidly with the aspect ratio ϵ, and it includes some additional stresses (e.g horizontal shear), so was considered useful by e.g Egholm et al 2012. The authors are concerned primarily with the first of these potential benefits, and, briefly, find that even in a case with a smooth

geometry and periodic boundary conditions, the SOSIA does not appear to be much (if any) improvement over the SIA.

The authors compute stress and velocity fields determined through their solutions to the SOSIA equations - eqs (15-19) which are expressed in terms of various derivatives and vertical integrals. These are compared with numerical solutions to the Stokes equations computed with the well-known Elmer/Ice code. Convergence with ϵ is no better than the SIA (or is actually worse, e.g fig 2a).

The authors then say that the cause of this result is the regularization of Glen's flow law. I think most people working in glaciology are familiar with the need for this sort of regularization : the effective viscosity becomes infinite at zero-rate of strain (so at the top of the ice sheet in these problems). The usual regularization consists of adding a positive number σ^2_{res} to the second invariant of the strain rate (σ^2). The authors work out the various terms in their equations (15-19) to find long expressions that contain terms including both σ_{res} and $\frac{1}{\sigma_{\text{res}}}$. That means that σ^2_{res} must be chosen with care and the correct choice turns out to be problem dependent. Even when it is chosen with care, the rate of convergence with ϵ seems to be improved only over a small region around $\epsilon = 0.1$, below which the rate is no better than the SIA

I am not at all familiar with the details of the SOSIA and I found this paper quite instructive. I also found the arguments convincing, though as an earlier reviewer notes, that does depend on the authors having derived the rather long expressions correctly. That reviewer urged the authors to show that was the case somehow (e.g, using a symbolic algebra package), and I would like to echo that.

1 Specific comments

1. What is the numerical error in (a) the MATLAB implementation of 15-19, which presumably involves numerical integrals and derivatives, and (b) the Elmer/Ice

solutions? The authors do say that they ensured the results were mesh-independent, but I am sure they know that this can never really be the case. Elmer/Ice, I think, would solve the Stokes eqns on a mesh with spacing h with an error $O(h^2)$, but perhaps the leading error term is actually something like $\alpha\epsilon^k h^2$, in which case the plots might be dominated by the Elmer/Ice error. Presumably, by mesh-independent the authors mean that they have determined that α (or its equivalent) is small, but I would like to see more detail. For example, there could be log-log plots of $||v_{\mathsf{Stokes,h}} - v_{\mathsf{Stokes,2h}}||$ against ϵ, or just some statement about the size of these numbers if there is no dependence on ϵ. The same goes for eqs 15-19

2. If the authors have correctly identified σ^2_{res} as the source of SOSIA's limitations, then the problems should vanish for Newtonian ($n = 1$ in Glen's law) fluids . I think the earlier reviewer made this point too. I would like to see a repeat of plot 2a (say) for $n = 1$. At the same time it isn't obvious to me that the terms in say, eq 19 or A6, with coefficients of σ^2_{res} vanish when $n = 1$, but can that be made obvious somehow? If the expressions had been derived for n in general rather than $n = 3$ it might be a bit clearer. I realize that is harder said than done, but the numerical test I suggest above would at least demonstrate the point for eqn. 19
3. p 4290: *There seems to be a common perception that the boundary layer is thin...* This needs a bit of explanation and some citations. Are other authors routinely saying this, or, inadvertently, something that is equivalent to this? If so, we need to know how

2 Minor typographical issues

1. p 4303 : *According to the classical theory SIA would be correct, SOSIA should...* doesn't make grammatical sense, although I can see what is meant