

Rimbay – a multi-physics 3-D ice-dynamics model for comprehensive applications: model-description and examples

1 Answer to Reviewer S. L. Cornford

Many thanks for the review and the suggestions to improve the manuscript. In particular for the hard-to-spot errors in the equations. We answered all items below in detail.

1.1 Issues related to scientific presentation

1. The title: does it make sense to refer to RIMBAY as a multi-physics model? That term usually seems to apply to applications that provide numerical methods useful across a wide range of physics, COMSOL multi-physics being a well-known example. I realise that RIMBAY includes a choice of approximation to the Stokes problem at its core, but that seems to be described by comprehensive. *The comprehensive refers to the variety of applications rather than possible approximations. However, we agree with the reviewer, that multi-physics might be misreading, as RIMBAY has a different focus than e.g., COMSOL or Elmer. Therefore we replaced the expression multi-physics with multi-approximation in the title and everywhere in the manuscript.*
2. p 3290, line 26: Therefore, the imminent climate change will have profound impact on society. Is this justified by the references? I'm not saying that this won't come to pass, but I don't think ice sheet modelling papers should contain speculative remarks like this. *We added a reference as suggested.*
3. p 3291, lines 16- : I agree with Helene Seroussi's comment, that there should be Discussion Paper some more (a few lines) discussion of the capabilities of other ice sheet models. *Following the reviewer's and Helene Seroussi's suggestion we added a paragraph discussing established ice sheet models to the final conclusion section.*
4. p 3303, lines 14-17: Equations 10,14,16 are not linear. I think the authors know what they want to say (the equations are non-linear because of the form of the viscosity, and often the basal traction law too), but just need to remove the first linear *Corrected as suggested.*
5. p 3304, lines 15. Is convergence of the non-linear system really faster. If Newton's method were used, the (inner) linear systems would look like

$$\begin{pmatrix} A & A' \\ B & B' \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} \quad (1)$$

for the hydrostatic and SSA cases. RIMBAY (I think) is attempting to solve this linear system by solving

$$Au = pAv \quad (2)$$

then

$$Bv = qBu \quad (3)$$

once for each cycle of the outer iterations (or more times? this needs to be clearer). Either way, this a quasi-Newton (Picard) method that is often outperformed. It is even surprising to me that the linear system (1) would converge more quickly by iteration of (2) and (3). Id like to see some evidence of this, or a citation to such evidence.

- According to Pattyn (2003), paragraph 16, just before equation 48, the successive solving for u and v needs less computational time than solving the system at once. We reformulated this to *faster convergence* in our manuscript. We are not sure if the reviewer might have interpreted this expression as *less iteration cycles* which is not meant.
- The implicit question of the reviewer he refers to with (*I think*) should be answered by our statement ... *it is sufficient to solve the system of linear equations for u and v successively, instead of solving both equations at once..*
- As explained on p3305 lines 9–14, we don't use a Picard-iteration but a *unstable manifold correction*.
- We hope Fig.2 illustrates sufficient, how variables are iteartively solved within RIMBAY.

6. Section 4.2 : Is the C-grid chosen to avoid checker-board pressure/velocity fields that appear in the A-grid? If so, could we see some solutions where the A-grid is problematic and the C-grid is not?

Historically, the code was originally written on an A-Grid. To avoid the checker-board effect some numerical diffusion has to be introduced, when solving the ice sheet evolution equation. However, this methode is ill-posed for ice shelves, which have a very smooth surface, as can be seen on p3308 line 6–7. Therefore we implemented the staggered C-Grid to overcome this limitation. Finally, we added a method to solve the ice sheet evolution equation on a C-Grid, even if the velocities are calculated on an A-Grid as an additional option. However, we consider these technical issues far beyond the scope of this manuscript.

7. Section 4.3.1 and 4.3.3. Surely the central-difference scheme (eq 33) is unconditionally unstable for pure advection on both A and C grids (not just the A grid), just as it is even if the velocity is a known constant

To our knowledge a central-difference scheme for $\frac{\partial H}{\partial x}$ is only unconditionally unstable if implemented as *explicit*-scheme with H^t . But we implemented an *implicit*-scheme (eq. 33) with H^{t+1} , which stabilises the numercial solution.

8. p 33089. Useful as the first-order upwind scheme is, it does not avoid numerical diffusion - rather, it is notorious for it. The text suggests the opposite, presumably

inadvertently, because the phrase to overcome the restrictions involved with the numerical representation of Eq. (35) which comes after noting the diffusion terms in eq 35/36.

The *restriction*, we refer to is not the diffusivity, but the reciprocal of (∇S) . To clarify this we reformulated the sentence: ... *to overcome the restrictions involved with the numerical representation with respect in $(\nabla S)^{-1}$ of Eq.(35), we implemented ...*

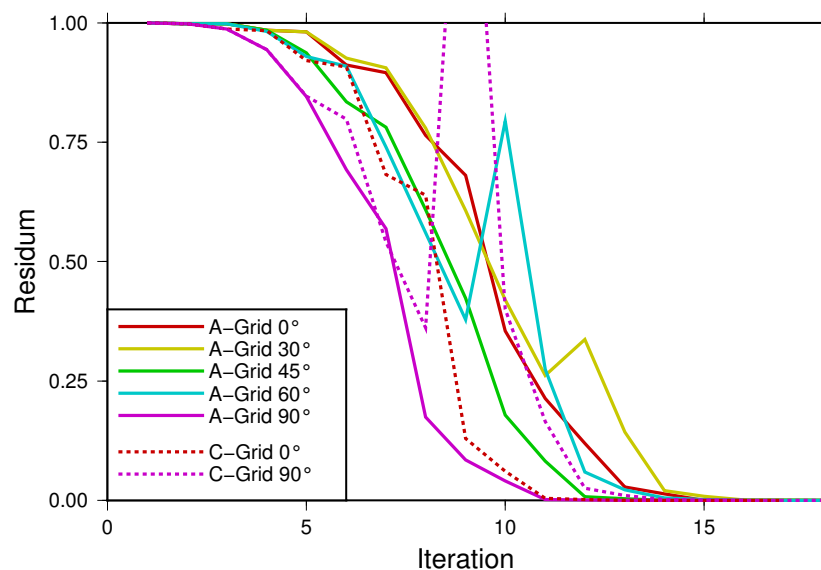
9. p 3310 line 5, Gladstone 2010 is not a full Stokes model (it is SSA), It is also a flow-line model - there are other adaptive models that treat the same sort of 3D problems as RIMBAY included in the MIS3D exercise (Pattyn et al 2013)
 Reformulated

10. p 3311 line 5: I dont like the invented word schoofism. I appreciate that the authors want to refer to the heuristic condition based on Schoof 2007 later, but I dont think that a phrase like imposing the heuristic condition outlined in section 4.4 is too unwieldy. Other authors that use this condition (Pollard and DeConto for example) dont introduce such a word.

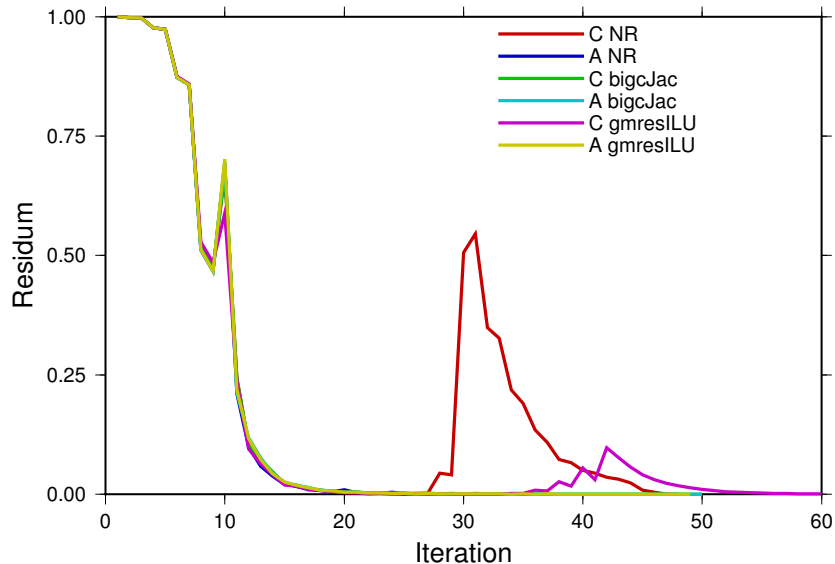
According to the reviewers suggestions, we removed the invented expression *shoofism* from the manuscript.

11. section 6.2 : These are interesting examples, and I would like to see some discussion (with a graph of residual vs iteration) of RIMBAYs solver performance in these cases. Ice shelves are tougher than grounded ice, because the linear systems are poorly-conditioned when $\beta = 0$. The tabular iceberg example should be ill-posed, because any constant velocity can be added to a solution (u, v) and still satisfy the governing equations, unless the velocity is imposed at one point - is this the case?

We didn't imposed a specific velocity at one point. We started the integration from $(u, v) = (0, 0)$ and the A-Grid solver converges quite fast. However, the C-Grid solver works only for 0° and 90° in this case. This figure shows the residuum during the integration:



A second example shows the convergence for a 5 km resolution version of the example shown in Fig. 7 for the A- and C-Grid and for different solvers. (NR is the `linbcg` solver from *Numerical-Recipes*, while `bigcJac` is the bi-conjugate gradient solver with a jacobian preconditioner and `gmresILU` is the generalized minimal residual method with a incomplete LU preconditioner, both from the LIS-package.)



From our point of view a discussion of this convergence issues would go beyond the scope of the manuscript, so we leave it as it is.

12. section 6.3 : The text says In consideration of the approximations and the low horizontal resolution, RIMBAY was able to keep up with the other 16 numerical models. I think the authors need to replace this with a more quantitative statement, comparing the RIMBAY results with the comparable models (ie those that use the heuristic, DPOx and VUBx, I think?). For example, does it produce the same steady-state configuration? Are the perturbed amplitudes and time-scales similar? Likewise, it would be interesting to know why RIMBAY produced less smooth grounding lines - is this something to do with the numerical treatment, e.g sub-grid interpolation or something like that.

However, we can not be sure why RIMBAY shows a different grounding line migration than DPOx and VUBx without comparing the numerical codes in detail, which would reach far beyond the scope of this study. We can only speculate, that the details in the numerical implementation of Schoof's heuristic rule (which is not justified anyway) are the source of these differences.

We revised the whole section thoroughly: First we added an additional experiment, which basically mimics the flow-line MISMIP benchmark, described in Patryn (2012). This experiment shows that RIMBAY is able to reproduce the semi-analytical solution based on the of the boundary layer theory of Schoof (2007) very well.

We also agree with the reviewer, that a more quantitative statement about the MISMIP-3D benchmark is reasonable. Therefore, we added a paragraph, describing more details.

13. section 6.4 : It is interesting that the grounding line advances when HOM or FS is used in place of the heuristic condition has it reached steady state?

Yes, we ensured that steady state was reached before we changed the model physics. We added this information to the manuscript.

If so, this wouldnt (or shouldnt) be the case for a straight grounding line, but it is harder to say what should happen in this case, especially if steady state has not been reached. For example, the MIMSIP3D perturbation caused the grounding line to advance at first from the starting point, but had the slippy spot remained in place for long enough, the GRL should eventually migrate upstream of the starting point. Softening the ice (by switching to HOM or FS) could lead to a similar transient, perhaps.

If possible (and I know that the CPU cost might be large), I would like to see this experiment expanded to help readers decide whether the results are correct or an artefact of the numerical treatment. For example, spin the model up with the heuristic condition and a uniform basal traction coefficient $C = C$ (as for the original MIMSIP3D), and report the position of the grounding line - does it agree with the formula in Schoof 2007? Then, switch to FS and/or HOM in a region around the grounding line. Does it retreat or advance at first? What is the steady state, if that can be computed in reasonable time, if not, does the grounding line ever change direction? Do the results change if the higher-order region is larger?

We do not have the resources to performed new 3D-experiments for this section. However, we extended the newly added MIMSIP-experiment (Pattyn, 2012) from the previous section, by a HOM/FS-domain in the vicinity of the GRL and discussed its results in a newly added paragraph.

1.2 Typographical and minor grammatical errors

1. p 3290, line 11 : FullStokes, should be full Stokes
OK
2. p 3291 line 27 : [multiple items] has been improved, → have been improved
OK
3. p 3292 line 21 : fulfil this needs → fulfil these needs
Corrected on p 3292 line 1
4. p 3293: lines 17 : programmes → programs.
OK
5. eq 7: the strain rate is rendered as e, but in the next lines is $\dot{\epsilon}$
Changed to $\dot{\epsilon}$ for consistency.
6. p 3926: $\epsilon_{xx}^2 + \epsilon_{yy}^2 + \epsilon_{zz}^2 \rightarrow \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$ (the resulting expression, eq 9, is correct)
Corrected

7. p 3303: rule of thump → rule of thumb
OK
8. p 3304, footnote ; swopped → swapped
OK
9. p 3308, line 25 ; missing factors of 1/2 in $U_{i,j} = 1(U_{i,j} + U_{i,j+1})$ in e.g $U_{i,j} = (U_{i,j} + U_{i,j+1})$ should be
OK
10. p 3308, line 3 ;than less ice → then less ice
OK
11. p 3312 line 13 level of classification: I think this is just an English language oddity, but I havent previously seen the organisation of C++ code into class hier archies described as classification.
We understand that this sounds *odd*, and we are glad, that the reviewer perfectly understood the meaning of our sentence. We would appreciate any suggestion of a native speaker to replace the expression *level of classification* by a more common phrase.
12. p 3312 line 20: It handles Makefiles and attends dependencies between different source (and header) files automatically. sounds a bit awkward, though I know what is meant
We reformulated this phrase to: *It generates system- and environment-dependent Makefiles automatically and attends dependencies between different source (and header) files.*