Geoscientific
Model Development
Discussions

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Interactive comment on "A bulk parameterization of melting snowflakes with explicit liquid water fraction for the COSMO model version 4.14" *by* C. Frick et al.

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Reply to Referee #2

Reviewer Comments are marked by **RC** and Author Reply by **AR**.

We thank anonymous referee #2 especially for the detailed comment on the capacitance of melting snowflakes. Consequently, we corrected our formulation of the

capacitance, leading to a modified melting integral and parametrization, and resimulated our case studies. A detailed description of the corrected calculation of the capacitance has been included in the paper and the new results (which are qualitatively very similar to the old ones) are presented.

Major Comments

1. • **RC** Calculation of capacitance

   The authors have chosen to express the analysis in terms of the maximum geometrical dimension of the mass equivalent dry snowflake, D_s, assuming a density relation of $m = 0.069 \cdot D_s^2$. Equally the authors could have chosen the diameter of the mass equivalent melted sphere, D_{eq}. The rationale would be the same, i.e. the diameter is constant following a particular particle throughout the melting process (as is the total mass of the particle). In contrast, the actual melted diameter would change (decrease) through the melting process due to increasing density, from the dry snowflake diameter, D_s, to the melted sphere diameter, D_{eq}.

   The use of a constant diameter assumption (in this case D_s) has a number of consequences. Firstly, there is a discontinuity at the point where all snow has turned to rain, which the authors point out for the assumption of size distribution on p2939. Secondly, particle characteristics that depend on diameter, such as capacitance can be incorrectly calculated for melting particles if not carefully accounted for.

   The capacitance is a term in the melting rate, and for a melting particle, is defined as a function of D_s and an increasing function of meltwater fraction, l, (Eqn 9) so for a melting particle with constant D_s, the capacitance increases with increasing meltwater fraction. Assuming constant density throughout the melting process D_{eq} is proportional to $D_s^{2/3}$, so the

capacitance also increases with constant D_{eq}, which is plotted in Figure 1a. In fact, the capacitance should *decrease* as the particle melts due to an increase in the density and decrease in melted diameter. Eqn 9 should include a modification term for the particle density, which was taken into account in Mitra et al. (1990), but isn't here. M90 assumed a linear increase in density from 0.02 for a dry snowflake to 1 for a raindrop (linear with melted water fraction). This will then lead to a smooth transition to the capacitance of a raindrop.

The result of this change will be a lower melting rate as the melting proceeds, which will change the results in all subsequent simulations and figures.

- **AR** We carefully had a look at our calculation of the capacitance and replaced the incorrectly applied diameter of the dry snowflake D_s by the diameter of the melting snowflake D_m as suggested by Mitra et al. (1990). A detailed description of the calculation has been added to the paper:

  "For the calculation of the capacitance M90 applied the approximation for an oblate spheroid. The axis ratio is assumed to be 0.3 for a dry dendritic crystal, and 1.0 for a raindrop. The axis ratio for melting snowflakes is approximated by a linear interpolation, i.e.,

  $$a(l) = 0.3 + 0.7\, l. \quad (8)$$

  and the capacitance is then given by (Pruppacher and Klett,1997, p. 547, Eq. (13-78)),

  $$C_m(D_s, l) = \alpha_{\mathsf{cap}}(l)\, \frac{D_m(D_s,l)}{2}\, \frac{\sqrt{1-a(l)^2}}{\arcsin\sqrt{1-a(l)^2}} \quad (9)$$

  with $C_m(D_s, 0) = C_s$ and $C_m(D_s, 1) = C_r$. D_m is the maximum dimension of the melting snowflake, which can be calculated as follows

  $$D_m(D_s, l) = \left(\frac{6m_s}{\pi a(l)\rho_m(D_s,l)}\right)^{1/3} \quad (10)$$

  assuming an oblate spheroid shape of the melting snowflake (see above) and in agreement with Eq. (8) of M90. Here ρ_m is the density of the melting snowflake. As suggested by M90 we interpolate $\rho_m(D_s, l)$ between the density of liquid water, $\rho_w = 1000\,\mathrm{kg\,m^{-3}}$, and the density of the dry snowflake $\rho_s(D_s, l)$:

  $$\rho_m(D_s, l) = \rho_s(D_s, l) + (\rho_w - \rho_s(D_s, l))l. \quad (11)$$

  For the density of a dry snowflake with the axis ratio of the melting snowflake it follows from the assumption of the oblate spheroid shape that

  $$\rho_s(D_s, l) = \frac{6m_s}{\pi D_s^3 a(l)} \quad (12)$$

  but only till a maximum value of $\rho_s = 500\,\mathrm{kg\,m^{-3}}$ because higher densities are not reasonable for snowflakes. The empirical correction factor $\alpha_{\mathsf{cap}}(l)$ in Eq. (9) is about 0.8 for dry snowflakes and for melting snowflakes M90 again suggest a linear interpolation, i.e.,

  $$\alpha_{\mathsf{cap}}(l) = 0.8 + 0.2\, l. \quad (13)"$$

  Equation numbers correspond to the new version of the manuscript. The resulting C_m is shown in Figure 1 below. Consequently, the new capacitance has been introduced to the parametrization, leading to a modified melting integral presented in Figure 2. We resimulated the presented case studies and found only minor differences for the results. The new melting integral and the new results have been included in the paper.

2. - **RC** Calculation of ventilation coefficient
     For the ventilation coefficient, it is not clear how the Reynolds number is specified as a function of l. Do you use Eq 13 with interpolated terminal

velocities between a dry snowflake and a rain drop calculated from Eq 11? A bit more detail would be appropriate. It is not so clear why there is so little dependence of the ventilation coefficient on equivalent diameter from rain to snow given the large change in terminal fall velocity - is this because the smaller melted drop size compensates exactly for the increased terminall fall speed? Figure 1(c) is very different to the equivalent plot in SZ99 (fig 2) which has the ventilation coefficient increasing significantly for increasing meltwater fraction. If there is a good reason for the differences, this should be explained.

[Note the empirical terminal fall velocity formulation in Fig 1(b) does look reasonable, and is consistent with SZ99 Fig 1. Might be a good idea to separate this section with subtitles, i.e. a) Capacitance, b) Terminal fall speed c) Ventilation coefficient?]

- **AR** Yes, the smaller drop size compensates for the increased terminal fall velocity, because for precipitation-sized particles at terminal fall velocity the Reynolds number is to a good approximation only a function of particle mass m as already pointed out with help of Eq. (13) in the original manuscript (corresponding to Eq. (16) in the new version). For spherical particles of arbitrary density this can easily be proven. At terminal fall speed friction equals gravity

$$m\frac{dv}{dt} = mg - \frac{1}{2}C_D\rho_l v^2 A = 0 \qquad (1)$$

where g is the gravitational acceleration (we neglect buoyancy for simplicity), C_D the drag coefficient, ρ_l the density of air, $v = v_T$ the terminal fall velocity of the particle and A its cross sectional area. For large particles we can assume that the drag coefficient is constant, $C_D = C_\infty$, and, because we assume spherical particles, we find $A = \pi/4\,D^2$ for the cross

C1337

sectional area. This leads to the well know result that

$$v_T = \sqrt{\frac{8}{\pi}\frac{mg}{C_\infty\rho_l D^2}} \sim \sqrt{D} \qquad (2)$$

in the limit of large Reynolds numbers and therefore

$$Re = \frac{v_T D}{\nu} = \sqrt{\frac{8}{\pi}\frac{mg}{C_\infty\rho_l \nu^2}} \qquad (3)$$

which proves that Re is only a function of mass m and independent from the particle density or size.

Usually we associate faster falling particles with higher Reynolds numbers, but this is of course only true if the particle size does not change. Comparing snowflakes and rain drops it can be quite illuminating to recognize that a large raindrop, say of 1 mm diameter, has a fall speed of approximately 10 m/s, a large snowflake of similar mass may have a geometric diameter (maximum dimension) of 5 mm and a fall velocity of 2 m/s and a lower density snowflake of the same mass might be 1 cm in size and have a fall speed of 1 m/s. All three particles obviously have the same Reynolds number. Therefore it is not far fetched to assume that the Reynolds number, and therefore the ventilation coefficient does not change strongly during the melting process.

Quantitatively this can be discussed and analyzed based on the aerodynamic theory of Böhm (1989, 1992) as well as Khvorostyanov and Curry (2002, 2005). Figure 3 at the end of this document shows the Reynolds number and Figure 4 the drag coefficient for raindrops and dry snow based on the snowflake geometry used in the manuscript. The detailed calculations further support our arguments, e.g., that the Reynolds number is large enough so that C_d can be approximated by the asymptotic value C_∞, and therefore the Reynolds number is approximately only a

C1338

function of mass (or the equivalent diameter D_{eq}). The plots also show that for large particles the drag coefficient of snow is actually larger than that of the corresponding raindrop. This is due to the roughness of the ice surface as assumed in the Khvorostyanov and Curry (2005) parameterization.

Unfortunately, we were not able to get a hand on the thesis of Papagheorghe (1996). Therefore the derivation of the relations of Szyrmer and Zawadzki (1999) remains unclear to us and it is difficult and maybe inappropriate to speculate where the different behavior in their Fig. 2 comes from. One issue in their approach might be that the empirical fall speed relation is a power law relation. Power laws are used quite often in empirical fall speed formulas, but have the disadvantage that they do not lead to the correct asymptotic behavior for large particles, i.e., the terminal fall velocity does not approach a constant value as it should occur based on aerodynamic theory. Therefore the extrapolation of power law relations to large particle size is very questionable, especially for the formula of Langleben (1954) which has a quite large power-law exponent of 0.61.

Another issue in Szyrmer and Zawadzki (1999) is that the chosen fall speed relation of Langleben (1954) is based on a snowfall event with a mix of dendrites and aggregates of plates. Is seems virtually impossible to come up with a snowflake geometry that is consistent with this fall speed relation and has a physically reasonable asymptotic behavior for large particle sizes. Note that Langleben (1954) used the classic measurement techniques with angora wool to capture the snowflakes and filter paper to estimate the melted diameter or mass. The geometric diameter or maximum dimension is not available from their measurements. Having a consistent geometry assumption is of course crucial when estimating the Reynolds number and the ventilation coefficient for an observed fall speed relation.

Figure 3 below supports our assumption that the variations in Reynolds
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number are very small during the melting process. Therefore we use the simple linear interpolation

$$Re_m = \ell\, Re_d + (1 - \ell)\, Re_s \qquad (4)$$

between the Reynolds number of the dry snowflake Re_s and the raindrop Re_d to calculate the ventilation coefficient. Alternatively, we could use the empirical fall speed relation of Mitra et al. (1990), for Eqs. (11) and (12) in the original version (now Eqs. (14) and (15)), and combine this with an estimate of the maximum dimension of the melting snowflake, as we do now for the capacity in the revised version. We think that the interpolation of the Reynolds number is the more robust approach and therefore we decided to use this formulation and avoid the estimate of the maximum dimension for the calculation of the ventilation coefficient. Hence, Eqs. (11) and (12) in the original version (corresponding to Eqs. (14) and (15) in the new version) are only used for the sedimentation of melting snowflakes, but not for the ventilation coefficient.

Of course, it may look paradox that the Reynolds number does change so little during melting, although the geometry changes dramatically. This can only be understood by the boundary layer of the particle which, at these high Reynolds numbers, smoothes out the details of the snowflake geometry. But given the complicated nature of snowflakes it would be desirable to have more detailed lab measurements of the particle properties, like the ventilation coefficient, during the melting process.

Minor Comments and grammatical suggestions

1. • **RC** Although I realise the model version "version 4.14" in the title has been
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requested by the journal, I don't think it is necessary or appropriate in this case? The paper is not a description of this particular model, but rather a description of a parametrization that is more generally applicable.

- **AR** We agree in the point that the parameterization is more generally applicable. In our opinion, the mention of the version is not necessary and we dropped it again, hoping that the journal will not insist on its position.

2.
- **RC** Abstract, p2928, line 12-13 I would suggest a slight reordering of the sentence to "For the bulk parameterization, a critical diameter is introduced which increases during the melting process. It is assumed that particles smaller than this diameter have completely melted..."
- **AR** We reordered the sentence.

3. p2929

- **RC** line 18, Correct "Szyrmer and (1999)"
- **AR** Fixed.
- **RC** line 22, Why "potential melting" and not just "melting"?
- **AR** Corrected to just "melting".
- **RC** line 28, "is called the melting layer"
- **AR** Corrected.

4. p2931

- **RC** line 11, "...and the subject of future investigation."
- **AR** Corrected.

5. p2932

- **RC** The text describes exponential or Gamma, but Eq. 3 describes only an exponential. Given the discussion here, it would be more appropriate to put the equation for a Gamma distribution (i.e. include $f(D_s) = N_0 D_s^\mu e^{-\lambda D_s}$ and then reword the text to include $\mu = 0$ for exponential in the text. Also be consistent using either "inverse exponential" or "exponential". The differences are really for the small end of the particle size spectrum and so it depends to some extent on the application. For mass changes, the mass-weighted part of the spectrum dominates and therefore an exponential is a reasonable assumption.
- **AR** We enlarged and rephrased the entire paragraph describing Eq. (3) of the manuscript:

$$"f(D) = N_0 D^\mu exp(-\lambda D) \quad (3)$$

where λ is the slope and N_0 the so-called intercept parameter of the distribution. First, the diameter D has to be specified. Most measurements show an exponential or Gamma distribution w.r.t. equivalent diameter D_{eq} (Gunn and Marshall, 1958; Brandes et al., 2007), but others, e.g., Field et al. (2005) find exponential distributions in geometric diameter D_s. The preference for one or the other might simply be due to different measurement devices. Most bulk parameterization use Eq. (3) with geometric diameter D_s which simplifies the formulation of the collection rates. We follow the latter approach and use D_s in Eq. (3). Second, the exponent μ has to be considered. For the calculation of mass changes, an exponential distribution, i.e., $\mu = 0$, is a reasonable assumption for the number density distribution because the mass-weighted part of the spectrum is more dominant. Therefore, we apply an exponential distribution ($\mu = 0$) in geometric diameter D_s for the number density distribution of snowflakes."

- **RC** For Eq. 4, for completeness should really include definition of rho symbol, i.e. density of air.
- **AR** Clarified.

6. p2933

- **RC** Equation 5, maybe you could put $dL_s/dt = -dm_w/dt = ...$ to link with Eq. 2, i.e. snow loss is meltwater gain.
- **AR** Done.
- **RC** line 24, Szyrmer reference missing something.
- **AR** Fixed.

7. p2934

- **RC** line 22, the "size of the mass equivalent dry snowflake D_s depends itself on the mass equivalent diameter of the melting snowflake, D_{eq} and l" (the meltwater fraction). However, it appears from the text that D_s is only a function of D_{eq}, as both D_s and D_{eq} are assumed to be constant as the particle melts, i.e. constant density.
- **AR** Sentence has been deleted.

8. p2935

- **RC** line 18 "terminal fall velocities of the mass equivalent dry snowflake", but vs and vr in Eqn 11 are expressed as functions of D_s and D_r, which are the maximum dimension even though these are functions of D_{eq}). Some inconsistency in notation/description here.
- **AR** We rephrased the paragraph describing Eq. (11) in the original version (corresponding to Eq. (14) in the new version):
  "Another important result of M90 is that the terminal fall velocity of a melting snowflake can be parameterized by

  C1343

  $$v_m(D_s, l) = v_s(D_s) + [v_r(D_r) - v_s(D_s)]\,\Psi(l) \quad (14)$$

  where v_s and v_r are the terminal fall velocities of the mass equivalent dry snowflake and rain drop, which we calculate following Khvorostyanov and Curry (2005). Both depend on the corresponding maximum dimensions which are in fact functions of the mass equivalent diameter D_{eq}. For the calculation of the maximum dimension of the rain drop from D_{eq} we follow Khvorostyanov and Curry (2002). For dry snowflakes D_s is calculated by using $m_s = (\pi/6)\,\rho_w D_{eq}^3$ and $D_s = (m_s/\alpha)^{1/2}$ with $\alpha = 0.069$ and, in addition, a cross sectional area of $A = 0.45\,\pi/4\,D_s^2$ is assumed following Field (2008)."

9. p2936

- **RC** line 2, is the D in cross-sectional area, D_s or D_{eq}?
- **AR** Clarified as D_s.
- **RC** line 13, Split the sentence, i.e. "...Reynolds number deviates from M90. Instead we are consistent..."
- **AR** Corrected.
- **RC** Eq 13. define ν_a, i.e. kinematic viscosity
- **AR** Fixed.

10. p2937

- **RC** Eq 15. should be $L_{s,i} = \rho q_{s,i}$
- **AR** Corrected.

11. p2939

- **RC** line 4, remove comma after "both,"

  C1344

- **AR** Corrected.

12. p2940

   - **RC** Eqns 21,22, Would help to point out here you are using $m_i = \alpha \cdot D^2$
   - **AR** Clarified.

13. p2942

   - **RC** line 11, "That makes it possible..."
   - **AR** Fixed.

14. p2943

   - **RC** line 3, commas..."To approximate, e.g. the melting integral, we chose..."
   - **AR** Corrected.
   - **RC** line 20, "According to the limitations..."
   - **AR** Corrected.

15. p2945

   - **RC** line 2, remove "to do"
   - **AR** Corrected.
   - **RC** line 16, as $q_{s,w}$ uses a "new generalized tracer implementation", does this use the same advection scheme as $q_{s,i}$? If not, then what impact does this have?
   - **AR** The tracer algorithm uses the same advection scheme. This fact is now mentioned in the text:
     "For both mixing ratios, $q_{s,w}$ and $q_{s,i}$, the same advection scheme is applied."

16. p2946

   - **RC** line 20, "at 850hPa"
   - **AR** Corrected.

17. Figure 3

   - **RC** Units for (a) and (b)?
   - **AR** Both, the melting integral and the conversion coefficient, are dimensionless. This is now mentioned in the Figure caption:
     "The melting integral and the conversion coefficient are dimensionless quantities.".

18. Figure 4

   - **RC** Could a box be added to the plot to show the area in Figure 5 for those not so familiar with the geography of Germany.
   - **AR** A box has been added to Figure 4 showing the area of Figure 5. This is mentioned in the Figure caption:
     "The black dot marks Dresden and the black rectangle the area shown in Figure 5."

19. Figure 5

   - **RC** Contour labels are incorrect colours, meltwater is red, 0° is green and cross section is black. Lines on the figure could be clearer, but maybe in the final version it will be a bit bigger?
   - **AR** Fixed.

20. p2948

- **RC** line 4, "bulk liquid water fraction". Be clearer that this is the fraction of the snow that is meltwater so that the following discussion is clear. Maybe "meltwater fraction" is a better phrase?
- **AR** We now use *"meltwater fraction"* in this context.

21. p2950

- **RC** line 5, "this might be due to the fact...". Can you not be more certain about this through diagnosis of what the scheme is doing?
- **AR** We now say: *"This is due to the fact ..."*
- **RC** line 14/15, "explicitly predicting" → "explicit prediction of"
- **AR** Done.
- **RC** line 19, "to receive"? Do you mean "to determine"?
- **AR** Corrected.

22. p2951

- **RC** Line 1 "The liquid water fraction..." Again be clearer that this is the meltwater/snow fraction. Maybe "meltwater fraction" is a better phrase?
- **AR** Done.
- **RC** line 21,22 The last sentence reads a little oddly to finish on: "could be ideal", "some assumptions". I would suggest something like: "A comparison with radar data would allow an assessment of the vertical structure of the simulated melting layer, which is sensitive to the assumptions made in the snow melting scheme" or: "A comparison with radar data would allow the accuracy of the vertical structure of the simulated melting layer and assumptions in the snow melting scheme to be assessed."

C1347

- **AR** We rephrased the last sentence to:
  "A comparison with radar data would allow an assessment of accuracy for the vertical structure of the simulated melting layer, which is sensitive to the assumptions made in the snow melting scheme."

Additional references

Böhm, H. P.: A general equation for the terminal fall speed of solid hydrometeors, J. Atmos. Sci., 46, 2419-2427, 1989.

Böhm, J. P.: A general hydrodynamic theory for mixed-phase microphysics. Part I: drag and fall speed of hydrometeors, Atmos. Res., 27, 253-274, 1992.

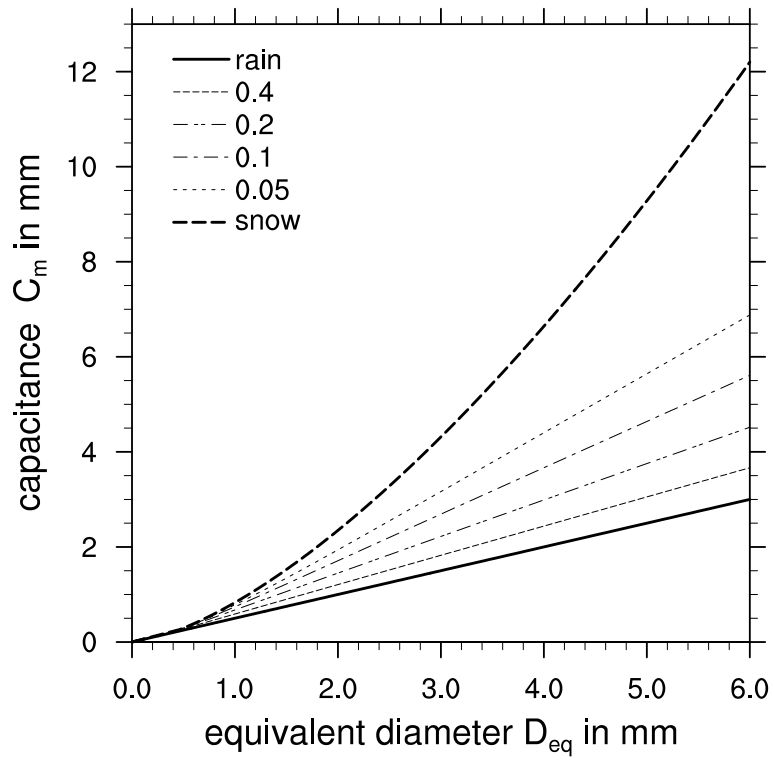Langleben, M. P.: The terminal velocity of snowflakes. Quart. J. Roy. Meteor. Soc., 80, 174–181, 1954.

Papagheorghe, R.: Modelisation de la couche de fonte. M. S. thesis, Dept. of Earth Sciences, University of Quebec at Montreal, 120 pp., 1996.
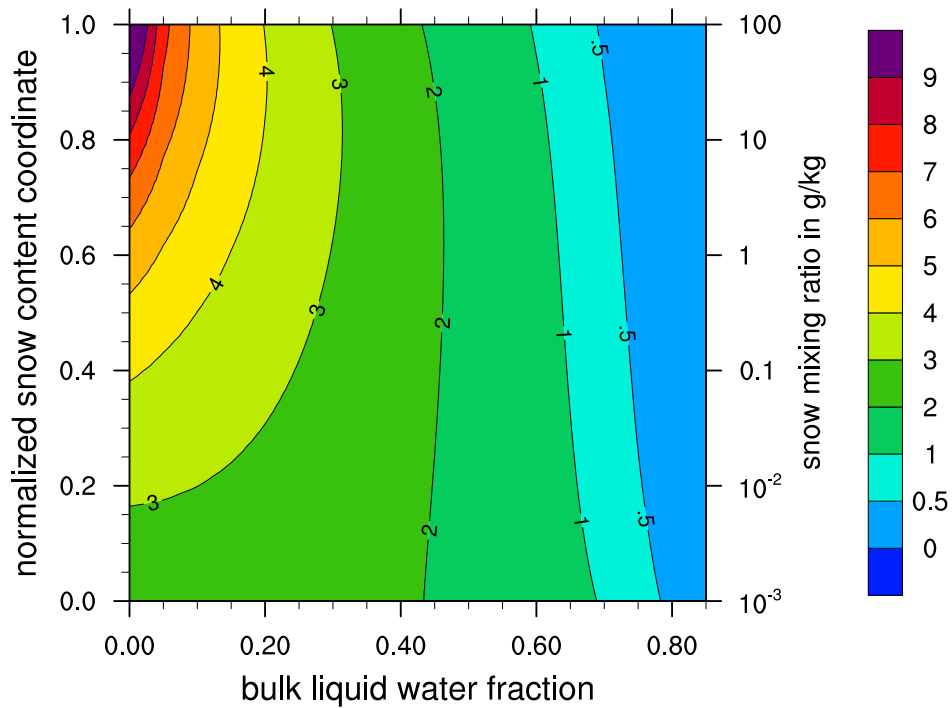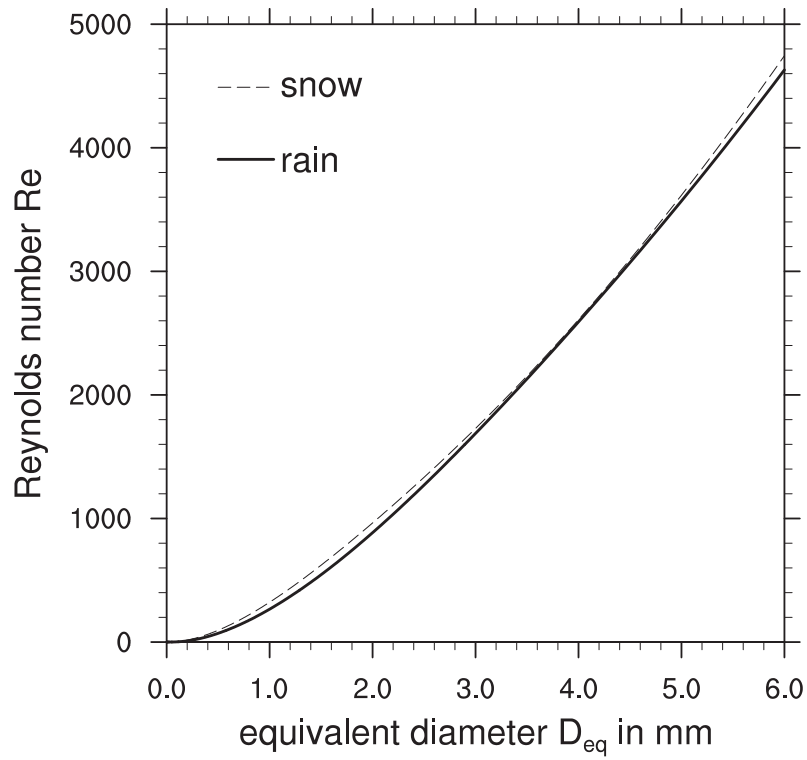
Fig. 1. The capacitance of a snowflake during the melting process for different liquid water fractions using the corrected formulation as presented in Eq. (9) in the new version.
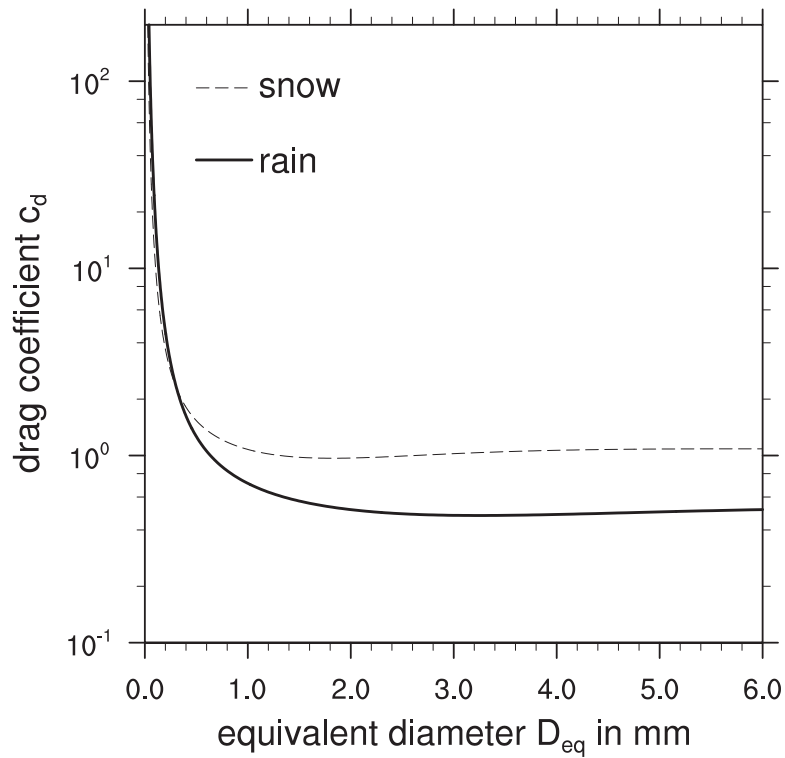
Fig. 2. The melting integral using the corrected capacitance as given by Eq. (9) in the new version, see Figure 3a of the discussion paper for a comparison.

Fig. 3. The Reynolds number as a function of equivalent diameter for rain drops (solid) and dry snowflakes (dashed).

Fig. 4. The drag coefficient as a function of equivalent diameter for rain drops (solid) and dry snowflakes (dashed).