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Interactive comment on "Accuracy of the zeroth and second order shallow ice approximation – numerical and theoretical results" *by* J. Ahlkrona et al.

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I object to the underlying premise of the "Second Order SIA". Namely, the work to date has simply assumed that all derivatives exist classically and has never addressed existence or uniqueness of solutions, yet it is a serious problem because the extra derivative does not exist for most problems of practical interest. Specifically, anything with a boundary or with irregular bed topography. Colleagues I have talked to that worked with SOSIA themselves have all found numerical artifacts around such features.

For example, isothermal SIA predicts a Vialov-Nye margin shape between $h(x) = x^n/(2n+2)$, which ranges between $x^{1/4}$ for $n = 1$ and $x^{1/2}$ for $n \to \infty$. If the con-

stitutive relation is regularized to allow diffusion creep (as in Goldsby and Kohlstedt), then the asymptotics are for $n = 1$ in SIA (where diffusivity goes to 0 at the boundary). Formally evaluating the first term of Equation 18 on a flat bed yields

epsilon$^2 t_{xz}^2 = -\rho g \partial_x[-1/6(z-h)^3(\partial_{xx}h)h] +$
$= C\partial_x[(z-x^{1/4})^3 x^{-7/4} x^{1/4}] + dotsb$
$= C\partial_x[(z-x^{1/4})^3 x^{-3/2}] +$
$= C[-3(z-x^{1/4})^2 x^{-9/4} - 3/2(z-x^{1/4})^3 x^{-5/2}]$

which results in a very singular integral appearing in Equation 19 near the boundary (z from 0 to $x^{1/4}$ as $x \to 0$). Computed velocity aside, it's reasonable to ask for the energy production (product of stress and strain rate) to be finite. With power-law rheology, the viscous energy production is somewhere between the second power (linear rheology) and a higher power of the stress. Integrating the expression above over a strip containing the boundary yields infinite energy production. Maybe steep boundary singularities do not appear when SOSIA is evolved, but steep boundaries are real, at least in the sense that Stokes and observations also show steep boundaries.

The derivative/regularity problem also exists for non-smooth bathymetry, and we all know that bathymetry does not even have continuous derivatives at essentially any scale. Indeed, models with higher derivatives are more noisy when under-resolved, which is always the case in computational geoscience. Indeed, one of the most important realizations in computational science is that models are *always* under-resolved since there is always a "next scale" that we are trying to represent. It is important that models with higher order derivatives posess sufficient regularity to stay within the space where those derivatives make sense (at least weakly), cf. elastic beam/shell and Cahn-Hilliard equations.

Finally, I assert that "including more derivatives" is *not the answer* to higher fidelity models. This statement matches the conclusion of nearly every other field in computational continuum mechanics; additional derivatives appear to model fundamentally "higher order" processes, but not as additional terms from a small parameter expansion. Integration is good, as is spatial coupling when models are truly non-local (e.g., slippery bed).

For these reasons, I do not think it is useful to continue publishing papers on SOSIA without first addressing its fundamental regularity problems that arise in so many common circumstances that they cannot be shrugged away as an academic technicality. Otherwise, it is just a distracting exercise in tedium. (And no, I don't buy the idea that we should have one approximation for use in C^{∞} domains away from boundaries and locations of less regular bathymetry/slipperyness while using other discretizations in the rest of the domain.)

Interactive comment on Geosci. Model Dev. Discuss., 6, 4281, 2013.