Dr James Annan

Editor of Geoscientific Model Development

Brussels, February 24, 2014

Dear Editor

A detailed response to the comments of the referees of our manuscript entitled "A 24-variable low-order coupled ocean-atmosphere model: OA-QG-WS v2", is provided below. The content of the manuscript has considerably benefited from their constructive comments.

Hoping that this new manuscript version is now suitable for publication in Geoscientific Model Development,

Yours sincerely

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Response to the first referee.

Thank you very much for your constructive comments. Please find below the response to your specific comments.

1. WS means "Wind Stress". I have explained that in the last paragraph of page 4.
2. Yes indeed the evolutions of the time averages were computed with different initial conditions in phase space. We agree with you that the convergence is not yet completed and we will modified the text according to your recommendation. See paragraph 2 page 13. It is now written as:
   "The temporal variations of these mean values are illustrated in Fig. 3, for Θ^*=0.077 and 0.14 starting from two different initial conditions. The convergence is very slow due to the long term natural variability of the ocean embedded in this system. The presence of different attractors cannot be confirmed or excluded at this stage, due to the blurring of the large natural variability of the system. This analysis would need even longer model integrations, with a higher order numerical scheme in order to better control the numerical error as suggested by the anonymous referee."
   At the same time, we have tested another scheme and we have obtained a different evolution of the mean as illustrated in Figure D (embedded 8th order Runge-Kutta Prince-Dormand method with adaptive step size and 9th order error estimate). But the convergence is not completed in this case either. Longer integrations would also be needed.
3. One important feature of the coupled model is to display a set of very small amplitude Lyapunov exponents. As you mentioned it is hard to distinguish between the different exponents close to 0 in Figure 5a. In fact for small values of the thermal forcing parameter, Θ^*, the solution are stationary stable solutions with very small amplitude (negative) Lyapunov exponents. When it is increased, periodic and quasi-periodic (2-Torus) solutions appear up to 0.065. 2-torus solutions are also appearing between 0.087 and 0.095 for the parameters explored. The small amplitude exponents are associated with the presence of the ocean whose response time scale is long. Thank you very much for pointing out the theorem of Newhouse, Ruelle and Takens. Indeed there are only one or two exponents very close to 0 in the periodic and quasi-periodic regimes. I discuss this point in more details at paragraph … : "For values of Θ^* smaller than 0.055, stable steady states are found with a set of 4 negative Lyapunov exponents of very small amplitude (e.g. for Θ^*=0.02, σ_1=-0.00128, σ_2=-0.00128, σ_3=-0.00133, σ_4=-0.00133 day^{-1}) and the next ones with amplitudes 1000 times larger. At Θ^*=0.055, a periodic solution emerges with a first exponent equal to σ_1=-1.1 10^{-8} day^{-1}. For larger values up to Θ^*=0.065, quasi-periodic solutions (2-Torus) appear, as well as for the parameter values explored between 0.087 and 0.095. Between 0.065 and 0.087, chaotic solutions separated by periodic windows are prevailing. Beyond 0.095, the dynamics become chaotic and no periodic solutions were found for the parameter range explored."
4. Thank you very much for drawing my attention toward these references. Indeed these will certainly be useful in our future investigations. We add a sentence at the end of the aforementioned paragraph as " A detailed analysis of the transitions from quasi-periodic

motions to chaotic behaviors will be investigated in the future as in recent works (Broer et al, 2011; Sterk et al, 2010; among others)."

Minor points:

1. We added a table with the list of parameters.
2. Thank you for pointing out this confusion. We added a sentence at the end of Section 2.2.

All the other corrections have been made.

Response to the second Referee, Anna Trevisan.

Dear Dr Trevisan

Thank you very much for your comments on our manuscript. Please find a detailed response to your specific comments below.

1. You are right. More information should be added concerning the results obtained with previous low-order coupled ocean models. This further highlights the similarities between the present work and the previous ones, with a marked difference with the results of Nese and Dutton (1993) and much more similarities with the work of van Veen (2003). In the former, the activation of the dynamics within the ocean leads to an increase of predictability. This feature contrasts with our results but could probably be associated with the way the heat is transported in the ocean basin and then transferred toward the atmosphere in their model.
   In the work of van Veen, the ocean plays a "passive" role when the atmosphere is in a chaotic regime, while it plays more active role in setting up the (coupled) dynamics when close to the periodic windows of the atmospheric model. This aspect is in agreement with our results, indicating that in a fully chaotic regime, the presence of the ocean is not increasing in a substantial way the amplitude of the Lyapunov exponents, but plays a more important role close to the periodic windows of the system as illustrated in Figure 6. Figure A shows the temporal evolution of the solution A_2 for two different values of δ and for a value of Θ^*, illustrating the crucial role played by the coupling close to the onset of chaos.
   Both models (Nese and Dutton, and van Veen) are quite different in their conception and it is therefore difficult to conclude why their results are quite different, and at the same time it is difficult to compare with our results which are only based on a mechanical coupling between the two model components. The introduction of a thermodynamic equation would be necessary to clarify this point and will be the subject of a future extension. This aspect will be discussed in more details in paragraph 5, section 3.2:
   "Their results are most probably associated with the way heat is transported in the ocean basin and then transferred toward the atmosphere in their model, a feature not present in our model."
   And at the end of paragraph 4, page 16:

"Interestingly, the results confirm the tendency already reported in van Veen (2003), indicating that the presence of the ocean has a stronger influence on the dynamics of the atmosphere when close to periodic windows."

We also add more information on the results of the two previous works (Nese and Dutton, and van Veen) in the introduction (3rd paragraph):
"The coupled model developed by Nese and Dutton (1993) was used to evaluate the impact of the ocean transport on the predictability of the coupled system. They have found that when the ocean dynamics is activated, an increase of predictability is realized."
And also :
"In this model, a systematic bifurcation analysis has been undertaken and compared with the bifurcation structure of the atmosphere only. In particular it was shown that the ocean plays an important role close to the bifurcation points of the model, but much less in the chaotic regime. In the latter case the ocean integrates the rapid fluctuations of the atmosphere in a quite passive manner without providing a strong feedback toward the atmosphere".

2. Indeed the values of the Lyapunov exponents obtained in the present work are different to the ones found in Trevisan et al (2001). The main reason is the difference of parameter settings chosen (but which were fixed in the range suggested by previous authors, Charney and Straus, 1980 and Reinhold and Pierrehumbert, 1982), in particular the aspect ratio of the domain. Moreover there is no orography in the present version, contrarily to the previous works of Reinhold and Pierrehumbert (1982).
   To try to understand the properties of the increase of Lyapunov instability as a function of the coupling parameter, we have computed the mean absolute amplitude of these vectors along the different modes of the coupled system. Figure B displays the result for the first (backward) Lyapunov vector corresponding to the dominant Lyapunov exponent. The ten first points correspond to the barotropic atmospheric variables, the ten next ones to the baroclinic variables and the last four to the ocean variables. Clearly the projections along the atmospheric variables do not change as a function of the coupling δ, but well the projection along the ocean variables. A similar picture is found for the other backward Lyapunov vectors. This suggests that the increase of Lyapunov instability is mainly associated with an increase of the projection of the Lyapunov vectors along the ocean variables, and not the baroclinic instability within the atmosphere. This aspect is worth investigating further in the future by investigating the properties of the characteristic vectors (also called covariant vectors) of the system which are (non-orthogonal) intrinsic directions of instability (e.g. Legras and Vautard, 1995). The figure and the comments on that feature are introduced at the end of section 3.
3. Concerning the link between the local Lyapunov exponents and the ocean variables, there is no clear relation as illustrated in Figure C, in which the local Lyapunov exponents corresponding to the first Lyapunov vector are plotted as a function of the second ocean variable (similar pictures are found for the other variables). A link is only visible between these local quantities and the dominant modes of the atmosphere, ψ_1 and Θ_1. As in the previous point, a better characterization of the instability properties of the flow and their link with the underlying variables would be worth analyzing based on the characteristic vectors.

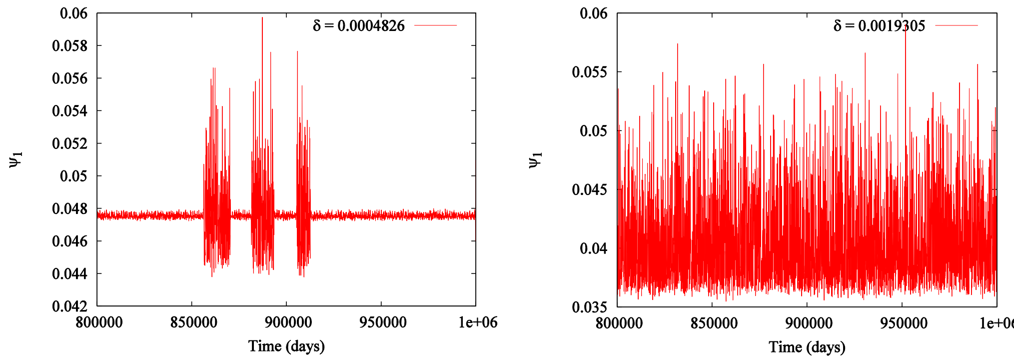

Figure A: Trajectories of Ψ_1 for two values of the coupling parameter and for Θ*=0.10.

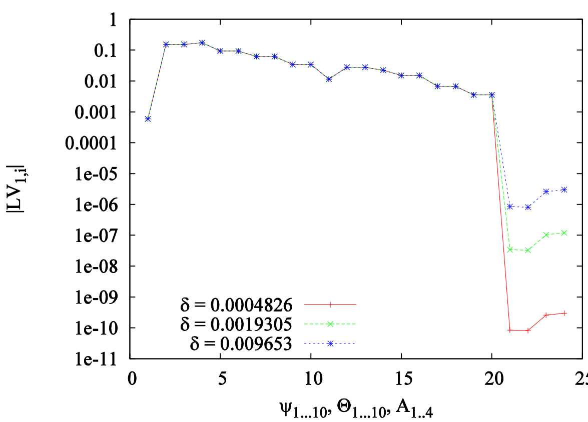

Figure B: Mean absolute amplitude of the first (backward) Lyapunov vector along the variables of the system.

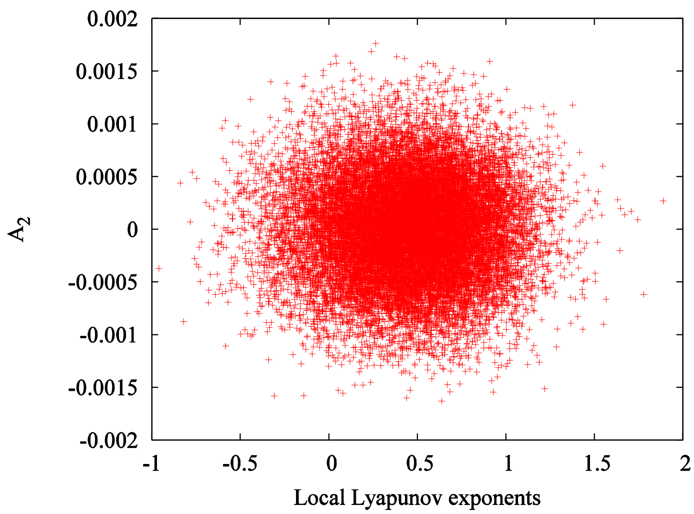

Figure C: 2-D representation of the variable A_2 as a function of the local Lyapunov exponents as obtained with the parameters of Fig. 1 of the manuscript.

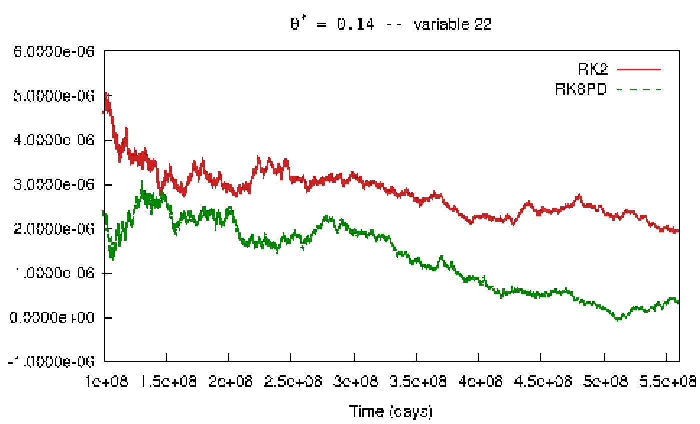

Figure D: Evolution of the temporal average for A_2 with two different numerical schemes. The green one corresponds to an embedded 8th order Runge-Kutta Prince-Dormand method with adaptive step size and 9th order error estimate.