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March 25, 2014

Dear Geoscientific Model Development Editor,

Enclosed please find our revised manuscript titled “Earth Orbit v2.1: a 3D Visualization and Analysis Model of Earth's Orbit, Milankovitch Cycles and Insolation.” Our detailed responses to the reviewer comments are included here, and they are also posted in the online Interactive Discussion.

All relevant manuscript changes are mentioned in the detailed responses to the reviewers. We also provide here a version of the manuscript (without the figures) that tracks all relevant changes, i.e. all changes are annotated so they can be quickly located. Note that Fig. 5 has been replaced completely (it is referred to as Fig. 1 in the Interactive Discussion responses, attached below). Some other figures have been edited in response to the reviewer comments as well.

Thank you for your time and consideration. Please let me know if anything else is needed.

Sincerely,
Tihomir S. Kostadinov

Author Response to Reviewer Comments
Original reviewer comments are included below. Our responses are in *bold italics.*

March 20, 2014

Note: The revised model files are attached as supplement.

Responses to Anonymous Referee #1
Comment: This paper does not provide new scientific breakthrough however it has two important contributions. On the one hand it gives an accurate and detailed overview of the science behind the computation of the solar energy received at the top of the atmosphere (insolation). This first part of the paper is more theoretical. All the different parameters involved in the computation of the insolation are clearly identified and explained. Some more critical aspects are also pointed out. A second part of the paper is devoted to the explanation of the software allowing the users to make this computation easily. The interface and the output are described in detail. Moreover, the figures and graphs displayed have also an educational dimension. At last a section is devoted to the validation of the tool.

General comments
Comment: 1. Paillard, D., L. Labeyrie and P. Yiou (EOS, 1996) developed a tool including the computation of the orbital parameters and the insolation (although it is not the major purpose of the tool). This tool also includes a graphical interface. I would like to suggest the authors to mention this work and to point out their additional contribution.
Response: We thank the reviewer for this comment. We now mention the AnalySeries software in the text and emphasize that the Earth orbit model presented here is developed independently of AnalySeries or other similar efforts. Earth Orbit v 2.1 uses first principles and its own internal geometry. The unique contributions of our model are detailed in a paragraph in the Introduction, but specifically as relates to AnalySeries, the additional contributions/differences are as follows: at the heart of the model of our model is a 3D visualization of Earth's orbit that is geometrically and astronomically accurate and has pan-tilt-zoom capabilities. In the GUI and text we focus on detailed understanding of the geometry of the orbit, and the effect of Kepler's Laws. Our GUI is meant to be very user-friendly and one of our main goals is use in educational settings. We also focus on the user's ease of selecting a demo mode with user-chosen orbital parameters that can be greatly exaggerated to visualize imaginary extreme orbits. The user can easily create many imaginary orbits and study them in 3D. This is an important additional contribution compared to AnalySeries. We focus on attractive, color-enhanced 3D visualizations. Our source-code is open and platform-independent and allows more advanced users and students to study and modify it. Finally, we note and acknowledge that AnalySeries has a somewhat different focus and many more capabilities that are not included in Earth Orbit v2.1 (such as spectral analyses, SSA, MTM; choosing seasonal insolation, defining time as true longitude vs. a date). In summary, we posit that the main additional contribution here is the 3D pan-tilt zoom visualization of the actual orbit, the ability to create imaginary orbits, and the higher level of user-friendliness and applicability to educational settings. We now point the reader to AnalySeries so they can compare and verify our solutions and have access to many more insolation computation options.

Comment: 2. The authors propose two starting dates for the calendar, either vernal equinox (20 March) or perihelion (3 January). The first one is commonly used while the second one is hardly used for the paleo purpose. Could the authors elaborate on the significance of choosing one or the other. Although I

think that astronomers commonly use the second choice, I can hardly imagine how it can be used for computation of past insolation.
Response: The reviewer is correct that the calendar start date of Jan. 3 can be confusing and is not likely to be used in paleoclimate studies. We nevertheless elect to include that option because it is in our view of great educational value, and the model's target audience is much wider than paleoclimatologists. The ability to illustrate the relativity of the calendar with respect to the physical reality of the orbit is important. Users can see that it really matters whether we select to fix perihelion or the equinox as the calendar start date. We note that insolation time series are indeed computed only for the calendar start date fixed at vernal equinox being March 20. This is stated in the GUI next to the button plotting insolation time series, to avoid confusion.

Comment: 3. This is a technical comment but I think it is really very important. Through the paper and even in the software, time units are sometimes yr and sometimes kyr. This is very confusing. I urge the authors to use one OR the other (not both).
Response: This has been fixed and all relevant units are now thousands of years (kyr).

Comment: 4. Insolation is depending on the latitude, the day in the year and the time. The authors provide insolation computation for a given time and the corresponding figure, either absolute values or deviation from present. They also provide insolation computation for a given latitude and the corresponding figure for the absolute values (but not for the deviation from the present). At last, there is no computation and no figure in the case of a given day, which is scientifically very important as well. Therefore I was wondering whether the 'missing' possibilities could be added or will be added in forthcoming releases.
Response: The insolation for a given day at a given latitude is indeed computed and as is stated in the text, this is the fundamental quantity from which the other insolation quantities are computed. This value is always displayed and updated in the GUI ancillary outputs. A paleo-time series of insolation on a given date and latitude is indeed computed and displayed (Fig. 4A in the paper). There are multiple possibilities for further options of plotting and displaying absolute values and anomalies on various time and space scales. We do intend to expand these options in future releases, as the author suggests. We also state in the text that for best options for insolation computations on various scales, and time intervals defined in various ways, the solutions of Berger et al. (2010) and Laskar et al. (2004) should be used. We now also point the user to AnalySeries, which also offers more insolation computation capabilities.

Comment: 5. Along the same line, it would be interesting to add the possibility to compute the insolation integrated over several days.
Response: We thank the reviewer for this suggestion. More insolation computation options are planned for future releases (see above response to comment #4).

Specific comments
Comment: P5949 – l17-28 : the discussion about the period of insolation variations is a bit fuzzy. Short period (11-yr sunspot cycle) and multi-millennial variability are discussed. How-ever, the short term variability of the orbital parameter is not mentioned. Moreover, the amplitude of these variations and their relative importance in insolation changes is not discussed.
Response: Bertrand et al. (GRL, 2002) discuss these high frequency orbital fluctuations in the context of their effect on insolation and climate. They conclude that high-frequency orbital variability is of very low amplitude and its effect on insolation and surface temperature is negligible, essentially

equivalent to model noise. The intrinsic solar variability (11-yr cycle) is shown to have a larger effect on both insolation and climate. This is now discusses briefly in the manuscript.

Citation: Bertrand, C., M. F. Loutre, and A. Berger, High frequency variations of the Earth's orbital parameters and climate change, Geophys. Res. Lett., 29(18), 1893,10.1029/2002GL015622, 2002.

Comment: P5950 – l2 : : : : the longitude of perihelion relative to the moving vernal equinox : : :
Response: Fixed.

Comment: P5950 – l8 : kyr should be used (and maybe defined) instead of Ky.
Response: Fixed.

Comment: P5950 – l17 : ": : : derived for several tens of million years : : :". Of course the mathematical computation can be done over such period. However, it would be more interesting to given an order of magnitude for time interval of validity/reliability of the solutions. Berger's (1978) is definitely much less than several tens of million years.
Response: We have now specified the period of validity for both the Berger and the Laskar solutions, which corresponds to the allowable years since J2000 that can be selected in the GUI.

Comment: P5952 – l4 :"insolation computation logic". I do not understand what the authors are referring to.
Response: This was paraphrased to state "no insolation computation code". We mean that we have not borrowed any code or derived formulae from the above solutions; rather we start from first principles.

Comment: P5955 – equation 1. There is some potential confusion here. The authors mention that the model uses a heliocentric Cartesian coordinate system. However, the equation is the equation of the ellipse in a polar coordinate system with one of the foci at the origin. Moreover, it seems (although I may be wrong) that the authors discuss several coordinate systems, depending on what they are computing.
Response: This paragraph has been rewritten to properly reflect that Eq. 1 is in polar coordinates, which are then converted to Cartesian heliocentric for plotting, because the main model coordinate system is Cartesian heliocentric. We also explicitly state that the Earth itself is parameterized in its own coordinate system which is then oriented properly in 3D and translated to Earth's chosen position on the orbit, and the plotted.

Comment: L5961 – l25 : please remind the reader what J2000 means.
Response: J2000 is now defined in the Introduction and readers are also reminded of the definition here.

Comment: P5963 – l3 : 'Figure 2b illustrate an imaginary orbit : : :' It would be nice to discuss further this orbit. It is indeed very surprising at first to see that July 1 occurs already during Fall.
Response: We have added a detailed discussion of this imaginary orbit. We have also added a third example orbit, for the case of 10 kyr in the future, to illustrate what happens to the real orbit due to precession effects. We agree this adds an important component to the discussion.

Comment: P5964 – l2-6 : I wouldn't have added some data at this stage. The software is indeed very interesting for the computation of the orbital parameters and the insolation, but the data (whatever

they are) are a completely different story, very complex as well. In particular the chronology of the data is a full story by itself. On the other hand I can hardly see the added value of these specific data. Why not choosing other data? For example, Lisiecki and Raymo (2005) provide a much longer climate record.
Response: We have now added a separate button in the GUI for optional plotting of paleoclimate data in its own separate figure window. In addition to the EPICA ice core data, we have now added the Lisiecki and Raymo (2005) benthic stack and the Zachos et al. (2001) oxygen isotope data sets, both of which go further back in time than the EPICA data. We have also added discussion in the text addressing the fact that it is provided here for information and educational purposes only and is not a focus of the model at this stage, and the chronology of the data itself needs to be treated with caution. In the future, some added functionalities may use the data more extensively. We now end this paragraph with the following statement: "These paleoclimatic data are included for convenience of the user and no further interpretation or analyses are provided. Users are cautioned that the interpretation of these paleoclimatic signals and their uncertainties, time-resolution and chronology (age models) is fairly complex and beyond the scope of this model. They are provided here for illustrative purposes only, e.g. it enables users to easily visualize the last few glacial-interglacial cycles (and the mid-Pleistocene transition to 100-kyr cyclicity, see Introduction), or to visually correlate these paleoclimatic time series with the corresponding Milankovitch parameter and insolation curves."

Comment: P5966 l7 : "+10000 yr since present". Does it mean in the future?
Response: Yes. This usage is now avoided in the text and it is clearly stated in the GUI that negative years are in the past and positive – in the future.

Comment: P5966 – l18 : the assumption already discussed should be reminded instead of quoting the section where they are discussed.
Response: This sentence has been modified to remind the reader what the section referred to is discussing.

Comment: P5966 – l23 : The time interval of reliability of the solutions should be reminded here.
Response: Done.

Comment: P5967 – l27 : "K-12 classroom". I do not know what it is. Does it correspond to the age of some pupils/students?
Response: This has been rephrased so it is understandable to an international audience.

Comment: Table 1 and figure 1 : the value of the AU is not the same in the table and in the figure.
Response: We have fixed this and the Table 1 value is the same as the one used in the model/displayed in the GUI.

Comment: Figure 5 : Does the authors really mean 1σ computed over three data points? Is it meaningful? Wouldn't it better to give the values for each of the three dates? Or (if possible) make the computation over 365 days.
Response: We have now performed a much more extensive validation (our model with La2004 orbital parameters vs. Laskar's software) for two dates and three latitudes over the entire period of 200 kyr in the past to 200 kyr in the future, with a 1 kyr step. We also superimpose the difference between the Be78 and La2004 solutions as computed by our model. We show the results as absolute difference and percent difference. Results indicate that generally inter-solution differences are larger than the model validation differences. This is now discussed in the text, and the validation figure has been replaced with the figure attached here (Fig. 1).

Comment: Figure 6 : What causes the discontinuity? Is it related to February 29?
Response: The short answer is yes. The discontinuities are caused by two factors. First, the Meeus (1998) data discontinuity in the rate of change and standard deviation of declination are caused by the way Feb. 29 is treated in the leap year – it is removed from the data and dates are counted in the four-year average, not days of year. Thus, for the mean declination for July 1 across all 4 years, all July 1 values were averaged, including the one for the leap year. Second, the Earth orbit model treats March 19 as a longer day to account for the fact that the sidereal year is ~365.25 days. This is the case when the calendar is fixed to start on vernal equinox on March 20 (when declination in the model is always exactly 0 degrees). Thus, the modeled declination has a discontinuity in its rate of change on March 19. Because of the above, the Meeus (1998)-based curves (green and red) exhibit a discontinuity on Feb.28/March 1, and the validation difference curve (black curve) exhibits discontinuities both on Feb.28/March 1 and March 19/March20. We note that these discontinuities are very small. A brief discussion of this is added in the main text.

Author Response to Reviewer Comments
Original reviewer comments are included below. Our responses are in *bold italics.*

March 20, 2014

Note: Some responses here refer to our responses to Reviewer #1. Please also see the figure attached to those comments. The revised model files are included as supplement to our response to reviewer #1 as well.

Responses to Anonymous Referee #2

This paper presents a very useful tool for visualising and analysing the Earth's orbit. The Earth Orbit Model will have wide applications for both science research and education. The paper is presented in two parts; initially an overview of orbital fluctuations or the Milankovitch cycles is given. The software is then explained in detail and a brief section on model validation is included. In general the paper is well written and provides enough detail for the end user to appropriately use the model.

Specific comments on the manuscript
Comment: 1. There are inconsistencies in the usage of the terms Ky and yr that should be addressed.
Response: These inconsistencies have been addressed and kiloyear (kyr) is used everywhere to mean one thousand years.

Comment: 2. pp 5950 (8): The authors mention the 100 Ky problem briefly. The text would benefit from a little more explanation as to the nature of the problem and its significance.
Response: We now have expanded the discussion of the 100-kyr/mid-Pleistocene transition problem, including stating the problem and briefly summarizing the currently proposed solutions.

Comment: 3. Although it is interesting to see the EPICA/deuterium data included in the model, there are climate records that span greater intervals of Earth history that would also be useful to compare to the orbital parameters. For example, going back 5 Ma, the benthic oxygen isotope curve of Lisiecki and Raymo (2005) could be added. Or to go back even further in time, one of the Zachos compilations would be a useful comparison. It might also be better to allow the user the option to display the data-model comparison or not, as looking further back in time than the EPICA record, one currently ends up with an empty plot at the bottom of the screen.
Response: See our response to the Reviewer #1 comment "P5964 – l2-6." Thank you for suggesting the Lisiecki and Raymo(2005) and the Zachos(2001) data sets, which we have now included as a separate optional figure in the model.

Comment: 4. True anomaly is first mentioned on pp 5955 (9 & 22 & 25), however the term is only described on pp 5957. I think the reader would benefit from description/definition of the true anomaly in the first instance that the term is used.
Response: We now define true anomaly in Sect 2.1 and mean anomaly in Sect. 2.2, where their definitions belong best. These terms are still mentioned a paragraph earlier when we discuss the solar constant, because we believe it's important to point out that averaging over angle vs. over time yields different results. The definitions of these terms do not belong there, however, so we clearly point the reader to the immediately following definitions.

Comment: 5. pp 5961 (7-18) – The authors discuss the effect of the varying length of the seasons over geologic time scales. I am not sure if I have understood correctly, but is this essentially the same problem as has been identified within the palaeoclimate community as pertaining to the definition of the calendar (i.e. fixed-day/angular or classical)? If this is the same problem, it might be useful to the reader to have a few sentences relating the author's description of the problem to other published efforts to understand the effect of this problem, when considering different orbits over time (e.g. Jousamme and Braconnot, 1997; Chen et al., 2010 – Climate Dynamics)
Response: Yes, this is the same problem. We have edited and expanded this section significantly and the fixed-day calendar vs. the fixed-angular calendar issue is now discussed at length in the context of Joussaume and Braconnot, 1997 and Chen et al. (2010), using consistent terminology.

Comment: 6. Validation of insolation output. It is great to see the authors validate their independent calculations in such a way. The authors state: "Validation is excellent; all test cases result in differences less than 1Wm-2 (Fig. 5)". It might be easier for the reader to determine the effectiveness of the insolation solution by comparing the magnitude of the disparity to the kind of magnitude in discrepancy that one might expect using alternative astronomical solutions for that time (e.g. BL78, BL92 and Laskar, 2010). In other words, the different astronomical solutions may lead to an insolation discrepancy of X Wm-2 at certain points in time, with the differences becoming larger in the further back in time (on the whole). How do the differences shown here compare to the inter-solution differences one might expect?
Response: See the response to Reviewer #1 Comment to Figure 5, as well as the attached figure, showing the much more extensive validation, as well as the inter-solution comparison you have suggested.

Comment: 7. pp 5965 (7) – I find the term +10 000 yr since present difficult to understand, is this 10 000 years in the future? Perhaps rephrase.
Response: This text has now been replaced. This usage is avoided in the text and clearly explained in the GUI.

Comment: 8. 5965(23) – Define UT
Response: UT (Universal Time) is now defined in the Introduction.

Comment: 9. Figure 2. It might be useful for the reader to have a little more description as to what the lines on the orbital configuration mean – give details of the black and the red lines. Maybe additionally show a Palaeo case and describe what it means when you are looking at July and it falls in the wrong season with regards to present day. Also, at what latitude is figure 2a representing for the 16 September and using which calendar start date. It would be good for the figure caption to contain enough information that the GUI user could reproduce it with ease. The text on the figure in this case is a little small and difficult to read.
Response: We have significantly expanded the caption of Figure 2 to explain all the elements of the plot. An additional real case for 10 kyr in the future has been shown (roughly equivalent in terms of precession to 10 kyr in the past), and then the demo exaggerated case is shown. These are discussed in more detail in the main text as well.

Comment: 10. Figure 5. Could the authors comment as to why the error bars are so much larger at 80_S 10,000 years ago than for any of the other cases?

Response: The validation has now been replaced with a much more extensive one – See the response to Reviewer #1 Comment to Figure 5, as well as the attached figure. The large number in question here is 1) not large compared to some others in the new validation figure, 2) not large at all in terms of percentage difference(not shown). Note that this particular case is no longer a part of the validation, but to address your specific question, insolation at polar latitudes can be very sensitive to the exact orbital parameters, because at low solar altitudes above the horizon, the derivative of the cosine of solar zenith angle is large. Day length itself becomes a value of large uncertainty in some cases. So larger differences are expected in this case due to the exact treatment of these issue and numerical procedures used. The above does not necessarily explain why the case for 10kyr in the future exhibits larger differences than the present, but it demonstrates that the exact values of insolation are highly sensitive there.

Comment: 11. Figure 6. It is not clear from the figure caption or the text what causes the discontinuity in the green and black lines in figure 6. Is this related to the leap year considered? Could the authors elaborate on this please?
Response: See our answer to the Figure 6 comment of reviewer #1. We have now added a brief discussion in the text addressing these discontinuities. They are due indeed to the treatment of Feb 29 in the leap year and the nature of March 19 (the day before equinox) in the model year (which uses the sidereal orbital period, and is neither leap nor common).

Specific comments pertaining to the GUI
Comment: 1. In the Milankovitch Orbital Parameters section the input requirements for the choice of year is in years since J2000, however, in the Time Series and Insolation Plotting Options section, the input should be in thousands of years since J2000. This is slightly confusing and it should be considered whether the inputs should both be in the same units. Additionally an example alongside the input box of what numbers (+/-) would be necessary input values to calculate insolation properties for e.g. 20,000 ka BP or 2 Ma ago, would be beneficial to the user.
Response: The units have been changed consistently to kyr everywhere where relevant in the GUI and text, and an example input is stated in the GUI just above the input boxes for the Laskar or the Berger solutions.

Comment: 2. The authors refer to the Astronomical Unit in Table 1 as a constant/variable model input parameters. From the GUI the AU changes with every solution requested – that AU varies might not be obvious to the reader. Perhaps therefore, the authors should list in the table which values are constant assumptions and which vary depending upon the GUI inputs.
Response: The semi-major axis is a prescribed model constant and it does not change with the different solutions or in the demo mode. It is almost exactly equal to 1 AU. This has now shown in a less ambiguous way in the GUI. The Table 1 values were slightly different from the GUI, which has now been fixed. The table also clearly lists which values are constants. The text (Sect.2, paragraph 1) also explains that the semi-major axis determines the orbital sidereal period, which is also a prescribed constant, i.e. Kepler's 3rd law is implemented implicitly by using these consistent constants.

Earth Orbit v2.1: a 3D Visualization and Analysis Model of Earth's Orbit, Milankovitch Cycles and Insolation

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T.S. Kostadinov dedicates this paper to his Mother, who first sparked his interest in the magnificent night sky and the science of astronomy.

Version 2 – revised after reviewer comments

March 25, 2014

Abstract

Milankovitch theory postulates that periodic variability of Earth's orbital elements is a major climate forcing mechanism, causing, for example, the contemporary glacial-interglacial cycles. There are three Milankovitch orbital parameters: orbital eccentricity, precession and obliquity. The interaction of the amplitudes, periods and phases of these parameters controls the spatio-temporal patterns of incoming solar radiation (insolation) and the timing of the seasons with respect to perihelion. This complexity makes Earth-Sun geometry and Milankovitch theory difficult to teach effectively. Here, we present "Earth Orbit v2.1": an astronomically precise and accurate model that offers 3D visualizations of Earth's orbital geometry, Milankovitch parameters and the ensuing insolation forcing. The model is developed in MATLAB® as a user-friendly graphical user interface. Users are presented with a choice between the Berger (1978a) and Laskar et al. (2004) astronomical solutions for eccentricity, obliquity and precession. A "demo" mode is also available, which allows the Milankovitch parameters to be varied independently of each other, so that users can isolate the effects of each parameter on orbital geometry, the seasons, and insolation. A 3D orbital configuration plot, as well as various surface and line plots of insolation and insolation anomalies on various time and space scales are produced. Insolation computations use the model's own orbital geometry with no additional *a-priori* input other than the Milankovitch parameter solutions. Insolation output and the underlying solar declination computation are successfully validated against the results of Laskar et al. (2004) and Meeus (1998), respectively. The model outputs some ancillary parameters as well, e.g. Earth's radius-vector length, solar declination and day length for the chosen date and latitude. Time-series plots of the Milankovitch parameters and some relevant paleoclimatological data can be produced. Both research and pedagogical applications are envisioned for the model.

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1 Introduction

The astrophysical characteristics of our star, the Sun, determine to first order the continuously habitable zone around it (Kasting et al., 1993; Kasting, 2010), in which rocky planets are able to maintain liquid water on their surface and sustain life. The surface temperature of a planet depends to first order upon the incoming flux of solar radiation (insolation) to its surface. Additionally, energy for our metabolism (and most of modern economy) is obtained exclusively from the Sun via the process of oxygenic photosynthesis performed by green terrestrial plants and marine phytoplankton. The high oxygen content of Earth's atmosphere, necessary for the evolution of placental mammals (Falkowski et al., 2005), is due to billions of years of photosynthesis and the geological burial of reduced carbon equivalents (Falkowski et al.,2008a; Falkowski et al., 2008b; Kump et al., 2010). Thus, the Sun is central to climate formation and stability and to our evolution and continued existence as a species.

The temporal and spatial patterns of insolation and their variability on various scales determine climatic stability over geologic time, as well as climate characteristics such as diurnal, seasonal and pole to Equator temperature contrasts, all of which influence planetary habitability. Insolation can change due to changes in the luminosity of the Sun itself. This can happen due to the slow increase of solar luminosity that gives rise to the Faint Young Sun Paradox (Kasting, 2010; Kump et al., 2010), or it can happen on much shorter time scales such as the 11-yr sunspot cycle (Fröhlich, 2013; Hansen et al., 2013). Importantly, insolation is also affected by the orbital elements of the planet. According to the astronomical theory of climate, quasi-periodic variations in Earth's orbital elements cause multi-millennial variability in the spatio-temporal distributions of insolation, and thus provide an external forcing and pacing to Earth's climate (Milankovitch, 1941; Berger 1988; Berger and Loutre, 1994; Berger et al., 2005). These periodic orbital fluctuations are called Milankovitch cycles, after the Serbian mathematician Milutin Milanković who was instrumental in developing the theory (Milankovitch, 1941). Laskar et al.(2004) provide a brief historical overview of the main contributions leading to the pioneering work of Milanković. There are three Milankovitch orbital parameters: orbital eccentricity (main periodicities of ~100 and 400 kyr (1 kyr = one thousand years)), precession (quantified as the longitude of perihelion relative to the moving vernal equinox, main periodicities 19 and 23 kyr) and obliquity of the ecliptic (main periodicity 41 kyr) (Berger 1978a). Obliquity is strictly speaking a rotational, rather than an

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orbital parameter; however, we refer to it here either as an orbital or Milankovitch parameter, for brevity.

The pioneering work by Hays et al. (1976) demonstrated a strong correlation between these cycles and paleoclimatological records. Since then, multiple analyses of paleoclimate records have been found to be consistent with Milankovitch forcing (e.g. Imbrie et al., 1992; Rial, 1999; Lisiecki and Raymo, 2005). Notably, the glacial-interglacial cycles of the Quaternary have been strongly linked to orbital forcing, particularly summertime insolation at high Northern latitudes (Milankovitch, 1941; Berger 1988; Berger and Loutre, 1994; Bradley, 2014 and refs. therein). Predicting the Earth system response to orbital forcing (including glacier growth and melting) is not trivial, and there are challenges in determining which insolation quantity (i.e. integrated over what time and space scales) is responsible for paleoclimate change, e.g. peak summer insolation intensity, or overall summertime integrated insolation at Northern latitudes (Imbrie et al., 1993; Lisiecki et al., 2008; Huybers, 2006; Huybers and Denton, 2008; Bradley, 2014). Moreover, some controversies related to the astronomical theory remain, notably the 100-kyr problem, or the so-called mid-Pleistocene transition. This refers the fact that the geological record indicates that the last 1,000 kyr have been dominated by 100-kyr glacial-interglacial cycles, a gradual switch from the previously dominant 41-kyr periodicity. This transition cannot be explained by orbital forcing alone, as there was actually a decrease in 100-kyr variance in this eccentricity band (e.g. Imbrie et al., 1993; Loutre et al., 2004; Berger et al., 2005; Bradley 2014 and refs. therein). Current consensus focuses on the explanation that the mid-Pleistocene transition is due to factors within the Earth system itself, rather than astronomical factors – e.g. internal climate system oscillations, nonlinear responses due to the continental ice sheet size, or CO_2 degassing from the Southern Ocean (Bradley 2014, Sect. 6.3.3 and 6.3.4 and refs. therein). Finally, alternative astronomical influences on climate have also been proposed, such as the influence of the orbital inclination cycle (Muller and McDonald, 1997).

The Milankovitch cycles are due to complex gravitational interactions between the bodies of the solar system. Astronomical solutions for the values of the Milankovitch orbital parameters have been derived by Berger (1978a) and Berger (1978b), referred to henceforth as Be78 (valid for 1,000 kyr before and after present), and Laskar et al. (2004), referred to

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While most Earth science students and professionals are well aware of Earth's orbital configuration and the basics of the Milankovitch cycles, the details of both and the way the Milankovitch orbital elements influence spatio-temporal patterns of insolation on various time and space scales remain elusive. It is difficult to appreciate the pivotal importance of Kepler's laws of planetary motion in controlling the effects of Milankovitch cycles on insolation patterns. The three-dimensional nature of Earth's orbit, the vast range of space and time scales involved, and the geometric details are complex, and yet those same factors present themselves to computer modeling and 3D visualization. Here, we present "Earth Orbit v2.1": an astronomically precise and accurate 3-D visualization and analysis model of Earth's orbit, Milankovitch cycles, and insolation. The model is envisioned for both research and pedagogical applications and offers 3D visualizations of Earth's orbital geometry, Milankovitch parameters and the ensuing insolation forcing. It is developed in MATLAB® and has an intuitive, user-friendly graphical user interface (GUI) (Fig. 1). Users are presented with a choice between the Be78 and La2004 astronomical solutions for eccentricity, obliquity and precession. A "demo" mode is also available, which allows the three Milankovitch parameters to be varied independently of each other (and exaggerated over much larger ranges than the naturally occurring ones), so users can isolate the effects of each parameter on orbital geometry, the seasons, and insolation. Users select a calendar date and the Earth is placed in its orbit using Kepler's laws; the calendar can be started on either vernal equinox (March 20) or perihelion (Jan. 3). A 3D orbital configuration visualization, as well as spatio-temporal surface and line plots of insolation and insolation anomalies (with respect to J2000) on

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various scales are then produced. Below, we first describe the model parameters and implementation. We then detail the model user interface, provide instructions on its capabilities and use, and describe the output. We then present successful model validation results, which are comparisons to existing independently derived insolation, solar declination and season length values. Finally, we conclude with brief analysis of sources of uncertainty. Throughout, we provide examples of the pedagogical value of the model.

Various insolation solutions and visualizations exist (Berger, 1978a; Rubincam, 1994 (however, see response of Berger, 1996); Laskar et al., 2004; Archer, 2013; Huybers, 2006). Notably, the AnalySeries software (Paillard et al., 1996; Paillard, 2014) shares many of the functionalities presented here and offers many additional ones, such as paleoclimatic time-series analysis and many more choices for insolation computation. Importantly, the model presented here was developed independently from AnalySeries (or other similar efforts) and computes insolation from first principles of orbital mechanics (Kepler's laws) and irradiance propagation, using exclusively its own internal geometry. The only model inputs are the three Milankovitch orbital parameters, either real astronomical solutions (Be78 or La2004) or user-entered demo values. No insolation computation code from the above-cited existing solutions has been used, so comparison with these solutions constitutes independent model verification, referred to here as validation, because we consider the La2004 and Meeus (1998) solutions the geophysical truth (Sect. 5).

The unique contribution of our model consists of the *combination* of the following features: a) central to the whole model is a user-controllable, 3D pan-tilt-zoom plot of the actual Earth orbit, b) an interactive user-friendly GUI that serves as a single-entry control panel for the entire model and makes it suitable for use by non-programmers and friendly to didactic applications, c) the Milankovitch cycles are incorporated explicitly and insolation is output according to real or user-selected demo orbital elements, which d) allows users to enter exaggerated orbital parameters independently of each other and isolate their effects on insolation, as well as view the orbit with exaggerated eccentricity, e) the source code is published and advanced users can check its logic, as well as modify it and adapt it, and f) the software is platform-independent.

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The issue of climate change has come to the forefront of Earth science and policy and it is arguably the most important global issue of immediate and long-term consequences (e.g. IPCC, 2013). Earth's climate varies naturally over multiple time scales, from decadal to hundreds of millions of years (e.g. Kump et al., 2010). It is thus crucial to understand natural climate forcings, their time scales, and the ensuing response of the Earth system. In addition, detailed understanding of the Sun's daily path in the sky and the patterns of insolation have become important to increasing numbers of students and professionals because of the rise in usage of solar power (thermal and photovoltaic). We submit that the model presented here can enhance understanding of all of these important subject areas.

2 Key Definitions, Model Parameters and Implementation

The model input parameters, and their values and units, are summarized in Table 1. The following definitions, discussion and symbols are consistent with those of Berger et al. (2010). The reader is referred to their Figure 1. According to Kepler's First Law of Planetary Motion, Earth's orbit is an ellipse, and the Sun is in one of its foci (e.g. Meeus, 1998). Orbital eccentricity, *e* (Table 1), is a measure of the deviation of Earth's orbital ellipse from a circle and is defined as $e = \sqrt{1 - b^2 / a^2}$,where *a* is the semi-major axis (Table 1) and *b* is the semi-minor axis of the orbital ellipse (e.g. Berger and Loutre, 1994). The semi-major axis is about equal to 1 AU (Meeus, 1998; Standish et al., 1992) and determines the size of the orbital ellipse and thus the orbital period of Earth; it is considered a fixed constant in the model, as its variations are extremely small (Berger et al, 2010; Laskar et al, 2004, their Fig. 11). Various orbital period definitions are possible; here, the sidereal period is used as a model constant (Meeus, 1998). Thus, Kepler's Third Law of Planetary Motion is implicit in these two constant definitions and is not included explicitly elsewhere in model logic. The obliquity of Earth, ε, is the angle between the direction of its axis of rotation and the normal to the orbital plane, or the ecliptic (Table 1). Eccentricity and obliquity are two of the three Milankovitch orbital parameters.

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The third Milankovitch orbital parameter, precession, is the most challenging for instruction and visualization. There are two separate kinds of precession that combine to create a climatic effect – precession of the equinoxes (also termed axial precession), and apsidal precession, i.e. precession of the perihelion in the case of Earth's orbit. Axial precession refers to the wobbling of Earth's axis of rotation that slowly changes its absolute orientation in space with respect to the distant stars. The axis or rotation describes a cone (one in each hemisphere) in space with a periodicity of about 26,000 years (Berger and Loutre, 1994). This is the reason why the star α UMi (present-day Polaris, or the North Star), has not and will not always be aligned with the direction of the North Pole. Also, due to axial precession, the point of vernal equinox in the sky moves with respect to the distant stars and occurs in successively earlier zodiacal constellations. Axial precession is clockwise as viewed from above the North Pole, hence the North Celestial Pole describes a counter-clockwise motion as viewed by an observer looking in the direction of the North Ecliptic Pole. Precession of the perihelion refers to the gradual rotation of the line joining aphelion and perihelion, with respect to the distant stars (or the reference equinox of a given epoch) (Berger, 1978a, Berger and Loutre, 1994).

Axial precession and precession of the perihelion combine to modulate the *relative* position of the equinoxes and solstices (i.e. the seasons) with respect to perihelion, which is what is relevant for insolation and climate. This climatically-relevant precession is implemented in the model and is quantified via the longitude of perihelion, $\tilde{\omega}$, which is the angle between the directions of the moving fall equinox and perihelion at a given time, measured counterclockwise in the plane of the ecliptic (Berger et al., 2010). Because both perihelion and equinox move, the longitude of perihelion will have a different (shorter) periodicity than one full cycle of axial wobbling alone (Berger and Loutre, 1994). The direction of Earth's radius-vector when Earth is at fall equinox (~Sept. 22) is referred to as the direction of fall equinox above. This is the direction with respect to the distant stars where the Sun would be found on its annual motion on the ecliptic on March 20th – i.e. at vernal equinox. In other words, that is the direction of the vernal point in the sky (Berger et al. 2010, their Fig. 1 & Appendix B), the origin of the right ascension coordinate. This distinction between vernal equinox and the direction of the vernal point can cause confusion, especially since the exact definition of longitude of perihelion can vary (e.g. c.f. Berger et al., 1978a; Berger et al., 1993; Berger and Loutre 1994; Joussaume and Braconnot, 2007; Berger et al., 2010) and the longitude of perihelion can also be confused with the longitude of perigee, $\omega = \tilde{\omega} + 180^{o}$, which is the angle between the directions of vernal equinox and perihelion, measured

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counterclockwise as viewed from the North Pole direction, in the plane of the orbit (Berger et al, 2010). Here, we use the terminology and definitions of Berger et al. (2010).

The magnitude of the climatic effect of precession is modulated by eccentricity. In the extreme example, if eccentricity were exactly zero, the effects of precession would be null. Climatic precession, $e\sin\omega$, is the parameter that quantifies precession and determines season lengths, the Earth-Sun distance at summer solstice (Berger and Loutre, 1994) and various important insolation quantities (Berger et al., 1993, their Table 1). This interplay between eccentricity and precession presents an important way to introduce both concepts pedagogically and to test student comprehension.

The solar "constant", S_o, is defined here as the total solar irradiance (TSI) on a flat surface perpendicular to the solar rays at a reference distance of exactly 1 AU (Table 1). As Berger et al. (2010) note, due to eccentricity changes, the mean distance from the Earth to the Sun over a year is not constant on geologic time scales. It also matters how this mean distance is defined – e.g. over time (mean anomaly) vs. over angle (true anomaly). True and mean anomaly are defined below in Sect. 2.1 & 2.2, respectively. If S_o is defined to be the irradiance from the Sun at the mean Earth-Sun distance, then it is indeed not a true constant. As used here, S_o is a true model constant as long as the luminosity of the Sun itself is assumed constant. The default value is chosen to be 1,366 W m^{-2} (Fröhlich, 2013). Recent evidence suggests that the appropriate value may actually be about 1,361 W m^{-2} (Kopp and Lean, 2011). Users can change the value of S_o independently of other model inputs in order to study the effects of changes in absolute solar luminosity – e.g. in order to simulate the Faint Young Sun (e.g. Kasting, 2010) or the sunspot cycle (e.g. Hansen, 2013).

2.1 Model Coordinate System; Sun-Earth Geometry Parameterization; Solar Declination

According to Kepler's First Law of Planetary Motion, Earth orbits the Sun in an ellipse, and the Sun is in one of the ellipse's foci. The heliocentric equation of the orbital ellipse in polar form is given by (Meeus, 1998; his Eq. 30.3):

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$$|\mathbf{r}(\nu)| = \frac{a(1-e^2)}{1+e\cos\nu} \qquad (1)$$

In the above, the Sun is at the origin of the coordinate system, *a* is the semi-major axis of the orbital ellipse, *e* is eccentricity, ν is true anomaly, and **r** is Earth's instantaneous radius-vector, i.e. the vector originating at the Sun and ending at the instantaneous planetary position. True anomaly, ν, is the angle between the directions of perihelion and the radius-vector, subtended at the Sun and measured counter-clockwise in the plane of the orbit (e.g. Meeus (1998), his Ch. 30; Berger et al. (2010) their Fig. 1). The true longitude of the Sun (or simply true longitude) is equal to Earth's true anomaly plus the longitude of perigee (Berger et al., 2010, their Eq. 6). True longitude is the angle Earth has swept from its orbit, subtended at the Sun, since it was last at vernal equinox. Mean longitude is the longitude of the mean Sun, in an imaginary perfectly circular orbit of the same period, i.e. mean longitude is proportional to the passage of time, much like mean anomaly (See Sect. 2.2 below).

In the Earth Orbit v2.1 model, given a user-selected calendar date, true anomaly, ν, is determined by solving the inverse Kepler equation (see Section 2.2 below). The Earth's radius vector is then solved for using Eq. 1 above. Because the main model coordinate system is heliocentric Cartesian, the (**r**,ν) pair of polar coordinates is then transformed to Cartesian (x,y) for plotting. The Earth is initially parameterized as a sphere in its own geocentric Cartesian coordinate system in terms of its radius and geographic latitude and longitude (corresponding to the two angles of a spherical coordinate system). The Earth's coordinate system's x and y axes are in the plane of the Equator (shown as a black dotted line, Fig. 2), and its z-axis is pointing towards the true North Pole and is coinciding with Earth's axis of rotation; these axes are also plotted in black dotted lines, the z-axis is lengthened toward North so that it pierces Earth's surface and is labeled, since this is critical in the definition and understanding of the seasons. Earth is plotted as a transparent mesh so that important orbital elements can be seen through it at various zoom levels (Fig. 2). The color scale of Earth's mesh is just a function of latitude and no day and night sides are explicitly shown. Earth's radius is not to scale with the orbit itself or with the Sun's radius. Thus, the center of Earth has its true geometric orbital position (and is the tip of its instantaneous radius-vector); however the surface of the sphere in the model is arbitrary and must not be interpreted as the true surface onto which insolation is computed, for example. The insolation

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computations (Sect. 2.3) are geocentric. The Sun is also plotted (not to scale) as a sphere centered at the origin of the main model coordinate system.

The Earth is oriented properly in 3D with respect to the orbital ellipse by using a rotation matrix to rotate its coordinate system. The 3D rotation matrix is computed using Rodrigues' formula (Belongie, 2013) for 3D rotation about a given direction by a given angle. The direction about which Earth is rotated is determined by a vector which is always in the orbital plane (**k**-component is zero), and the **i** and **j** components are determined by the longitude of perihelion. The angle by which Earth is rotated is determined by obliquity. Thus, the rotation matrix is a function of two of the three Milankovitch parameters and is a valuable and useful instructional tool/concept for lessons in geometry, mathematics, astronomy, physical geography, and climatology. At this point the Earth is correctly oriented in 3D space with respect to its orbit and the distant stars. Earth is then translated to its proper instantaneous position on its orbit by addition of its radius-vector to all relevant Earth-bound model elements (which are then plotted in the main heliocentric coordinate system).

Declination is one of the two spherical coordinates of the equatorial astronomical coordinate system. It is measured along a celestial meridian (hour circle) and is defined as the angle between the celestial Equator and the direction toward the celestial object (Meeus, 1998). Solar declination varies with the seasons, due to obliquity. It is zero at the equinoxes, reaches a maximum of $+\varepsilon$ at summer solstice and a minimum of $-\varepsilon$ at winter solstice. Solar declination determines the length of day and the daily path of the Sun in the sky at a given latitude, i.e. its altitude and azimuth above the horizon as a function of time. Thus, solar declination determines instantaneous and time-integrated insolation. In turn, solar declination and its evolution over the course of a year are a function of the orbital elements; thus it provides the mathematical and conceptual link between the Milankovitch orbital elements and insolation and climate. Here, we compute instantaneous solar declination using the angle between the direction of the North Pole and Earth's radius-vector, calculated using their dot product. Thus, we explicitly compute solar declination from the geometry of the model and it is a model *emergent property* rather than prescribed *a-priori*; therefore, this also applies to insolation computations (Section 2.3 and Sect. 5).

2.2 Implementation of Kepler's Second Law of Planetary Motion

The heliocentric position of a planet in an elliptical orbit at a given instant of time is given in terms of its true anomaly, ν – see Eq. 1 and Section 2.1 above. True anomaly can also be thought of as the angle (subtended at the Sun) which the planet has “swept” from its orbit since last perihelion passage. Kepler's Second Law of planetary motion states that the planet will “sweep” equal areas of its orbit in equal intervals of time and governs the value of true anomaly as a function of time (e.g. Meeus, 1998; Joussaume and Braconnot, 1997). At non-zero eccentricity, ν is not simply proportional to time since last perihelion passage (time of flight) expressed as a fraction of the orbital period in angular units. The latter quantity is called mean anomaly, M. Kepler's Second Law is used to relate M and ν, using an auxiliary quantity called eccentric anomaly, E. E and M are related by Kepler's Equation (Meeus, 1998):

$$E = M + e \sin E, \tag{2}$$

where e is orbital eccentricity. When E is known, ν can be solved for using (Meeus, 1998):

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \tag{3}$$

The forward Kepler problem consists of solving for time of flight, M, given the planetary position, ν. This is straightforward by first solving for E in Eq. 3 and using it to solve for M in Eq. 2.

However, in the most intuitive case, which is implemented here, the user enters a desired date, and the position of the planet has to be determined from the date, i.e. time of flight/mean anomaly M is given, and true anomaly has to be determined. This is referred to as the inverse Kepler problem and amounts to solving for E in Eq. 2 and then for ν in Eq. 3. Solving for E is not straightforward, as no analytical solution exists. Numerous numerical methods exist for the solution of the inverse Kepler problem. Here, the binary search algorithm of Sinnott et al. (1985) is used, as given in Meeus (1998). It has the advantage of being computationally efficient, which becomes important when time series of insolation is the desired model output.

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It also has the distinct advantages of being valid for any value of eccentricity and converging to the exact solution to within the user machine's precision.

2.3 Implementation of Insolation Computation

Instantaneous insolation at the top of the atmosphere (TOA) can be computed as:

$$S(h,\mathbf{r}) = S_o\left(\frac{|\mathbf{r}_o|}{|\mathbf{r}|}\right)^2 \sin h, \tag{4}$$

Where $|\mathbf{r}|$ is the length of the radius-vector of Earth expressed in AU, and h is the altitude of the Sun above the horizon (e.g. Berger et al., 2010). Eq. 4 is an expression of the inverse square law and Lambert cosine law of irradiance. The radius-vector length is computed in the model for the chosen date (and not for every instant) using Eq. 1. S_o is the TSI at $|\mathbf{r}_o|$ = 1 AU by definition (Sect. 2). In this equation insolation, S, is defined as the total (spectrally integrated) solar radiant energy impinging at the TOA on a unit surface area parallel to the mathematical horizon at a given latitude at a given instant. S carries the units of S_o, here W m^{-2}. S needs to be integrated over time and/or space in order to compute insolation quantities of interest. Here, the main discrete time step over which S is computed and output is one 24-hour period, i.e. daily insolation.

Daily insolation is a function of latitude, date, and S_o. The date is associated with a given true anomaly for a given calendar start date and orbital configuration (Joussaume and Braconnot, 1997; Sect. 2.3.1). This determines the current solar declination and the length of the radius-vector of Earth, i.e. the Sun-Earth distance. The user inputs the desired latitude, date and TSI, and the rest of the quantities are computed from the model geometry. Solar declination and the latitude determine the daily evolution of solar altitude, h, as a function of time, as follows (e.g. Meeus 1998):

$$\sin h = \sin\delta \sin\phi + \cos\delta \cos\phi \cos t \tag{5}$$

In the above equation δ is solar declination, ϕ is geographic latitude on Earth, and t is the hour angle of the Sun. δ is assumed constant for the day of interest, and t is a measure of the progress of time. Half the day length, t_s, (i.e. the time between local solar noon and sunset), is determined by setting $h = 0^{o}$ in Eq. 5:

$$t_s = \mathrm{acos}(-\tan\phi\tan\delta) \tag{6}$$

Eq. 5 is integrated over time from solar noon to sunset in order to compute the time-average of the sine of the solar altitude for the given date and latitude:

$$sinh\_ave = \frac{1}{t_s}\int_0^{t_s}(\sin\delta\sin\phi + \cos\delta\cos\phi\cos t)dt \tag{7}$$

Eq. 7 is integrated numerically with a very small time step of about 10 s. Because the altitude of the Sun is symmetric about solar noon, it is sufficient to integrate only from solar noon to sunset time. Daily insolation is then computed by using the time-averaged *sinh_ave* quantity in Eq. 4. The results are scaled by multiplying by the actual day length and dividing by 24 hours. The resulting quantity represents the mean daily insolation over a full day, which is the standard value used in astronomical, climate and paleoclimate science (e.g. Laskar 2014). If this daily insolation is multiplied by 24 h (in seconds), total energy receipt for that day (in J m^{-2}) can be calculated.

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At high latitudes, there are periods of the year with no sunset or no sunrise. These cases depend on the relationship of latitude and solar declination (e.g. Berger et al., 2010). They are handled separately by either integrating Eq. 7 over 24 hrs, or, in the case of no sunrise (polar night), assigning a value of exactly 0 W m^{-2} to daily insolation.

2.3.1 Integrating Insolation over Longer Time Periods - Caveats

Because of the varying eccentricity and longitude of perihelion, there is no fixed correspondence between true anomaly and any one single calendar date, even if one were to define a fixed calendar start date. True anomaly and longitude are the astronomically rigorous ways to define a certain moment in Earth's year and seasons (e.g. Berger et al., 2010). If one wishes to make insolation comparisons between different orbital configurations, one must define strictly a calendar start date, and even then insolation will be in phase for different geological periods only for that date (Joussaume and Braconnot, 1997). Thus, the question "What is insolation on June 20?" is ill posed, unless one defines strictly what is meant by the

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date of June 20. The problem persists if one wishes to compare insolation integrated over periods of time longer than a day, because over geologic time scales, absolute values and the interval of time true anomalies "swept" between two classical calendar dates are not constant. Thus, there are two ways to define a calendar – the classical or fixed-day calendar, in which month lengths follow the present-day configuration and the date of vernal equinox is fixed, or a fixed-angular calendar, which defines months beginning at certain true longitudes (function of true anomaly and the precession phase, see also Sect 4.2. below) and they can therefore have different number of days depending on the orbital configuration (Joussaume and Braconnot, 1997; Chen et al. 2010). The time intervals between solstices and equinoxes also varies, because of varying eccentricity and because these intervals happen in different places in the orbit with respect to perihelion. Thus season lengths vary over geologic time. Earth orbit v2.1 outputs season length in the main GUI to emphasize this important fact. Earth Orbit v2.1 uses the classical calendar dates (24 hr periods) as the user time input, rather than true anomaly or true longitude. This choice is much more intuitive to non-experts, and serves the educational purposes of the model best. The user has as a choice of calendar start date (Sect. 3) and true solar longitude is output (Sect. 4.2) to remind users of the above considerations. The effect of calendar choice on insolation phases and comparisons and on climate models is discussed at length by Joussaume and Braconnot (1997), Timm et al. (2008) and Chen et al. (2010).

The time step of integration can also influence the results of insolation computations, e.g. if annual insolation is averaged with a 5-day step, results are substantially different from the case when a 1-day step is used (not shown). For this reason, the model computes annually averaged insolation at a given latitude by using 1-day steps of integration. Finally, we note that the daily insolation computations of the model are robust and validated (Sect. 5.1); however, the model currently has limited functionality for making comparisons of insolation integrated over longer time periods over different geologic scales. In order to make such comparisons, the use of the elliptical integrals method of Berger et al. (2010) is recommended, as well as the Laskar et al. (2004) methods, both of which come with accompanying software (Laskar 2014) and Berger (2014), respectively). In addition, users are referred to the latest version of the AnalySeries software package (Paillard et al., 1996;

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Paillard, 2014) for additional insolation and time series options. All of the above can also be used for verification of the output of the model presented here.

3 Model User Interface

The Earth orbit model is provided as supplementary material (Appendix A). The model is developed and runs in MATLAB®. All model control is realized via a single, user-friendly GUI panel (Fig. 1). Users are presented with a choice between the Be78 and the La2004 astronomical solutions for eccentricity, obliquity and precession. A "demo" mode is also available. If a real astronomical solution is chosen, users are asked to input a year before or after present (defined as J2000, i.e. January 1, 2000 at 12 noon UT, see *Introduction*) for which they wish to run the model. The GUI only allows users to choose years within the respective solution's validity: the Be78 solution is available for 1,000 kyr before and after present (J2000), whereas the La2004 solution is available for 101,000 kyr in the past and 21,000 kyr in the future. The La2004 solutions are provided by Laskar (2014) in tabulated form in 1 kyr intervals (specifically at http://www.imcce.fr/Equipes/ASD/insola/earth/La2004/index.html). The Be78 solutions are obtained by transcribing code from NASA GISS (see *Acknowledgements*). The model looks up the values of eccentricity, obliquity and precession for the chosen year and solution (using linear interpolation between tabulated years if necessary), and these values are used in subsequent visualizations and analyses. If the user chooses the "demo" mode, they select, independently of each other, the values of the Milankovitch parameters, which can be greatly exaggerated. In this way users can isolate the effects of each parameter on orbital geometry, the seasons, and insolation. The "demo" mode is central to the pedagogical value and applications of the model because it allows users to build and visualize an imaginary orbit of, for example, very high eccentricity while keeping obliquity fixed. Moreover, it will output all subsequent parameters, such as solar declination, day length, radius-vector length, based on this exaggerated imaginary orbit.

Users input the desired calendar date, geographic latitude on Earth (positive degrees in the Northern Hemisphere and negative degrees in the Southern Hemisphere), and desired value of TSI. The calendar date defaults to the current date, latitude defaults to 43^{o} N, and TSI defaults to 1,366 W m^{-2} (Sect. 2). Two choices of calendar start date are available: either fix

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vernal equinox to be at the beginning of March 20th (default), or fix perihelion to be at the beginning of Jan. 3rd. The availability of this choice complicates interpretation of model output; however it has high instructional value. It illustrates that the choice of calendar start date and a calendar system is a human construct, accepted by convention; it is based on the actual year and day length but is relative. This can also help test knowledge of the concepts explained in Sect. 2.3.1. The effect of the different choice of calendar start date is most apparent at exaggerated eccentricities and/or at longitudes of perihelion that are very different from the contemporary value. Insolation time series output (Sect. 4) is only computed for the calendar being fixed to vernal equinox on March 20th.

4 Model Output

4.1 Graphical Output

The main output of the model is a 3D plot of Earth's orbital configuration. Fig. 2A illustrates the orbital configuration using the contemporary values of the Milankovitch parameters (the La2004 solution for J2000 is shown), for September 16. The current phase of the precession cycle is such that Northern Hemisphere winter solstice occurs shortly before perihelion (longitude of perihelion is ~102.9^{o}). This results in Northern hemisphere spring and summer being longer than the respective fall and winter (as shown in the GUI, Fig. 1). Fig 2B illustrates the orbital configuration also on Sept. 16 and using the La2004 solution, but for 10 kyr in the future. Since this represents about a half of a precession cycle, the timing of the seasons is approximately 180^{o} out of phase with respect to the contemporary configuration (the longitude of perihelion is ~279.2^{o}, and Northern hemisphere summer occurs near perihelion and is the shortest season). Because we chose to fix the calendar start date such that vernal equinox is always on March 20, and the eccentricity is fairly low, the date Sept. 16 still occurs near the fall equinox, like in the contemporary example. However, because the length of time passing between vernal equinox and fall equinox is now shorter, Sept 16 almost coincides with fall equinox, unlike the contemporary case. Of course obliquity and eccentricity have also changed 10 kyr in the future, but unlike the longitude of perihelion, their changes are small in absolute terms, and thus this cannot be readily visualized by comparing Figs. 2A and 2B. This is one reason why it is very useful to have the ability to choose arbitrary independent value of the Milankovitch parameters in the demo mode,

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constructing an imaginary orbit. Fig. 2C illustrates one example of such an imaginary orbit with greatly exaggerated eccentricity (0.6) and obliquity (45^{o}) and longitude of perihelion of 225^{o}, i.e. very different from the J2000 values. This imaginary orbit illustrates that the date July 1 can occur in the Fall, due to the large eccentricity and the specific phase of precession chosen. Spring lasts only ~20 days in this configuration because it occurs during perihelion passage, where the planet is much faster according to Kepler's Second Law, as compared to aphelion passage (Fall lasts ~229 days in this configuration). Summer lasts about 58 days. Thus, July 1 occurs during the Fall season, counterintuitively. Importantly, such an exaggerated eccentricity means that the planet is very close to the Sun during perihelion, and some really high insolation values can occur even at modest solar declinations (e.g. for March 29, at 43^{o}N, solar declination is ~27^{o}, daylength is ~16 hours, and daily insolation is 3,307 W m^{-2}, far exceeding any contemporary value anywhere on Earth). The reason is that the Sun-Earth distance then is only 0.4 AU, and the distance factor becomes a first-order effect on insolation, whereas it is a second-order factor in the real Earth orbit configuration (angle being the first-order factor, see Eq. 4).

The plots of Fig. 2 have pan-tilt-zoom capability, so users can view the orbital configuration from many perspectives; this is at the core of the pedagogical value of the model. The plot is updated with the current parameter selections by pressing the "Plot/Update Orbit" button. Finally, note that the orbits of Fig. 2 can be viewed from any angle and the apparent eccentricity of the orbits also changes with the view angle and the projection onto a 2D screen. This should not be confused with the intrinsic orbital eccentricity, which can be also judged by the relative distance of the orbital foci (marked with an 'x') from the ellipse's center (the intersection of the semi-major and semi-minor axes, red lines in Fig. 2)

Users are presented with several options of plotting insolation as function of time and latitude. First, insolation can be plotted for a single year (using the currently selected Milankovitch parameters) as a function of day of year and latitude (Fig. 3A, upper panel). Insolation anomalies with respect to the J2000 La2004 orbital configuration are also plotted, using S_o = 1,366 W m^{-2} (Fig. 3A, lower panel). Anomalies are especially useful when analyzing the effect of changes in insolation on the glacial-interglacial cycles. For example, the anomalies at 65^{o}N during summer months 115 kyr before present (Fig. 3A, lower panel) suggest the

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inception of glaciation (e.g. Joussaume and Braconnot, 1997), as these areas were receiving about 35-40 W m^{-2} less insolation than they are receiving now. The data in these plots is computed with a step of 5 days and 5 degrees of latitude. Multi-millennial insolation time series can also be plotted in a 3D surface plot as a function of year since J2000 and day of year, at the selected latitude. Users select the start and end years for the time series. The data for these plots are computed for steps of 1 kyr and one day (for day of year). An example of the output is provided in Fig. 3B.

Several time series line plots are also produced. Insolation time series are plotted for the currently selected latitude; both the currently selected date and the annual average are shown (Fig. 4A). A multi-panel plot (Fig. 4B) allows the comparison of the three Milankovitch parameters. Precession is visualized as the longitude of perihelion, as well as the climatic precession parameter, $e\sin\omega$ (Berger and Loutre, 1994). A separate GUI button allows users to optionally produce time series plots of several paleoclimatic data sets (Fig. 4C). The top panel shows the EPICA CO_2 (Lüthi et al., 2008a; Lüthi et al, 2008b) and deuterium temperature (Jouzel et al., 2007a; Jouzel et al., 2007b) time series which go back to ~800 kyr before present. The bottom panel of Fig. 4C shows two benthic oxygen isotope (δ^{18}O) data set compilations – the Lisiecki and Raymo (2005) benthic stack (Lisiecki, 2014) and the Zachos et al. (2001) data (Zachos et al., 2008). These data sets go back to 5,320 kyr and 67,000 kyr before present, respectively. To first order, higher δ^{18}O values are associated with higher continental ice sheet volumes and lower benthic ocean water temperatures (Zachos et al., 2001). For this reason, the y-axis of the lower panel of Fig. 4C is inverted, so that higher values of EPICA CO2 and temperature from the uper anel of Fog. 4C can be easily associated with lower δ^{18}O values. These paleoclimatic data are included for convenience of the user and no further interpretation or analyses are provided. Users are cautioned that the interpretation of these paleoclimatic signals and their uncertainties, time-resolution and chronology (age models) is fairly complex (e.g. Bradley, 2014) and beyond the scope of this work. They are provided here for illustrative purposes only, e.g. it enables users to easily visualize the last few glacial-interglacial cycles (and the mid-Pleistocene transition to 100-kyr cyclicity, see *Introduction*), or to visually correlate these paleoclimatic time series with the corresponding Milankovitch parameter and insolation curves.

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4.2 Numerical/Ancillary Output

Ancillary data (and their units) are output in the main GUI window (Fig. 1) and are updated every time the Earth orbit plot (Figs. 2) is re-drawn (Sect. 4.1), i.e. every time the "Plot/Update Orbit" button is pressed. Variables that are output in the main GUI are as follows: solar declination, insolation at the TOA for the chosen date and latitude, day length, Sun-Earth distance, length of the seasons (as defined in the North hemisphere (NH)), the longitude of perigee, and true and mean longitude of the Sun. As a reminder, the longitude of perigee is the angle between the directions of vernal equinox and perihelion and true longitude is the angle Earth has swept from its orbit, subtended at the Sun, since it was last at vernal equinox; mean longitude is proportional to time instead (for detailed definitions, see Sect 2 and 2,1 above). Users also are given the option of saving the data used to make the insolation plots in Fig. 3 in ASCII format. The first row and column of these files list the abscissa and ordinate values of the data, respectively.

5 Model Validation

5.1 Insolation Validation

Daily insolation is the most important model output from climate science perspective and is the fundamental discrete time unit at which the model calculates energy receipt at the TOA. Daily insolation was validated against the results of Laskar et al. (2004), as provided in Laskar (2014) (specifically, the pre-compiled Windows package at http://www.imcce.fr/Equipes/ASD/insola/earth/binaries/index.html). In both the Earth Orbit model and the Laskar software, the La2004 solution for the orbital parameters was used, and the default model solar constant (Table 1) was used. Laskar (2014) defines March 21 as vernal equinox, whereas Earth Orbit v2.1 fixes vernal equinox on March 20 for insolation time series. This was taken into account in this validation. Two dates were tested – March 21 and June 20 (according to the Earth Orbit v2.1 calendar; this corresponds to 1^{o} and 90^{o} mean longitude for the Laskar (2014) software), at three latitudes – 20 S, 45 N and 65 N. The entire time series from 200 kyr in the past to 200 kyr in the future (present = J2000) were tested with a time step of 1 kyr. Validation is excellent; virtually all test cases result in differences in insolation of less than 1 W m^{-2} for March 21 and less than 2 W m^{-2} for June 20, respectively (Fig. 5A and 5B, solid lines with dots), which corresponds to less than 0.5% of the absolute

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values (Fig. 5C and 5D, solid lines with dots). Importantly, these differences are generally much smaller or of the same order of magnitude as the corresponding differences between the Be78 and La2004 astronomical solutions as computed by Earth Orbit v2.1 (Fig. 5, dashed lines). Furthermore, these differences are generally smaller than the uncertainty resulting from varying estimates of the TSI (e.g. Fröhlich 2013 vs. Kopp and Lean, 2011, see Sect. 2); also, these differences are smaller than the total contemporary anthropogenic radiative forcing on climate due to fossil fuel emissions (IPCC, 2013; their Fig. SPM. 5).

The Earth Orbit v2.1 model uses its own internally constructed orbital geometry and first principles equations to compute insolation. There is no additional *a-priori* prescribed constraint to the model other than the orbital elements astronomical solution and the semi-major axis and orbital period (Sect. 2, 2.3; Table 1). Therefore the validation presented here is an independent verification of the model's geometry and computations, taking the Laskar (2014) values as truth. Sect. 6 discusses sources of model uncertainty which can explain some of the small differences observed.

5.2 Solar Declination Validation and Season Length Validation

Solar declination was validated against the algorithms of Meeus (1998). The model year is neither leap, nor common (Table 1) and is thus not equivalent to any single Gregorian calendar year. In order to validate declination at all dates, the Meeus (1998) algorithm was used to compute solar declinations for 12 UT on each date of four years (2009-2012, 2012 being leap) and average the declinations for each date (not day of year, Fig. 6). These averages were then compared to the solar declination output by the model for that date. Results indicate differences are always less than ~0.2^{o} (Fig. 6, black line). By construction, model solar declination on March 20th will always be exactly zero degrees. In reality, the exact instance of vernal equinox varies year to year, so these validation differences are expected. Importantly, the differences between the model and the 4-yr averaged Meeus (1998) declinations are consistently smaller than the daily rate of change of declination (Fig. 6, green curve), as computed from the Meeus (1998) data. Additionally, these differences are of a similar magnitude to the standard deviation of declination between these four years for each date (Fig. 6, red curve). Thus the solar declination validation is excellent and model configuration for each date is representative of a typical generic Gregorian calendar date. The

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discontinuities in the Meeus (1998)–derived curves in Fig 6 (red and green) are due to omitting Feb 29, 2012 when averaging declination values for each date. The discontinuities in the Earth Orbit v2.1 to Meeus (1998) comparison curve (Fig. 6, black curve) are due to the above, plus the fact that the length of the model year is equal to the sidereal orbital period and thus March 19 is a longer "day" in the model year, since calendar start is fixed as vernal equinox on March 20 (also see Sect. 6 below). Finally, season lengths are an excellent method to validate the geometry of the model, because they test that the model is correctly computing a given time of flight on the orbit for a section of the orbit that corresponds to a given season, and generally not coinciding with special points such as perihelion. Season lengths agree to within 0.01 days with the tabulated values of Meeus (1998) (his Table 27F).

6 Sources of Uncertainties

Assumptions and approximations in the model and the underlying astronomical solutions propagate to uncertainties in the model outputs, such as declination and insolation. Some of these assumptions were already discussed, such as calculating insolation for a given calendar date vs. true anomaly interval (fixed-date vs fixed-angle calendars), and choosing integration steps for insolation time series (Sect. 2.3.1). The calendar bias discussed in detail in Sect. 2.3.1 means that if one compares insolation over geologic time on a given classical calendar date, e.g. Sept. 16, which occurs a given number of 24-h periods after the fixed vernal equinox, one is not necessarily comparing insolation at the same true longitude. The same argument is valid for an arbitrary interval of time longer than a day and shorter than a full orbital cycle. This calendar bias creates the artificial North-South tilt observed in insolation anomalies (Chen et al., 2010), which is also exhibited by the Earth Orbit v2.1 model output (Fig. 3A, second panel). This is expected because Earth Orbit v2.1 uses the classical calendar dates, which are more user-friendly.

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Next, we draw the users' attention to a few additional sources of uncertainty. Determination of some of these uncertainties is outside the scope of this work; however, users can run sensitivity analyses using the model in order to quantify them. Importantly, uncertainties in the astronomical solutions that are used as input to the model will propagate to insolation computations. There are differences between the different astronomical solutions (e.g. Fig. 5). Accuracy is highest near the present time and degrades further into the past or future

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(Laskar, 1999; Laskar et al, 2004). Chaotic components of planetary orbital motions introduce an uncertainty that increases by an order of magnitude every ten million years, making it impossible to obtain astronomical solutions for the Milankovitch parameters over period longer than a few tens of millions of years (Laskar et al., 2004). As a reminder, the Be78 solution is valid for one million years in the past or future, whereas the La2004 solution is valid from 101 million years before present to 21 million years in the future; however, solutions for times further back in time than 50 million years before present should be treated with caution (Laskar et al., 2004)..

Due to the gravitational interaction of Earth and other solar system bodies, in particular Jupiter, Venus and the Moon, high frequency variability (time scales of years to centuries) of the Milankovitch parameters is superimposed on the long-term low-frequency Milankovitch cycles. An example of such variability is the nutation in obliquity with a period of ~18 yr. These high frequency fluctuations also lead to insolation changes. Bertrand et al. (2002) used results from the VSOP82 planetary position solution (Bretagnon, 1982) and a simple climate model to demonstrate that the amplitudes of these high-frequency variations and the effect on insolation and surface temperature is negligible (equivalent to model noise) as compared to the 11-yr Sun cycle or the low-frequency trends.

The model is prescribed the sidereal year as the orbital period (Table 1), which is slightly longer than the tropical year (Meeus, 1998). The difference is of the order of 0.01 days. The use of these two different period definitions leads to negligible differences in solar declination on a given date (not shown), much smaller than the validation differences of Fig. 6. We conclude that the choice of orbital period does not influence the insolation computations significantly.

A single value for solar declination and the radius-vector length is used in the computation of daily insolation (Sect. 2.3). In reality, these quantities change continuously, instead of having discrete values. This is likely to introduce small errors in insolation that will be smaller in magnitude than the difference in daily insolation between successive days. Sunrise and sunset times used in the insolation computation are referred to the center of the disk of the Sun and the mathematical horizon at the given latitude at the surface of Earth. Note also that irradiance is given at the top of the atmosphere (TOA), but all computations are geocentric, rather than topocentric, which should lead to negligible insolation differences.

Since the model year is not an integral number of days, if total annual insolation is computed by summing daily insolation values, the March 19 insolation needs to be scaled by 1.256363 to reflect the fact that this day is 24 x 1.256363 hrs long in the model (Berger et al. 2010). Here, we average daily insolation to output average annual insolation, so this correction is not applied.

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7 Concluding Remarks

We presented Earth Orbit v2.1, an interactive 3D analysis and visualization model of the Earth orbit, Milankovitch cycles, and insolation. The model is written and runs in MATLAB® and is controlled from a single integrated user-friendly GUI. Users choose a real astronomical solution for the Milankovitch parameters or user-selected demo values. The model outputs a 3D plot of Earth's orbital configuration (with pan-tilt-zoom capability), selected insolation time series, and numerical ancillary data. The model is intended for both research and educational use. We emphasize the pedagogical value of the model and envision some of its primary uses will be in the classroom. The user-friendly GUI makes the model very accessible to non-programmers. It is also accessible to non-experts and the primary and secondary education classroom, as minimal scientific background is required to use the model in an instructional setting. Disciplines for which the model can be used span mathematics (e.g. spherical geometry, linear algebra, curve and surface parameterizations), astronomy, computer science, geology, Earth system science, climatology and paleoclimatology, physical geography and related fields.

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The authors encourage feedback and request that comments, suggestions, and reports of errors/omissions be directed to tkostadi@richmond.edu

Appendix A: Code Availability & License

The files necessary to run the model "Earth Orbit v2.1" in MATLAB® are provided here as Supplement. In addition, model files are expected to be available on the website of the University of Richmond Department of Geography and the Environment (http://geography.richmond.edu), under the *Resources* category; documented updates may be posted there. Sources of external data files are properly acknowledged in the file header and/or the ReadMe.txt file, as well as in this manuscript. The GUI is raised by typing the name of the associated script ('Earth_orbit_v2_1.m') on the MATLAB® command line. The model has been tested in MATLAB® release R2013b on 64-bit Windows 7 Enterprise SP1 and Linux Ubuntu 12.04 LTS, but should run correctly in earlier versions of MATLAB® and on different platforms. The model is distributed under the Creative Commons BY-NC-SA 3.0 license. It is free for use, distribution and modification for non-commercial purposes. Details are provided in the ReadMe.txt file.

Supplementary material related to this article is available online at [To copy editor: Link here]

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References

Archer, D (2013), Orbital Forcing of Climate, http://forecast.uchicago.edu/Projects/orbits.doc.html, last accessed September 24, 2013.

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Belongie, Serge (2013), "Rodrigues' Rotation Formula." From *MathWorld* - A Wolfram Web Resource, created by Eric W. Weisstein. http://mathworld.wolfram.com/RodriguesRotationFormula.html, last accessed September 12, 2013.

Berger, A.L. (1978a), Long-term variations of daily insolation and Quaternary climatic changes, *J Atmos Sci*, 35: 2362-2367

Berger, André L. (1978b), A Simple Algorithm to Compute Long Term Variations of Daily Insolation, Institut D'Astronomie et de Géophysique, Université Catholique de Louvain, Louvain-la Neuve, No. 18.

Berger, A. (1988), Milankovitch theory and climate. *Reviews of geophysics*, *26*(4), 624-657.

Berger, A., & Loutre, M. F. (1992). Astronomical solutions for paleoclimate studies over the last 3 million years. *Earth and Planetary Science Letters*, *111*(2), 369-382.

Berger, A., Loutre, M. F., & Tricot, C, (1993). Insolation and Earth's orbital periods. *Journal of Geophysical Research*, 98(D6), 10341-10362.

Berger, A. and M.F. Loutre (1994), Precession, eccentricity, obliquity, insolation and paleoclimates, In: "Long Term Climatic Variations, Data and Modelling", J.Cl. Duplessy and M.T. Spyridakis (Eds), NATO ASI Series, vol. 22, pp. 107-151, Springer-Verlag Berlin Heidelberg.

Berger, A. (1996), Comments to “Insolation in terms of Earth's orbital parameters” by DP Rubincam—Theor. Appl. Climatol. 48, 195–202, 1994. *Theoretical and applied climatology*, *53*(4), 253-255.

Berger A., M.F. Loutre, and H. Gallee, (1998), Sensitivity of the LLN climate model to the astronomical and CO_2 forcings over the last 200 kyr, *Climate Dynamics*, 14, pp. 615-629.

Berger, A., J. L. Mélice, and M. F. Loutre (2005), On the origin of the 100-kyr cycles in the astronomical forcing, *Paleoceanography*, 20, PA4019, doi:10.1029/2005PA001173.

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Berger, A, Marie-France Loutre, Qiuzhen Yin (2010), Total irradiation during any time interval of the year using elliptic integrals, *Quaternary Science Reviews* 29 (2010) 1968-1982.

Berger, A (2014), ftp://ftp.elic.ucl.ac.be/berger/ellipt/, last accessed March 24, 2014.

Deleted: A

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Bertrand, C., M. F. Loutre, and A. Berger (2002), High frequency variations of the Earth’s orbital parameters and climate change, *Geophys. Res. Lett.*, 29(18), 1893,10.1029/2002GL015622, 2002.

Formatted: Font: Italic

Bradley, R.S. (2014) Paleoclimatology, Third Edition: Reconstructing Climates of the Quaternary, Academic Press/Elsevier.

Bretagnon P. (1982), Theorie du mouvement de l’ensemble des planetes. Solution VSOP82, *Astron. Astrophys.*, 30, 141–154, 1982.

Formatted: Font: Italic

Chen, G. S., Kutzbach, J. E., Gallimore, R., & Liu, Z. (2011), Calendar effect on phase study in paleoclimate transient simulation with orbital forcing, *Climate dynamics*, 37(9-10), 1949-1960.

Formatted: Font: Italic

Falkowski, Paul G., Miriam E. Katz, Allen J. Milligan, Katja Fennel, Benjamin S. Cramer, Marie Pierre Aubry, Robert A. Berner, Michael J. Novacek, and Warren M. Zapol (2005), The rise of oxygen over the past 205 million years and the evolution of large placental mammals, *Science* 309, no. 5744 (2005): 2202-2204.

Falkowski, P. G., & Godfrey, L. V. (2008a), Electrons, life and the evolution of Earth's oxygen cycle. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 363(1504), 2705-2716.

Falkowski, P. G., & Isozaki, Y. (2008b), Geology. The story of O_2. *Science*, 322(5901), 540-542.

Fröhlich, C. (2013)., Solar Constant: Construction of a Composite Total Solar Irradiance (TSI) Time Series from 1978 to present, Physikalisch-Meteorologisches Observatorium Davos, World Radiation Center, http://www.pmodwrc.ch/pmod.php?topic=tsi/composite/SolarConstant, last accessed Sept. 20, 2013.

Deleted:

Hansen, J., Sato, M., & Ruedy, R. (2013), Global Temperature Update Through 2012. NASA (http://www.nasa.gov/pdf/719139main_2012_GISTEMP_summary.pdf), last accessed March 24, 2014.

Deleted: GISS website

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Hays, J. D. , J. Imbrie and N. J. Shackleton (1976), Variations in the Earth's Orbit: Pacemaker of the Ice Ages, *Science:* 194 (4270), 1121-1132. DOI:10.1126/science.194.4270.1121.

Huybers, P. (2006). Early Pleistocene glacial cycles and the integrated summer insolation forcing. *Science*, *313*(5786), 508-511. Data and code posted at: http://www.ncdc.noaa.gov/paleo/pubs/huybers2006b/huybers2006b.html, last accessed September 24, 2013.

Huybers, P., & Denton, G. (2008), Antarctic temperature at orbital timescales controlled by local summer duration, *Nature Geoscience*, 1(11), 787-792.

Imbrie, J., E. A. Boyle, S. C. Clemens, A. Duffy, W. R. Howard, G. Kukla, J. Kutzbach, D. G. Martinson, A. McIntyre, A. C. Mix, B. Molfino, J. J. Morley, L. C. Peterson, N. G. Pisias, W. L. Prell, M. E. Raymo, N. J. Shackleton, J. R. Toggweiler (1992), On the structure and origin of major glaciation cycles: 1. Linear responses to Milankovitch forcing, *Paleoceanography*, 7(6), 701– 738.

Imbrie, J., A. Berger, E.A. Boyle,S.C. Clemens, A. Duffy, W.R. Howard, G. Kukla, J. Kutzbach,D. G. Martinson, A. Mcintyre, A.C. Mix, B. Molfino, J.J. Morley, L.C. Peterson , N.G. Pisias, W. L. Prell, M.E. Raymo, N.J. Shackleton , and J . R. Toggweiler (1993), On the structure and origin of major glaciation cycles 2. The 100,000-year cycle. *Paleoceanography*, *8*(6), 699-735.

IPCC, 2013: Summary for Policymakers. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Joussaume, S., and P. Braconnot (1997), Sensitivity of paleoclimate simulation results to season definitions, *J. Geophys. Res.*, 102(D2), 1943–1956, doi:10.1029/96JD01989.

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Deleted: Imbrie, J., Berger, A., Boyle, E. A., Clemens, S. C., Duffy, A., Howard, W. R., ... & Toggweiler, J. R.

Jouzel, J., et al. (2007a), EPICA Dome C Ice Core 800KYr Deuterium Data and Temperature Estimates. IGBP PAGES/World Data Center for Paleoclimatology Data Contribution Series # 2007-091. NOAA/NCDC Paleoclimatology Program, Boulder CO, USA.

Jouzel, J., V. Masson-Delmotte, O. Cattani, G. Dreyfus, S. Falourd, G. Hoffmann, B. Minster, J. Nouet, J.M. Barnola, J. Chappellaz, H. Fischer, J.C. Gallet, S. Johnsen, M. Leuenberger, L. Loulergue, D. Luethi, H. Oerter, F. Parrenin, G. Raisbeck, D. Raynaud, A. Schilt, J. Schwander, E. Selmo, R. Souchez, R. Spahni, B. Stauffer, J.P. Steffensen, B. Stenni, T.F. Stocker, J.L. Tison, M. Werner, and E.W. Wolff (2007b), Orbital and Millennial Antarctic Climate Variability over the Past 800,000 Years, *Science*, Vol. 317, No. 5839, pp.793-797.

Kasting, J. F., Whitmire, D. P., & Reynolds, R. T. (1993), Habitable zones around main sequence stars, *Icarus*, *101*(1), 108-128.

Kasting, J. (2010), *How to Find a Habitable Planet*. Princeton University Press.

Kopp, G., and J. L. Lean (2011), A new, lower value of total solar irradiance: Evidence and climate significance, *Geophys. Res. Lett.*, 38, L01706, doi:10.1029/2010GL045777

Kump, L. R., Kasting, J. F., & Crane, R. G. (2010), *The Earth System*, San Francisco: Prentice Hall.

Laskar, J. (1999), The limits of Earth orbital calculations for geological time-scale use. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, *357*(1757), 1735-1759.

Laskar, J., P. Robutel, F. Joutel, M. Gastineau, A. C. M. Correia, and B. Levrard (2004), A Long-term Numerical Solution for the Insolation Quantities of the Earth, *Astronomy and Astrophysics* 428, no. 1: 261-85. doi:10.1051/0004-6361:20041335.

Laskar, J., Fienga, A., Gastineau, M., & Manche, H. (2011), La2010: A new orbital solution for the long term motion of the Earth. *arXiv preprint arXiv*:1103.1084.

Laskar, J. (2014), Astronomical Solutions for Earth Paleoclimates, Institut de mecanique celeste et de calcul des ephemerides, https://www.imcce.fr/Equipes/ASD/insola/earth/earth.html, last accessed March 24, 2014.

Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic $\delta^{18}O$ records, *Paleoceanography*, 20, PA1003, doi:10.1029/2004PA001071.

Lisiecki, L. E., Raymo, M. E., & Curry, W. B. (2008), Atlantic overturning responses to Late Pleistocene climate forcings. *Nature*, *456*(7218), 85-88.

Lisiecki, L. (2014), LR04 Benthic Stack, http://www.lorraine-lisiecki.com/stack.html, last accessed March 24, 2014.

Loutre, M. F., Paillard, D., Vimeux, F., & Cortijo, E. (2004), Does mean annual insolation have the potential to change the climate? *Earth and Planetary Science Letters*, *221*(1), 1-14.

Lüthi, D., et al. (2008a), EPICA Dome C Ice Core 800KYr Carbon Dioxide Data. IGBP PAGES/World Data Center for Paleoclimatology Data Contribution Series # 2008-055. NOAA/NCDC Paleoclimatology Program, Boulder CO, USA.

Lüthi, D., M. Le Floch, B. Bereiter, T. Blunier, J.-M. Barnola, U. Siegenthaler, D. Raynaud, J. Jouzel, H. Fischer, K. Kawamura, and T.F. Stocker (2008b), High-resolution carbon dioxide concentration record 650,000-800,000 years before present, *Nature*, Vol. 453, pp. 379-382. doi:10.1038/nature06949

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Kopp, G., and J. L. Lean (2011), A new, lower value of total solar irradiance: Evidence and climate significance, *Geophys. Res. Lett.*, 38, L01706, doi:10.1029/2010GL045777.

Meeus, Jean (1998), *Astronomical Algorithms*, Willmann-Bell, Richmond, VA.

Milankovitch, M. (1941), *Kanon der Erdbestrahlung und seine Anwendung aufdas Eiszeitenproblem*, 633 pp., Ed. Sp. Acad. Royale Serbe, Belgrade. (English translation Canon of Insolation and Ice Age Problem, Israel program for Scientific Translation; published for the U.S. Department of Commerce and the National Science Foundation).

Muller, R. A., & MacDonald, G. J. (1997), Glacial cycles and astronomical forcing. *Science*, 277(5323), 215-218.

Paillard, D., L. Labeyrie and P. Yiou (1996), Macintosh program performs time-series analysis, *EOS Trans.*, AGU, 77: 379.

Paillard, D. (2014), AnalySeries pour MacOS, LSCE, http://www.lsce.ipsl.fr/Phocea/Page/index.php?id=3 (Accessed March 17, 2014)

Rial, J.A. (1999), Pacemaking the Ice Ages by Frequency Modulation of Earth's Orbital Eccentricity, *Science*: 285 (5427), 564-568. DOI:10.1126/science.285.5427.564

Rubincam, D. P. (1994), Insolation in terms of Earth's orbital parameters. *Theoretical and applied climatology*, 48(4), 195-202.

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Standish, E. M., Newhall, X. X., Williams, J. G., & Yeomans, D. K. (1992), Orbital ephemerides of the Sun, Moon, and planets. *Explanatory supplement to the astronomical almanac*, 279-323.

Sinnott, Roger W. (1985) Sky and Telescope (70), August 1985, p. 159.

Timm, O., A. Timmermann, A. Abe-Ouchi, F. Saito, and T. Segawa (2008), On the definition of seasons in paleoclimate simulations with orbital forcing, *Paleoceanography*, 23, PA2221, doi:10.1029/2007PA001461.

USNO (2013), Selected Astronomical Constants, 2013 in *The Astronomical Almanac Online*, US Naval Observatory, Washington, DC, http://asa.usno.navy.mil/SecK/Constants.html, last accessed Sept. 27, 2013.

Zachos, J., M. Pagani, L. Sloan, E. Thomas, and K. Billups, (2001), Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present, *Science*, Vol. 292, No. 5517, pp. 686-693, 27 April 2001. DOI: 10.1126/science.1059412.

Zachos, J., et al. (2008), Cenozoic Global Deep-Sea Stable Isotope Data. IGBP PAGES/World Data Center for Paleoclimatology Data Contribution Series # 2008-098. NOAA/NCDC Paleoclimatology Program, Boulder, CO, USA, http://www.ncdc.noaa.gov/paleo/metadata/noaa-ocean-8674.html, last accessed March 24, 2014.

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Tables

Table 1. Summary of constant and variable model input parameters.

Symbol	Constant/Variable	Value	Units	Reference	Notes
AU	Astronomical unit	149.597870700	10^6 km	USNO (2013)	constant
a	Semi-major axis	149.598261150	10^6 km	Standish et al. (1992)	1.00000261 AU (constant)
T	Sidereal orbital period	365.256363	Days	Meeus (1998)	constant
S_o	TSI at 1 AU	1,366*	W m^{-2}	Fröhlich (2013)	Also see Kopp and Lean (2011)
e	Eccentricity	0.01670236**	--	La2004***	--
ε	Obliquity	23.4393**	degrees	La2004	--
$\tilde{\omega}$	Longitude of perihelion	102.9179**	degrees	La2004	--

*Users can change this default value.

**Default J2000 value. Users can change these variables independently of each other or choose real astronomical solutions depending on the mode selected.

***La2004 refers to Laskar et al. (2004)

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Figure Captions

Figure 1. Main MATLAB® GUI window of Earth Orbit v2.1. Input and output displayed corresponds to the graphical output of Fig. 2A, i.e. contemporary (J2000) La2004 configuration for September 16, at 43^{o} N latitude.

Figure 2. A) Present (J2000) orbital configuration for September 16, using the La2004 solution and the calendar start date fixed at vernal equinox on March 20. The orbital ellipse is shown in blue, the semi-major and semi-minor axes (perpendicular to each other) are in red and the lines connecting the solstices and equinoxes (also perpendicular to each other) are shown in black. The perihelion point, as well as the equinoxes and solstices are labeled. The Sun is shown as a semi-transparent yellowish sphere centered at one of the orbital ellipse's foci, both of which are marked with an 'x' along the semi-major axis. The Earth is plotted with its center on the corresponding place along the orbit, and the angle it has swept since last perihelion passage (the true anomaly angle) is filled in semi-transparent light green. Earth's Equator is plotted as a solid black line, as is its axis of rotation, with the North Pole marked. The spheres of the Earth and the Sun are not to scale, the rest of the figure is geometrically/astronomically accurate and to scale. This plot is in 3D and has pan-tilt-zoom capability in the Earth Orbit v2.1 model. The corresponding GUI with numerical ancillary output is shown in Fig. 1 (for latitude 43^{o} N and S_o = 1,366 W m^{-2}), B) Real orbital configuration for September 16, 10 kyr in the future, using the La2004 solution and a March 20 equinox as calendar start date; and C) demo (imaginary) orbital configuration for July 1 (Vernal equinox fixed at March 20), Eccentricity = 0.6, Obliquity = 45^{o}, Longitude of perihelion= 225^{o}. The geometry is consistent with Berger et al. (2010), their Fig. 1, although it is being viewed in A) from the direction of fall equinox, as opposed to from the direction of spring equinox in their figure. The apparent eccentricity of the three orbits in Fig. 2 is also due to the view angle of the 3D plot and the respective projection onto a 2D monitor/paper; the intrinsic eccentricity can be judged by tilting the plot or observing the relative distance from the two foci (the Sun being at one of them) to the center of the ellipse, the intersection of the semi-major and semi-minor axes (red lines).

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Figure 3. A) A day of year-latitude insolation plot for 115 kyr before present (J2000) (upper panel) and the corresponding anomaly from J2000 (lower panel), using S_o = 1,366 W m^{-2}. B) Insolation time series at 65°N as a function of day of year, spanning 200 kyr before and after present (J2000). Negative years are in the past.

Figure 4. A) Insolation time series plot spanning 200 kyr before and after present (J2000) at 65° N on June 20th (blue) and annual average (red); B) Time series plots of Milankovitch orbital parameters spanning 500 kyr before and after present. Panels from top to bottom display eccentricity, obliquity, and longitude of perihelion and climatic precession; C) Time series plots of paleoclimatic data spanning one million years before and after present: EPICA ice core CO_2 and deuterium temperature (upper panel) and the Lisiecki and Raymo (2005) and Zachos et al (2001) compilations of benthic oxygen isotope ($\delta^{18}O$) data (lower panel). Note the y-axis of the $\delta^{18}O$ plot is inverted. Negative years for all Fig. 4 panels are in the past.

Figure 5. Absolute differences (W m^{-2}, solid lines with dots) between our insolation solution (using the La2004 astronomical parameters) and the Laskar (2014) insolation solution (also using the La2004 astronomical solutions; insolation provided by his Windows pre-compiled package at http://www.imcce.fr/Equipes/ASD/insola/earth/binaries/index.html) for March 21 (A) and June 20 (B). Differences between the Be78 and La2004 astronomical solutions (insolation for both computed by our model) are shown for comparison with dotted lines. Data are shown for three different latitudes – 20 °S (red), 45 °N (green), and 65 °N (blue). C) same as in A) but displaying percent insolation difference, D) same as in B) but displaying percent insolation difference. Earth Orbit v2.1 insolation computations use the model's own orbital geometry with no additional *a-priori* input other than the Milankovitch parameter solutions of La2004. Negative years are in the past. See Section 5.1 for details.

Figure 6. Solar declination validation: difference between solar declination as computed by the internal geometry of the Earth orbit model and mean actual declination from the years 2009, 2010, 2011 and 2012 as computed for 12:00:00 UT for every day with the algorithms in Meeus (1998) (black solid line). The rate of change of declination (green solid line) and the standard deviation of declination for each date for the four years (red solid line, N = 4 for

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each data point) are also shown for reference. The model computations were performed with the calendar start date fixed at vernal equinox of March 20th. Feb. 29th 2012 was removed from the analysis, so the abscissa corresponds to a given date, i.e. dates, not days of year were averaged for a given mean solar declination across the four years. Abscissa ticks represent the 15th of each month. If 0 UT is used for the Meeus computations instead, differences (black curve) have a different pattern and are larger, but never exceed ~0.4 degrees (not shown). See Sect. 5.2 for details