

## ***Interactive comment on “Improved convergence and stability properties in a three-dimensional higher-order ice sheet model” by J. J. Fürst et al.***

**T. Albrecht (Referee)**

torsten.albrecht@pik-potsdam.de

Received and published: 29 August 2011

general comments:

This paper presents an advanced numerical finite-difference discretization scheme for the force-balance equation in the higher order Blatter/Pattyn (LMLa) approximation.

It is advanced in a technical sense, that for the used solvers the convergence is significantly faster and more regular as for a scheme, that has been suggested by Pattyn (2003) for the ISMIP-HOM benchmark model intercomparison. Furthermore, much more precise solutions can be found, which really is an important issue in ice sheet modeling.

The main difference to the older scheme is, that the viscosity is here defined on a

C584

shifted/staggered grid with respect to the velocities. This allows for a smart discretization of the second-order derivatives within the compact stencil and an additional reduction of the truncation error compared to the DIR/Pattyn scheme, that is defined exclusively on a regular grid.

The general idea of defining the viscosity field on a staggered grid and enhancing the coupling by averaging adjacent velocities is not new and has already been used in similar ways, e.g., for the discretization of the force balance in Shallow Shelf Approximation (SSA). But in this specific case it is worthwhile to give a profound description of the exact formulation, which easily can be reproduced. What I really like is the formulation in terms of operators split up in single steps, which helps a lot in creating a clearly arranged structure of distinct cases for a complex set of equations.

From my point of view this paper is ready for publication after some minor revisions. My comments below focus on some aspects, which could be discussed in more detail or where I would wish a more precise reference.

specific comments:

section1:

p1573 l.12: “reduce numerical instability” Does this mean that the scheme is less unstable for a certain set of parameter or does it mean it is stable for a larger range of parameters?

p1573 l.18–19: “Decoupling of the solution in adjacent points using centred differences in the Stokes equation is an understood phenomenon” Could you give some more information for those who are not aware of this phenomenon. Is this discussed in Mattheij et al. (2005), what page?

section2:

p1574 l.21 and following: “The acceleration term in the force balance equation is in general omitted but not, as sometimes stated, because it is negligibly small. On the

C585

contrary, accelerations in fact reach large values but the time needed to adjust the velocity field and attain a new balance of forces is small." This is an interesting issue though just mentioned as a side information and not further relevant for the study itself. You argue that the time scale for the adjustment of the ice to accelerations is much shorter compared to glaciological relevant time scales. It think it is not trivial and worth to explain a little more or give a reference.

p1581 l.8 and following: The CFL-criterion is named here as a condition for stability. There are different definitions of this criterion but all of them are related with explicit time-marching schemes. What exactly is the condition here? Is it about a perturbation of the parameters, which doesn't blow up during the iterations? Is it a norm of the (dissipative) average? A page number of the cited book by Wesseling (2001) could be helpful. In general, stability issues for non-local problems are absolutely not trivial. "Condition numbers" may be useful measures for the sensitivity of the solution with respect to input coefficients and hence of the accuracy of the solution, both for prescribed viscosities and in the nonlinear case.

section3:

p1582 l.12 and following: "...to a numerical decoupling of the x- and y-direction of the force balance equation in the nonlinear iterations and consequently reduces the matrix size of the linear system by a factor 4." How does this decoupling look like in detail and what are the consequences? In Eq. 14 one still finds a dependence on both the old u and v-velocity components. Is there a reference, since this could be of common interest for other ice sheet modelers?

section5:

p1587 l.27 and following "For high precisions, the maximum becomes locally flat and even shows a local depression..." How broad is this depression in terms of grid lengths? As you showed in Eq. 12 there is a strong resolution dependency of the DIR and the convergence is not very regular. It is definitely an interesting issue and emphasizes

C586

the quality of the STAG scheme. But regarding the outline of Appendix C it is not clear to me, why this phenomenon is so important for this study. I would guess that for even coarser resolution also the STAG scheme may fail in reproducing ellipticity characteristics of the underlying PDE.

technical corrections:

Please consider these to be suggestions. It is not my intent to be pedantic, just to be helpful.

p1574 l.24: "small" -> "short"

p1576/77 Eq. 5 or 9: y' and zeta are switched in the last a\_x term in one of these equations

p1578 l.17-18: "The two first derivatives of the velocity field and the one of the surface elevation..." Maybe reformulate this passage to make sure what exactly you mean. (Similar issue in p1581 l.15)

p1578 l.21: "that" -> "which", there are only two operators

p1579 l.21: "right" -> "relevant", or "suitable" may fit better here

p1580 l.14: "Therefore..." suggests it would generally be a result of the use of staggering and the compact stencil, rather than an effect of the specific STAG scheme.

p1581 l.17: ".. to this..." -> maybe drop it or specify what you mean by "this"

p1582 l.10: Is there a certain reason for the \tilde over the u? To my understanding, the r stand for the current iteration of u.

p1582 Eq.14: Does the b\_x and b\_y correspond to those in Eq. A4? If not, maybe rename.

p1587 l.4 "shows" -> "show"

p1603 l.10 reference "Press et al" not in order

C587

