Geoscientific Model Development Discussions

Mass-flux subgrid-scale parameterization in analogy with multi-component flows: a formulation towards scale independence

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Multi-component analogue

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Title Page

Abstract	Introduction
Conclusions	References
Tables	Figures

| ◄◄ | ►► |
| ◄ | ► |

| Back | Close |

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Abstract

The mass-flux parameterization formulation is generalized by taking an analogy of the large-scale atmospheric flow with multi-component flows. This generalization permits to include any subgrid-scale variability into the mass-flux parameterization. Those in-
5   clude stratiform clouds as well as cold pools in the boundary layer.

An important finding under the present formulation is that the subgrid-scale quantities are advected by the velocities characteristic of given subgrid-scale components (sub-component flows), rather than by the large-scale flows as simply defined by grid-box average. This formulation, as a result, ensures the lateral interaction of subgrid-scale
10   variability crossing the grid boxes, which are missing in the current parameterizations, and leading to a reduction of the grid-size dependence in its performance. It is shown that the subcomponent flows are driven by subcomponent pressure gradients. The for-mulation, as a result, furthermore includes a self-contained description of subgrid-scale momentum transport.

15   The formulation is applicable to a situation in which the scale separation is still sat-isfied, but fractional areas occupied by individual subgrid-scale components are no longer small. A complete formulation is presented and various implementation issues are discussed. The present formulation is also expected to alleviate problems aris-ing from increasing resolutions of operational forecast models without invoking more
20   extensive overhaul of parameterizations.

The main purpose of the present paper is to appeal the importance of this new possibility suggested herein to the numerical weather forecast community with implica-tions for the other parameteizations (cloud fraction, mesoscale organization) as well as resolution-dependence of parameterizations.

1 Introduction

The present paper presents a generalization of the mass-flux parameterization formulation for representing nonconvective processes as well as convection. As a side product, we also provide an answer to the following question: Are the subgrid-scale parameterized variables advected by large-scale flows? This is one of the typical questions often asked in a context of operational implementation of a subgrid-scale process parameterization. The present paper shows that it is the corresponding subcomponent flow that advects a given subgrid-scale component variable, but not the whole large-scale flow. Importance of analogy of the mass-flux based parameterization with the multi-component flows is emphasized in order to better understand this conclusion.

Importance of the mass-flux convection parameterization, originally introduced by Ooyama (1971), Fraedrich (1973, 1974), and Arakawa and Schubert (1974) is hardly overemphasized. This approach is adopted by majority of current global and regional atmospheric models both for operational forecasts and climate studies. The present paper considers a particular limit in subgrid-scale parameterization under the mass-flux framework: whereas the individual subgrid-scale elements are much smaller than the grid-box size, a fractional area occupied by each category of subgrid-scale processes is no longer substantially smaller than the grid-box size.

An example of such a situation is shown in Fig. 1 taken from Fig. 2a of Yano et al. (2005a) with modifications. Here, we show a spatial distribution of five cloud categories over a typical size of a grid box for global climate modelling, 512 km × 512 km, simulated by a cloud-resolving model (CRM). The categories show (1) precipitating convection, (2) precipitating stratiform, (3) non-precipitating stratiform, (4) shallow clouds, and (5) ice anvils. The remainder, the category (6), is considered "environment". The CRM simulation is from a TOGA-COARE (Tropical Ocean Global Atmosphere Couple Ocean Response Experiment) period, representing a typical fully-developed marine-type deep convective system. We refer to Yano et al. (2005a) for details of the simulations as well as the categorization scheme.

Individual precipitating convective elements (category 1) occupy only very small fractional areas in the grid box, being consistent with the scale separation principle as assumed in the standard mass-flux parameterization. The stratiform-type clouds, on the other hand, tend to take larger fractional areas individually, and looking them as
5 a whole category, the occupied area is no longer substantially smaller than the grid-box size. Even for precipitating convection, the total fractional area is not as small as that for the individual convective elements, because they are numerous. Most importantly, the environment does not occupy majority of the grid box at all, but its fractional area is just comparable with any stratiform-type cloud regions.

10 The present paper is going to present a formulation for this type of situation in subgrid-scale parameterization under a mass-flux based approach. The situation to be considered is a drastic departure from the approximations adopted by the standard mass-flux subgrid-scale parameterization, as established by Arakawa and Schubert (1974): (1) fractional areas occupied by individual subgrid-scale components
15 (convective-plume types) are much smaller than unity; (2) the environment (non-convective area) occupies a majority of the grid-box domain; and (3) all the subgrid-scale components are exclusively surrounded by the environment.

In contrast, the situation considered in the present work is: (1) fractional areas occupied by individual subgrid-scale components are no longer much smaller than unity;
20 (2) the environment (non-convective area) no longer occupies a majority of the grid-box domain; and as a result, (3) subgrid-scale components are no longer exclusively surrounded by the environment, but more than often adjacent with the other subgrid-scale components.

Importance of such generalization in the mass-flux based parameterization cannot
25 be overemphasized. Most importantly, this kind of generalization is crucial in order to consider mesoscale convective organization (cf., Moncrieff, 1995) under the mass-flux framework. The mesoscale convective organization is clearly a structure comparable in size with the grid box even for a relatively low-resolution climate model. The proposed formulation provides a way of incorporating this processes as an integrated part of the

mass-flux formulation. The best effort so far towards this goal, in the author's best knowledge, is by Donner (1993), who indeed includes the mesocale effects as a part of his mass-flux convection parameterization. However, the mesoscale effect itself is not formulated by mass flux, but simply added as an appendix.

⁵ Applications of the proposed formulation are, however, not limited to the convective processes. The formulation can also be applied to various non-convective processes such as stratiform clouds, cold pools in the boundary layer. Thus, it includes the cloud-fraction parameterization within its scope, for example. As an important consequence of this modification, the "environment" no longer represents a special status but just ¹⁰ reduces to one of many possible subgrid-scale categories.

In order to perform a required modification to the mass-flux parameterization, a point of view taken is a one advocated by Yano et al. (2005a, 2010a) that the mass-flux convection parameterization can be considered as a consequence of an application of segmentally-constant approximation (SCA) to a full physical system. The idea of SCA ¹⁵ consists of subdividing a grid box domain into a number of constant-value segments in different sizes and shapes as suggested by Fig. 1. SCA can be considered as a geometrical constraint applied to a full physical system in order to construct the mass-flux based parameterization in a more general manner.

A full description of the SCA formulation is presented in Yano et al. (2010a) by taking ²⁰ a two-dimensional nonhydrostatic anelastic system as a full physical system. In this respect, the present paper generalizes the SCA system to a fully three-dimensional case. For ease of developing a formulation, we take the hydrostatic primitive equation system. Some justifications for this choice is given in the beginning of Sect. 2.2.

The mass-flux parameterization developed under the present formulation presents ²⁵ a special flavor with a close link to a traditional description of multi-component flows. This aspect is discussed in the first half of the next section. Remainder of the next section is devoted in introducing the primitive equation system that is taken as a starting point for applying SCA. Section 3 is devoted to a stepwise reduction of the full system under the application of SCA. The obtained set of equations is discussed in

Sect. 4. Discussions include not only various formulational details and possible applications, but further implications from the present work including construction of a scale-independent parameterization. The paper is concluded in Sect. 5. Some mathematical details are deferred to the Appendix.

2 Preliminaries

2.1 Analogy with multi-component flows

The basic idea of mass-flux parameterization may be understood as that of a multi-component system with each subcomponent designated by subscript j. A colloidal system such as milk is such an example. Milk consists of many "bubbles" of water and fats, which are not visible on a macroscopic scale (i.e., large scale *or* grid scale), but which should appear on the microscale (i.e., subgrid scale). Thus, in order to describe the macroscopic evolution of milk (flow), we have to specify the fractional volume occupied by water and fats, respectively, at every macroscopic point. The fractional volume is the counter concept for the fractional area, σ_j, occupied by the j-th subcomponent in the mass-flux formulation. From this perspective, a review on multi-component fluid systems given, for example, by Gyarmati (1970) is fruitful for better understanding the principle of mass-flux parameterization.

The situation can be understood under a mathematical symbolism followingly. Let the characteristic scales for the macroscopic and microscopic processes be ΔX and Δx, respectively. Separation between macroscopic and microscopic processes suggests

$$\Delta X \gg \Delta x.$$

The notion of the subgrid scale may be understood by introducing the grid-box size, L, explicitly to the above inequality. The grid-box size must be sufficiently smaller than typical macroscopic processes, because otherwise they are not numerically properly

represented. On the other hand, the grid-box size must be taken sufficiently larger than a typical scale for microscopic processes. Thus,

$$\Delta X \gg L \gg \Delta x. \tag{2.1}$$

However, the meteorological subgrid-scale parameterization problem is more complex than a standard multi-component flow, because there is an exchange of mass between the different subcomponents. For example detrained air from a cumulus cloud can turn into a part of a stratiform cloud. This is a situation normally not expected in fluid-mechanical multi-component flows: in milk, water always remains water and fats always remain fats. Chemical reactions are the only expected means that one subcomponent turns into another in these multi-component flows (cf., Gyarmati, 1970).

In order to fully take into account these complex processes in the subgrid-scales, the most straightforward mathematical approach would be to adopt that of the multi-scale analysis as pointed out by Majda (2007a, 2007b) and as applied by Xing (2009). Under this approach, the coordinates for the two scales are introduced, those, say, (x,y) describing the subgrid-scales and those (X,Y) describing the large-scale (grid-scale). General coordinates may be given by $(x + X/\epsilon, y + Y/\epsilon)$ with ϵ being a small parameter, which may be taken as $\Delta x/\Delta X$. By taking an asymptotic limit, $\epsilon \to 0$, the subgrid-scale variability described by the coordinates (x,y) shrinks into a single "macroscopic" point in respect to the large-scale (grid-scale) coordinates (X,Y). That is the basic notion behind the scale separation principle.

In the present paper, we take a slightly different approach, in which all the physical variables are consistently considered under averaging over the grid box. As a reminder for this averaging operation, we put a bar to the nabla, $\bar{\nabla}$, when the nabla operator is considered in terms of the grid-box average.

An important issue to keep in mind from the multi-component system point of view is that all the subgrid-scale component variables are also functions of large-scale (grid-scale) coordinates. In other words, they should vary from one grid box to another. Implication from the scale separation principle (cf., Yano , 2009) is that these subgrid-scale

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component variables must, furthermore, vary smoothly from one grid box to another, because otherwise smoothness of a large-scale (grid-scale) solution is not guaranteed. This scale separation issue turns out to be key in properly developing any parameterization.

2.2 A basic set of equations: hydrostatic primitive equation system

In order to develop a generalized mass-flux formulation in a heurestic manner from a basic set of equations, we adopt the hydrostatic primitive equation system as the latter. This choice may look hardly justifiable in light of the fact that subgrid-scale processes in concern could be highly nonhydrostatic. However, here, we emphasize that parameterization is only concerned with feedback of these subgrid-scale variability only to grid-box average. Most importantly, although individual subgrid-scale processes may be much faster than those of the grid scales, evolution of each subgrid-scale component defined in terms of an "ensemble" average over the grid box would have a characteristic time-scale comparable to those of the grid-scale processes. As a result, we can suppose that the nonhydrostatic effects, such as local acceleration of vertical velocity, would become small by taking grid-box average. For these reasons as well as for simplifying the formulation, we neglect these effects from onset. Recall that Yanai et al. (1973) also developed their mass-flux formulation for observational diagnoses from the primitive equation under pressure coordinate.

Notice that this argument applies only when the total fractional area, σ_j, occupied by a j-th category is the order of unity. When the fractional area is small, a simple scale analysis suggests that the subcomponent vertical velocity must be scaled by $w_j \sim \sigma_j^{-1}$, thus the local acceleration associated with vertical motion would no longer be negligible. An obvious such process is deep convective towers.

The primitive equation system under the pressure p vertical coordinate consists of the horizontal momentum equation

$$\frac{\partial}{\partial t}\boldsymbol{u} + [\nabla \cdot (\boldsymbol{u}\boldsymbol{u})] + \frac{\partial}{\partial p}\omega\boldsymbol{u} = -\nabla\phi + \boldsymbol{F}_u,$$ (2.2a)

the hydrostatic balance

$$\frac{\partial}{\partial p}\phi = -\alpha,$$ (2.2b)

the mass continuity

$$\nabla \cdot \boldsymbol{u} + \frac{\partial}{\partial p}\omega = 0,$$ (2.2c)

and the heat equation

$$\frac{\partial}{\partial t}\theta + \nabla \cdot (\boldsymbol{u}\theta) + \frac{\partial}{\partial p}\omega\theta + \omega\frac{\partial\theta_0}{\partial p} = Q.$$ (2.2d)

Note that the second term of the left-hand side of Eq. (2.2a) is a short-handed expression with the exact form defined by Eq. (A2) with $\sigma_j = 1$.

Here, $\boldsymbol{u}$ is a horizontal velocity, ω a vertical velocity (pressure velocity), ϕ the geopotential, and ∇ designates the gradient operator over a constant pressure surface. In the horizontal momentum equation (Eq. 2.2a), all the other force (e.g., Coriolis force) other than pressure gradient is simply designated together as $\boldsymbol{F}_u$.

In the hydrostatic balance, the perturbation specific volume α is related to the potential temperature perturbation θ by

$$\alpha = (R/p)(p/p_0)^\kappa\theta,$$

where R is the gas constant, $\kappa = R/C_p$ with C_p the specific heat with constant pressure, $p_0 = 1000\,\text{hPa}$ a reference pressure.

In the thermodynamic equation (Eq. 2.2d), Q is the total diabatic heating rate, θ_0 is the reference state for the potential temperature, which is assumed to be a function of pressure only. For economy of presentation, we omit the prime representing a deviation from a reference state. As a result, for example, the total potential temperature is given by $\theta_0 + \theta$.

We may further consider, for example, the mixing ratios, q_μ, for various water types (vapor, liquid cloud, precipitating water, etc.) with subscript μ designating water types:

$$\frac{\partial}{\partial t} q_\mu = -\nabla \cdot \boldsymbol{v} \, q_\mu - \frac{\partial \omega q_\mu}{\partial p} + S_\mu, \tag{2.3}$$

where S_μ is a source for the given water type. In general, we can write a prognostic equation for any physical variable, say φ in the form

$$\frac{\partial}{\partial t} \varphi = -\nabla \cdot \boldsymbol{v} \, \varphi - \frac{\partial \omega \varphi}{\partial p} + F \tag{2.4}$$

with F forcing (or source) of the given variable.

3 A general formulation for the subgrid-scale processes under SCA

Segmentally-constant approximation (SCA) represents an ensemble of subgrid-scale components, marked by an index j ($= 1, 2, \cdots$), within a grid box domain by approximating a full system with a corresponding ensemble of constant-value segments (cf., Yano et al., 2005a, 2010a). The j-th subcomponent occupies an area of S_j with a boundary designated by ∂S_j. As a result, the original full system reduces to a discrete set of equations describing evolution of these constant-segment values at each vertical level. These subcomponents may, for example, represent different cloud types as shown in Fig. 1 above. Thus, in case of Fig. 1, the six subcomponents are considered. Note that, as illustrated by Fig. 1, a subcomponent does not usually constitute a single enclosed

element, but many enclosed sub-elements. We assume a grid box with a side length L, and the grid-box area is designated by S ($= L^2$).

For the derivation of the generalized mass-flux formulation under SCA, we focus on a generic prognostic equation (Eq. 2.4) and the mass continuity (Eq. 2.2c). Both the momentum equation (Eq. 2.2a) and the water mixing-ratio equation (Eq. 2.3) can be considered as special cases of Eq. (2.4).

3.1 Prognostic equations

The equation for any prognostic variable φ for a j-th subcomponent is obtained by integrating Eq. (2.4) over an area S_j occupied by the j-th subcomponent. The result is given by Eq. (4.1) of Yano et al. (2005a) when the boundary ∂S_j for the j-th subcomponent with the other components does not move with time. A two-dimensional case with a moving subcomponent-boundary is given by Eq. (3.4) of Yano et al. (2010a). By generalizing these results, under a three-dimensional configuration with the moving subcomponent-boundary, a system under SCA is defined by

$$\frac{\partial}{\partial t}\sigma_j\varphi_j + \frac{\partial \sigma_j(\omega\varphi)_j}{\partial p} + \frac{1}{S}\oint_{\partial S_j} \varphi_{\mathrm{b},j}(\boldsymbol{u}^*_{\mathrm{b},j} - \dot{\boldsymbol{r}}_{\mathrm{b},j})\cdot d\boldsymbol{r} = \sigma_j F_j, \tag{3.1}$$

where the average over the j-the subcomponent (segmentally-constant values) is given by e.g.,

$$\varphi_j = \frac{1}{\sigma_j S}\int_{S_j} \varphi\, dxdy, \tag{3.2a}$$

$$(\omega\varphi)_j = \frac{1}{\sigma_j S}\int_{S_j} w\varphi\, dxdy, \tag{3.2b}$$

and a fractional area σ_j occupied by the j-th subcomponent is defined by

$$\sigma_j = \frac{1}{S}\int_{S_j} dxdy. \tag{3.2c}$$

Furthermore, $\dot{r}_{\mathrm{b},j}$ designates the rate of the movement of the subcomponent boundary, $\boldsymbol{u}^{*}_{\mathrm{b},j}$ is a normal velocity to the boundary defined by

$$\boldsymbol{u}^{*}_{\mathrm{b},j} = \boldsymbol{u}_{\mathrm{b},j} - \omega_{\mathrm{b},j}\frac{\partial \boldsymbol{r}_{\mathrm{b},j}}{\partial p} \qquad (3.3)$$

with $\boldsymbol{r}_{\mathrm{b},j}$ designating the position of the boundary. The subscript b designates the
5 values at the subcomponent boundary.

By following the standard mass-flux approximation (cf., Yano et al., 2004) we approximate the vertical flux by

$$(\omega\varphi)_{j} \simeq \omega_{j}\varphi_{j}.$$

Note that fluctuations within a subgrid-scale component-segment can easily be in-
10 cluded by re-writing it:

$$(\omega\varphi)_{j} = \omega_{j}\varphi_{j} + (\omega''_{j}\varphi''_{j})_{j},$$

where the double prime indicates a deviation of a variable from SCA within the given subcomponent segment, i.e.,

$$\varphi''_{j} = \varphi - \varphi_{j}.$$

15 We refer to Soares et al. (2004), and Siebesma et al. (2007) for the treatment of these fluctuation terms.

3.1.1 Horizontal divergence term

The divergence (contour integral) term in Eq. (3.1) can be separated into the three parts:

$$\oint_{\partial S_j} \varphi_{b,j}(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r} = \oint_{\partial S_j^+} \varphi_{b,j}(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r}$$

$$+ \oint_{\partial S_j^-} \varphi_{b,j}(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r} + \oint_{\partial S_{b,j}} \varphi_{b,j}(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r}. \tag{3.4}$$

The first two parts (∂S_j^+ and ∂S_j^-) are inside the grid box, where the last part ($\partial S_{b,j}$) is a contribution form the grid-box boundary.

The first two parts are furthermore separated into those where outflows ($(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r}_j > 0$: ∂S_j^+) and inflows ($(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r}_j < 0$: ∂S_j^-) are found at the subcomponent boundary. The inflow part may be further divided by adjacent subgrid-scale components. As a result, ∂S_j^- is given by a sum of sub-segments $\partial S_{j,i}^-$ adjacent with sub-components designated by the subscript i:

$$\partial S_j^- = \sum_{i=\{i\}_j} \partial S_{j,i}^-. \tag{3.5}$$

Here, $\{i\}_j \equiv \{i_1, i_2, \cdots\}$ designates a set of the subcomponents that are directly adjacent with the j-th subcomponent.

For both parts, we take an upstream approximation, as adopted by Asai and Kasahara (1967: their Eq. 3.29) as well as by Arakawa and Schubert (1974), thus

$$\varphi_{b,j} = \begin{cases} \varphi_j, & (\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j})\varphi \cdot d\boldsymbol{r}_j > 0, \\ \varphi_i, & (\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j})\varphi \cdot d\boldsymbol{r}_j < 0. \end{cases} \tag{3.6}$$

We furthermore, introduce the detrainment and the entrainment rates by

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$$d_j = \frac{1}{S} \oint_{\partial S_j^+} (\boldsymbol{u}_{b,j}^* - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r}, \tag{3.7a}$$

$$e_{j,i} = -\frac{1}{S} \oint_{\partial S_{j,i}^-} (\boldsymbol{u}_{b,j}^* - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r}. \tag{3.7b}$$

Substitution of Eqs. (3.6), (3.7a,b) into Eq. (3.4) shows that the inflow and outflow fluxes provide

$$\frac{1}{S} \oint_{\partial S_j^+} \varphi_{b,j} (\boldsymbol{u}_{b,j}^* - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r} = d_j \varphi_j, \tag{3.8a}$$

$$\frac{1}{S} \oint_{\partial S_{j,i}^-} \varphi_{b,j} (\boldsymbol{u}_{b,j}^* - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r} = -e_{j,i} \varphi_i, \tag{3.8b}$$

respectively.

Here, note that the detrainment and the entrainment rates, d_j and $e_{j,i}$, are only short-handed expressions for the more complete expressions in the right-hand side of Eq. (3.7a,b). The best procedure would still be to evaluate $\boldsymbol{u}_{b,j}^* - \dot{\boldsymbol{r}}_{b,j}$ explicitly as done in Yano et al. (2010a) under a two-dimensional framework. In the remainder of the paper, we simply treat these coefficients, d_j and $e_{j,i}$, as if given, accepting the extensive issues behind (cf., Sect. 4.3).

3.1.2 Contributions from the grid-box boundary

Calculation for the contribution from the grid-box boundary (last term in Eq. 3.4) is slightly more complicated. A relatively obvious constraint is that the total contribution

of the contour integral along the grid-box boundary is equal to the total divergence by Gauss' theorem: i.e.,

$$\frac{1}{S}\sum_j \oint_{\partial S_{b,j}} \varphi_{b,j}(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r} = (\overline{\nabla \cdot \boldsymbol{u}\varphi}). \tag{3.9}$$

Another important constraint is that the grid-box boundary is usually a rectangular shape without changing with height. The grid-box boundary does not usually move (as we assume here), either, so that $\boldsymbol{u}^*_{b,j} = \boldsymbol{u}_{b,j}$ and $\dot{\boldsymbol{r}}_{b,j} = 0$ at $\partial S_{b,j}$. The contribution from the grid-box boundary for a j-th subcomponent, as a result becomes

$$\oint_{\partial S_{b,j}} \varphi_{b,j}(\boldsymbol{u}^*_{b,j} - \dot{\boldsymbol{r}}_{b,j}) \cdot d\boldsymbol{r} = \oint_{\partial S_{b,j}} \varphi_{b,j}\boldsymbol{u}_{b,j} \cdot d\boldsymbol{r}.$$

For further reductions, we note that all the subgrid-scale variables, $\boldsymbol{u}_{b,j}$, $\varphi_{b,j}$ are smooth function of the large-scale coordinates, and the results of the integral should not change regardless of where these subcomponents are placed within the grid box. Consequently, the integral range can be modified into that over the whole grid box weighted by the fractional area, σ_j, occupied by the j-the subcomponent without loss of generality:

$$\oint_{\partial S_{b,j}} \varphi_{b,j}\boldsymbol{u}_{b,j} \cdot d\boldsymbol{r} = \oint_{\partial S} \sigma_j\varphi_{b,j}\boldsymbol{u}_{b,j} \cdot d\boldsymbol{r}.$$

The final result is obtained by taking the asymptotic limit, $L/\Delta X \to 0$, recalling Eq. (2.1). Then the application of the Gauss divergence theorem leads to

$$\frac{1}{S}\oint_{\partial S_{b,j}} \varphi_{b,j}\boldsymbol{u}_{b,j} \cdot d\boldsymbol{r} = \bar{\nabla} \cdot (\sigma_j\varphi_j\boldsymbol{u}_j). \tag{3.10}$$

Note that Eq. (3.10) is consistent with Eq. (3.9) under the relation:

$$\overline{\nabla \cdot \boldsymbol{v}\varphi} = \bar{\nabla} \cdot \overline{\boldsymbol{v}\varphi} = \bar{\nabla} \cdot \sum_j (\sigma_j\varphi_j\boldsymbol{u}_j).$$

However, note in general $\overline{\boldsymbol{u}\varphi} \neq \bar{\boldsymbol{u}}\bar{\varphi}$ as an important feature of multi-component flow.

Finally, substitution of Eqs. (3.8a,b) and (3.10) into Eq. (3.4) leads to

$$\frac{1}{S}\oint_{\partial S_j}(\boldsymbol{u}_{\text{b},j}^* - \dot{\boldsymbol{r}}_{\text{b},j})\varphi \cdot d\boldsymbol{r} = d_j\varphi_j - \sum_{\{i\}_j} e_{j,i}\varphi_i + \bar{\nabla}\cdot(\sigma_j\boldsymbol{u}_j\varphi_j), \tag{3.11}$$

and further substitution of Eq. (3.11) into Eq. (3.1) leads to

$$\frac{\partial}{\partial t}\sigma_j\varphi_j + \frac{\partial\sigma_j(\omega\varphi)_j}{\partial p} + d_j\varphi_j - \sum_{i=\{i\}}e_{j,i}\varphi_i + \bar{\nabla}\cdot(\sigma_j\boldsymbol{u}_j\varphi_j)$$

$$= \sigma_j F_j. \tag{3.12}$$

This is the prognostic equation for any physical variable, φ_j, for the j-th subgrid-scale component.

3.1.3 Turbulence effects

The upstream approximation (Eq. 3.6) may still be too simple from a physical point of view. The convective plume interface with the environment is often considered to be extremely turbulent associated by various fine-scale mixing (cf., Turner, 1986). A generalization of the above formulation can be made by adding a turbulent contribution φ'':

$$\varphi_{\text{b},j} = \begin{cases} \varphi_j + \varphi_{\text{b},j}'', & (\boldsymbol{u}_{\text{b},j}^* - \dot{\boldsymbol{r}}_{\text{b},j})\varphi \cdot d\boldsymbol{r}_j > 0, \\ \varphi_{j'} + \varphi_{\text{b},j}'', & (\boldsymbol{u}_{\text{b},j}^* - \dot{\boldsymbol{r}}_{\text{b},j})\varphi \cdot d\boldsymbol{r}_j < 0. \end{cases}$$

As a result, we need to add a new term

$$\frac{1}{S}\oint_{\partial S_j}\varphi_{\text{b},j}''(\boldsymbol{u}_{\text{b},j}^* - \dot{\boldsymbol{r}}_{\text{b},j})\cdot d\boldsymbol{r}$$

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to the above definitions of the lateral mixing (Eq. 3.11). A closed expression for this term is proposed, for example, by Asai and Kasahara (1967: their Eq. 3.13) as

$$\frac{1}{S}\oint_{\partial S_{j,i}} \varphi''_{b,j}(\boldsymbol{u}_H^* - \dot{\boldsymbol{r}}_b)\cdot d\boldsymbol{r} = -k_{j,i}(\varphi_i - \varphi_j)$$

with a constant $k_{j,i}$.

5 However, this generalization is inconsequential because the original formula (Eq. 3.11) is recovered by redefining the detrainment and entrainment rates as $d_j + \sum_{\{i\}_j} k_{j,i}$ and $e_{j,i} + k_{j,i}$ (cf., de Rooy and Siebesma, 2010). Thus, we no longer consider the eddy mixing effect explicitly in the following.

3.2 Horizontal momentum equation

10 Both the heat equation (Eq. 2.2d) and the water mixing-ratio equation (Eq. 2.3) can be cast into the form (Eq. 3.12) in a straightforward manner. The derivation for the horizontal momentum equation (Eq. 2.2a) is, however, slightly more involved due to the presence of the pressure-gradient force. For conciseness, we introduce an approximation

$$\frac{1}{S}\int_{S_j}(-\nabla\phi)dxdy \simeq -\bar{\nabla}\sigma_j\phi_j. \tag{3.13}$$

This approximation is justified only if the boundary of the j-th subcomponent is randomly inclined (or not at all inclined) within the grid box. Otherwise, the right-hand side should take the form given by Eq. (A1). Note than when the subgrid-scale structure is influenced by vertical wind-shear, for example, this assumption would no longer be 20 satisfied, and the full formula (Eq. A1) may be considered.

With the help of Eq. (3.13), the horizontal momentum equation under SCA is given by

$$\frac{\partial}{\partial t}\sigma_j\boldsymbol{u}_j + \frac{\partial}{\partial p}\sigma_j\omega\boldsymbol{u}_j + d_j\boldsymbol{u}_j - \sum_{i=\{i\}_j} e_{j,i}\boldsymbol{u}_i + \bar{\nabla}\cdot(\sigma_j\boldsymbol{u}_j\boldsymbol{u}_j) = -\bar{\nabla}\sigma_j\phi_j + \sigma_j\boldsymbol{F}_{u,j}. \tag{3.14}$$

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Note that the last term of the left-hand side is a short-handed expression with the exact form defined by Eq. (A.2).

3.3 Hydrostatic balance

The exact form of the hydrostatic balance under SCA is given by Eq. (A.3). We, again, propose to neglect the effects due to inclination of the boundary, $r_{b,j}$. Thus, it reduces to:

$$\frac{\partial}{\partial p}(\sigma_j \phi_j) = -\sigma_j \alpha_j. \tag{3.15}$$

3.4 Mass continuity

Mass continuity under SCA is obtained by directly averaging the mass continuity equation (Eq. 2.2c) over the subcomponent S_j in a similar manner as for obtaining Eq. (3.12) from Eq. (2.4):

$$\frac{1}{S}\oint_{\partial S_j} \boldsymbol{u}_{b,j}^* \cdot d\boldsymbol{r} + \frac{\partial}{\partial p}\sigma_j \omega_j = 0. \tag{3.16}$$

Another form of mass continuity is obtained by setting $\varphi_j = 1$, $F_j = 0$ in Eq. (3.12):

$$\frac{\partial}{\partial t}\sigma_j + \frac{\partial \sigma_j \omega_j}{\partial p} = e_j - d_j - \bar{\nabla}\cdot(\sigma_j \boldsymbol{u}_j), \tag{3.17}$$

where $e_j = \sum_{\{i\}_j} e_{j,i}$ is the total entrainment rate. Note that the eddy effects at the subcomponent boundary does not affect the mass continuity. Furthermore, a difference between Eqs. (3.16) and (3.17) gives

$$\frac{\partial}{\partial t}\sigma_j = \frac{1}{S}\oint_{\partial S_j} \dot{\boldsymbol{r}}_{b,j} \cdot d\boldsymbol{r}. \tag{3.18}$$

This has a simple geometrical interpretation that the rate of change of an area of a subgrid-scale component is defined by a rate of the change of the position of the boundaries of the given subgrid-scale component.

Equation (3.18) enables us to evaluate σ_j prognostically in time, provided that the right-hand side is given in a closed from. In order to obtain such an expression, we first re-write this term with the help of Eqs. (3.3) and (3.11):

$$\frac{1}{S}\oint_{\partial S_j} \dot{\boldsymbol{r}}_{b,j} \cdot d\boldsymbol{r} = -\frac{1}{S}\oint_{\partial S_j} \omega \frac{\partial \boldsymbol{r}_{b,j}}{\partial p} \cdot d\boldsymbol{r} + e_j - d_j.$$

Substitution of the above into the previous equation leads to:

$$\frac{\partial}{\partial t}\sigma_j = -\frac{1}{S}\oint_{\partial S_j} \omega \frac{\partial}{\partial p} \boldsymbol{r}_{b,j} \cdot d\boldsymbol{r} + e_j - d_j. \tag{3.19}$$

As before, we assume that inclination of the boundary is random over the grid box so that the integral term in the right-hand side of Eq. (3.19) does not contribute. As a result, Eq. (3.19) is approximated by

$$\frac{\partial}{\partial t}\sigma_j = e_j - d_j. \tag{3.20}$$

Furthermore, substitution of Eq. (3.20) into Eq. (3.17) leads to

$$\frac{\partial \sigma_j \omega_j}{\partial p} = -\bar{\nabla} \cdot (\sigma_j \boldsymbol{v}_j). \tag{3.21}$$

Thus, by assuming a random distribution of subgrid-scale components, we succeeded in separating the mass continuity into two independent equations for describing σ_j and ω_j.

Equation (3.20) may appear odd in first glance being not invariance under Galilean transformation. This virtual inconsistency is resolved by taking into account the fact that the detrainment and entrainment rates, defined by Eq. (3.8a,b), also change with Galilean transformation. The equation states that σ_j becomes constant in time when the entrainment and detrainment vanish under a given coordinate framework.

4 Discussions

4.1 Summary

A full description for representation of subgrid-scale processes under a mass-flux formulation in analogy with multi-component flow is obtained in the last section. This representation is obtained by a systematic application of SCA to the primitive equation system.

Under this representation, the horizontal momentum equation, the hydrostatic balance, the mass continuity for the j-th subgrid-scale component are given by Eqs. (3.14), (3.15), and (3.21), respectively. The equation for any physical variable, whose prognostic equation is given in the form (Eq. 2.4) in the full system, is given by Eq. (3.12). This includes the potential temperature as well as any water components. Additionally, this subgrid-scale representation includes an additional prognostic equation (Eq. 3.20) for the fractional area, σ_j, occupied by the subcomponent. This set of equations (Eqs. 3.14, 3.15, 3.21, 3.12, 3.20) constitutes a closed set once the entrainment and detrainment rates, $e_{j,i}$, d_j, defined by Eq. (3.7a,b), are specified.

Analogy of the derived mass-flux parameterization system with multi-component flows is evident. When entrainment and detrainment are turned off, the system essentially reduces to non-interacting multi-component flow system with the index j designating a subcomponent of the flow.

4.2 Comparison with the standard mass-flux formulation

It may be worthwhile to compare the present multi-component analogue system with the standard mass-flux formulation. The latter is simply given by

$$\frac{\partial}{\partial p}\sigma_j\omega_j\varphi_j + d_j\varphi_j - e_j\bar{\varphi} = \sigma_j F_j \qquad (4.1)$$

for any physical variable, φ, defined by Eq. (2.4), and with the mass continuity

$$\frac{\partial}{\partial p}\sigma_j\omega_j = e_j - d_j \qquad (4.2)$$

(cf., Eqs. 27–30 of Yanai et al., 1973; Eqs. 8 and 17 of Tiedtke, 1989). This set of equations are coupled with the equations, for exmaple, given by Eqs. (2.2)–(2.4) with
5 grid-box scale averaging, designated by the overbar above. Note that in the multi-component flow analogue, there is no corresponding equation for the grid-box mean.

The most notable difference of the present multi-component flow analogue from the standard formulation is that all the equations are given in prognostic form except for the hydrostatic balance (Eq. 3.15) and the mass continuity (Eq. 3.21). This is in contrast
10 with the standard mass-flux formulation that all the variables are defined in diagnostic manner, but except for the convective vertical velocity. Our multi-component flow analogue is a generalization of the standard mass-flux parameterization in the sense that Eqs. (3.12) and (3.17) reduce to Eqs. (4.1) and (4.2), respectively, in the asymptotic limit to $\sigma_j \to 0$ with the scaling, $\sigma\omega_j \sim O(1)$.
15 The standard mass-flux formulation requires to introduce a bottom boundary condition for the mass flux, $\sigma_j\omega_j$, in order to solve the mass continuity (Eq. 4.2). The condition is called the closure. On the other hand, the present multi-component flow analogue does not require a closure. The mass flux, $\sigma_j\omega_j$, is simply driven by the sub-components divergence, $\bar{\nabla}\cdot(\sigma_j\boldsymbol{u}_j)$. This version of mass continuity (Eq. 3.21) can be
20 vertically integrated simply by assuming the vanishing mass flux at the surface. All the other equations are integrated simply in time. However, this scheme requires an initialization, and this is considered a problem of data assimilation. Note that though the present formulation is prognostic, time integration should be performed with the standard model time step without time splitting as long as the fractional areas for subgrid-
25 scale components remain finite (cf., Sect. 2.2).

4.3   Remaining problems: entrainment and detrainment

The main remaining problem in the present formulation is defining the entrainment and detrainment rates. This is crucial, because entrainment-detrainment is the sole process that subgrid-scale components interact under the present formulation. The task is, however, more and more involved as more subcomponents are considered, because the entrainment rate, $e_{j,i}$, is given by a matrix: the entrainment rate from the i-th subcomponent to the j-th subcomponent must be defined, with all possible combination of indices i and j. Difficulty in specifying the entrainment-detrainment rate should not be underestimated, because this is already a formidable problem under the standard mass-flux formulation (cf., Turner, 1986; Yano and Bechtold, 2009; de Rooy and Siebesma, 2010). We have no precedent experience of extending this idea into a matrix formulation.

The most prudent approach would be, by following the standard method by e.g., Arakawa and Schubert (1974), Tiedtke (1989), and Bechtold et al. (2001), to describe the entrainment rates in terms of fractional entrainments, assuming that the entrainment rate is proportional to the mass flux. In describing the stratiform cloud, for example, an even simpler approach may be taken for the detrainment rate, simply assuming that the subcomponent dissipates into the environment with a constant characteristic time-scale. Partitioning of the detrained air into the adjacent subcomponents is totally an open question, too.

The best available approach to answer all these questions would be to employ cloud-resolving modelling and large-eddy simulations in a systematic manner. A standard methodology already exists for estimating the entrainment and detrainment rates (Siebesma and Cuijpers, 1995), which can easily be generalized for a matrix formulation.

4.4 Remaining problems: subcomponent prescription

As a whole, the present formulation contains the two obvious advantages: no need for the closure and triggering conditions. On the other hand, we have to face a new problem which may be called the "prescription" problem: how to teach each subgrid-scale subcomponent to function with the designated physical role?

Mathematically speaking, it is straightforward to integrate the system consisting of Eqs. (3.12), (3.14), (3.15), (3.20) and (3.21) in time from any initial conditions, by designating the subcomponents as convective updraft, downdrafts, stratiform clouds, etc. However, there is absolutely no guarantee that each subcomponent keeps behaving as we have initially designated. For example, the convective-updraft subcomponent may simply die out after an initial convective event, and nothing happens afterwards. The stratiform cloud may simply remain cloud free all through the simulation.

It is clear that additional assumptions must be introduced in order to maintain the proper function of each subcomponent: e.g., the convective-updraft subcomponent must be convectively more unstable than the other subcomponents. In order to maintain such a state, for example, surface flux must be preferably applied to the convective-updraft subcomponent so that convective updrafts are indeed induced under an favorable large-scale condition. A stratiform cloud could be maintained by entraining cloudy air preferably into it from the other subgrid-scale components such as convective up-drafts.

However, the most important point to keep in mind is that all the subgrid-scale components should never vanish by reaching a zero fractional area. Because once they disappear, we have to re-introduce them later. For this purpose, we also have to re-introduce a "triggering" condition to the formulation.

4.5 Remaining problems: deep convection

Finally, we may wish to treat deep convection separately from the present framework. Deep convection is usually well confined in space over a large-scale grid box. Thus,

it could be best to simply retain a standard mass-flux parameterization, before it could be fully reformulated under the present formulation without hydrostatic approximation. In this case, the mass continuity (Eq. 3.21) would be replaced by a traditional one, (Eq. 4.2), but under assumption that the convective updraft is surrounded by convective
5   downdrafts and stratiform clouds, for example.

    Alternatively, it may turn out that the present hydrostatic formulation is not a bad approximation for deep convection. The present formulation contains an advantage in the sense that deep convection is explicitly coupled with "large-scale" convergence described by the convective-component flows, but not the "total" large-scale convergence
10   as assumed in the traditional wave-CISK (Hayashi, 1970, 1971; Lindzen, 1974). The new configuration is likely to give new insights to this old idea. A linear stability analysis could be helpful for further elucidations.

4.6   Possible applications: stratiform cloud representation

The present formulation is relatively general and in principle, covers all types of subgrid-
15   scale processes under a mass-flux framework. However, probably the most practical first application would be to develop a cloud fraction parameterization (stratiform-cloud representation) under the present formulation in a stand-alone manner. The developed scheme is coupled with a existing deep-convection scheme, by also removing the hypothesis that the environment subcomponent covers majority of the grid box.
20   The cloud fraction is a major quantity to be evaluated in global models both for radiation and microphysics. Approaches based on probably density function (pdf) are increasingly becoming popular (e.g., Bony and Emanuel, 2001; Tompkins, 2002). However, the main difficulty with this approach would be to find a formulation for pdf from a physical principle without going through too many heuristic arguments and mathe-
25   matical assumptions. The present formulation, on the other hand, provides a cloud fraction more directly without introducing a pdf.

    It may be worthwhile to note in this context that Tiedtke (1993) lays down his basic formulation (his Eqs. 1–5) for his cloud scheme in terms of SCA, but without explicitly

introducing cloud area in his space integrations, except in his Eq. (4). Moreover, he does not pursue this SCA principle in consistent manner as in the present study. For example, flows associated with subgrid-scale clouds are not explicitly considered, as stated in the first paragraph of his Sect. 2.a.

5   Similarity of the two formulations may be seen by comparing his Eq. (7) with our Eq. (3.20). Various source and sink terms in his Eq. (7) are defined in terms of entrainment and detrainment processes, being consistent with our more systematic derivation. A major difference of his Eq. (7) from our Eq. (3.20) is a presence of a large-scale transport term, A, which should not exist. Our formulation is more consistent by considering

10 the values of physical variables for each subcomponent explicitly. Tiedtke (1993) only considers the grid-box averaged quantities: see his Eqs. (6), (9), (10).

Generality of the formulation for the subgrid-scale component fraction (e.g., cloud fraction) given by Eq. (3.20) cannot be overemphasized. Ultimately, any cloud-fraction parameterization must be consistent with Eq. (3.20), being based on a purely geo-

15 metrical argument. It should, however, be noticed that this formulation is not at all closed. The entrainment and detrainment terms, introduced by Eq. (3.7a,b), are even no longer necessarily the same physical processes as those assumed in convective-plume dynamics. Instead, they are merely a measure of lateral mixing over the cloud boundaries. However, the formulation gives an important point that it is a dynamical

20 mixing rather than a local physics (e.g., cloud physics), as assumed in Wilson et al. (2008), that defines the evolution of the cloud fraction.

4.7 Possible applications: mesoscale organization

The most challenging and attractive application would be the parameterization of mesoscale organized convection. The strategy would be conceptually along the line of

25 the archetype model proposed by Moncrieff (1981, 1992). Under the present formulation, the ides of archetype would be implemented by dividing the archetype structure into several subcomponents: mesoscale stratiform deck, mesoscale updraft and downdraft, etc.

A proper coupling of deep-convection parameterization and a SCA-based stratiform-cloud parameterization, as just discussed in the previous subsection, could already provide a reasonable representation of mesoscale convective organization. The SCA-based stratiform-cloud representation is physically consistent in such manner that it
5 could spontaneously generate a mesoscale downdraft just underneath under an appropriate environment. The role of the vertical wind shear is already partially taken into account by advecting each subgrid-scale subcomponent by the subcomponent flow. The key of this parameterization would be to mimic the shear intensification tendency over the mesoscale stratiform region by properly defining the entrainment-detrainment
10 rates.

4.8 Possible applications: subgrid-scale momentum transport

Parameterization of convective momentum transport always remains difficult due to a need for estimating the aerodynamic pressure influencing the convective-scale momentum in a closed form (e.g., Zhang and Cho, 1991; Wu and Yanai, 1994; Kershaw
15 et al., 1997). The present formulation provides a surprisingly clean solution to this problem of the subgrid-scale momentum by solving the ensemble-averaged subgrid-scale horizontal momentum equation (Eq. 3.14) explicitly. The ensemble-averaged subgrid-scale pressure, ϕ_j, can simply be evaluated by a hydrostatic balance (Eq. 3.15). This is another attractive feature of the present formulation.

20 ## 4.9 Further issues: towards the scale independence

It may be important to emphasize the present formulation is presented in a manner independent of the model resolution by strictly adhering to the scale-separation principle. Absence of both the closure and triggering, which could be scale dependent, is a particular advantage. More importantly, advection of all the subgrid-scale components by
25 their own flows much alleviates the current syndrome of subgrid-scale parameterization strongly depending on the model grid size. Arguably, such the grid-size dependence

stems from the fact that the traditional parameterizations operate totally independent of neighboring grid boxes. Advection of subgrid-scale components introduces direct interaction of subgrid-scale processes with neighboring grid boxes, leading to much less resolution-dependent behavior. A major remaining problem is the entrainment and the detrainment rates, which are likely to be scale dependent. A much careful investigation on this issue is warranted also for this reason (cf., Sect. 4.3).

4.10 Further issues: high-resolution limit

The present formulation does not directly address the more challenging issues of subgrid-scale representation when the scale separation breaks down (i.e., high-resolution limit). This is an urgent issue to be tackled seriously with current accelerating trend of further and further increasing horizontal resolutions of operational forecast models (cf., Yano et al., 2010b). Nevertheless, the present formulation could also be considered as a first step for developing a subgrid-scale representation in high-resolution limit by already taking into account the finite size of subgrid-scale components, but by strictly adhering to the scale separation principle,

As the horizontal resolution increases, lateral communication of subgrid-scale processes between the grid boxes becomes increasingly important (Yano et al., 2010b). At ECMWF (European Centre for Medium-Range Weather Forecasts), for example, this difficulty is partially overcome by artificially introducing a cellar automaton model (Shutts, 2005). The present formulation has a clear advantage in the sense that the subgrid-scale variables are explicitly advected from one grid box to another by following the subcomponent flow. In this respect, the present parameterization already embodies a key ingredient required for the subgrid-scale representation in high-resolution limit.

A more strident parameterization formulation for high-resolution limit would be derived by the same application of SCA, but to a nonhydrostatic system. Unfortunately, this formulation, to be presented in a separate paper, is much more involved. For this reason, the present formulation could be considered a good practical compromise

for alleviating issues arising with increasing resolutions of operational models without facing a more serious overhaul of the current parameterizations.

5 Concluding remarks

Implementation of the present scheme into an operational model is beyond the scope of the present paper. The task is so intensive that it would even be beyond the single author's efforts. The main reason for the publication of the present formulation paper is to appeal to the numerical weather prediction (NWP) community the need for developing such a scheme. The major obstacle in numerical implementation is a need for modifying the whole dynamical core of a model in order to make it possible to accommodate multiple-flow components as "large-scale" variables.

It is emphasized that the main proposal of the present formulation *does not* reside on simply advecting the subgrid-scale variables by large-scale flows. Such a proposal would be relatively easy to make, but the presnet careful formulational analysis shows that we have to do it differently: each subgrid-scale component must be advected by a flow specifically associated with the given subcomponent (i.e., the subcomponent flow, u_j). Unfortunately, that is exactly where the major coding modification is required.

Many details are left out in order to suggest general possibilities of the present formulation rather than being too specific. The author strongly believes that general proposal herein is an important step forward for including various subgrid-scale physical processes in systematic manner by overcoming defects of the current parameterizations. In this respect, the present paper provides a more specific proposal based on a general methodology proposed by Yano et al. (2005a).

The author believes that the formulation presented herein has extensive implications as discussed extensively in the last section. The main open question in the present formulation under the the multi-flow analogue is the treatment of the entrainment and detrainment rates (cf., Sect. 4.3). An alternative possibility is to evaluate subgrid-scale horizontal winds more directly under a SCA formulation. Though the

resulting formulation would be more involved in the latter case (cf., Yano et al., 2010a), it would contain much less assumptions than a entrainment-detrainment based formulation. Sooner or later, we would face a critical question of whether to retain the entrainment-detrainment hypothesis or to move beyond.

Appendix A

Mathematical details

A1 Horizontal momentum equation

The pressure gradient term under SCA without approixmation is obtained with the help of Leibnitz's theorem and it is given by:

$$\frac{1}{S}\int_{S_j}(-\nabla\phi)dxdy = -\bar{\nabla}\sigma_j\phi_{j\cdot} + \frac{1}{S}\oint_{\partial S_j}\phi\begin{pmatrix}\dfrac{\partial r_b}{\partial x}\\[2mm]\dfrac{\partial r_b}{\partial y}\end{pmatrix}\cdot dr \tag{A1}$$

A short-handed expression in Eq. (3.14) is defined by

$$\bar{\nabla}\cdot(\sigma_j\boldsymbol{u}_j\boldsymbol{u}_j) \equiv \begin{pmatrix}\dfrac{\partial}{\partial x}\sigma_j u_j^2 + \dfrac{\partial}{\partial y}\sigma_j v_j u_j\\[3mm]\dfrac{\partial}{\partial x}\sigma_j u_j v_j + \dfrac{\partial}{\partial y}\sigma_j v_j^2\end{pmatrix} \tag{A2}$$

with $\boldsymbol{u}_j = (u_j, v_j)$.

Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper

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4, 3127–3160, 2011

Multi-component analogue

J.-I. Yano

Title Page

Abstract   Introduction

Conclusions   References

Tables   Figures

◄◄   ►►

◄   ►

Back   Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

A2   Hydrostatic balance

The exact hydrostatic balance under SCA is given by

$$\frac{\partial}{\partial p}(\sigma_j \phi_j) - \frac{1}{S} \oint_{\partial S_j} \phi \left(\frac{\partial \boldsymbol{r}_{\mathrm{b},j}}{\partial p} \right) \cdot d\boldsymbol{r} = -\sigma_j \alpha_j \tag{A3}$$

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Multi-component analogue

J.-I. Yano

Back | Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

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Multi-component analogue

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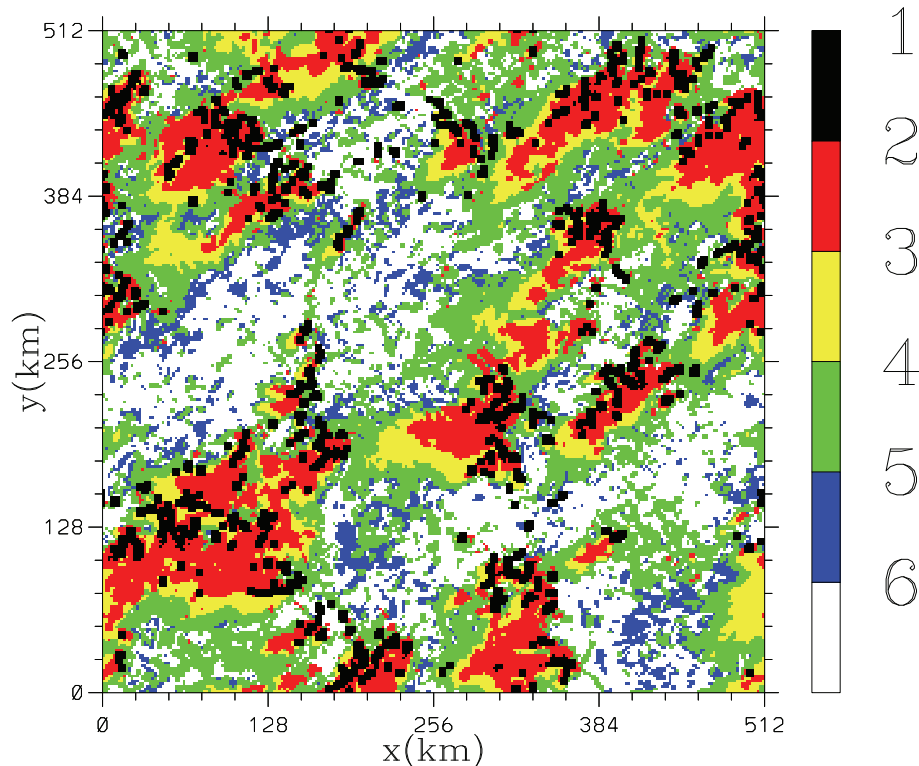
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Fig. 1. Spatial distribution of six subgrid-scale categories over a domain of the sizes, 512 km × 512 km, simulated by a cloud–resolving model (CRM). The categories are (1) precipitating convection, (2) precipitating stratiform, (3) non-precipitating stratiform, (4) shallow clouds, (5) ice anvils, and (6) environment. The CRM simulation is from the TOGA-COARE (Tropical Ocean Global Atmosphere Couple Ocean Response Experiment) period, representing a typical fully-developed marine-type deep convective system. See Yano et al. (2005a) for details of the simulations as well as the categorization scheme. (From Fig. 2a of Yano et al., 2005a with modifications).

Title Page

Abstract | Introduction

Conclusions | References

Tables | Figures

|◄ | ►|

◄ | ►

Back | Close

Full Screen / Esc

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Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper