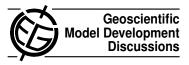
Geosci. Model Dev. Discuss., 3, C165–C170, 2010 www.geosci-model-dev-discuss.net/3/C165/2010/ © Author(s) 2010. This work is distributed under the Creative Commons Attribute 3.0 License.



## Interactive comment on "Efficient approximation of the incomplete gamma function for use in cloud model applications" by U. Blahak

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### Response to anonymous referee #2

### Response to major comment #1:

The referee is certainly right that there are many other processes involing some sort of spectral cut off, whose parameterization could possibly be improved by explicitly using incomplete gamma functions. Corresponding remarks have been added at the end of the first paragraph of section 1 and at the beginning of the last paragraph on page 454 (referring to the original manuscript).

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However, the author does not know about extensively much one- or two-moment bulk schemes applying variable shape parameters  $\mu$  and  $\nu$  of the (generalized) gamma distribution, and hence, a variable a. Only recently a variable (diagnostic)  $\mu$ -parameter is used in a few schemes for some of the processes and some of the hydrometeor species (two new references have been added: Milbrandt, 2005b; Seifert, 2008; see below). Nevertheless, the original statement "However, because for many cloud microphysical process parametrizations in the context of bulk (moment) approaches the parameter a is fixed during subsequent function evaluations, ..." has been slightly relaxed to "However, if the parameter a is fixed during subsequent function evaluations (as is the case for many cloud microphysical process parametrizations in the context of bulk approaches), ...".

Additionally, the issue about the appearance of "fixed" or "not fixed" x and a has been clarified by an added paragraph below Eq. 5:

"Eq. 5 is a simple but instructive example of a bulk cloud microphysical process parameterization, that comprises two typical features regarding the parameters *a* and *x* of the incomplete gamma function. First, the parameter *a* depends on the shape parameters  $\mu$  and  $\nu$  of the assumed distribution Eq. (2) and not on  $N_0$  and  $\lambda$ . Now, in most of the established one- or two-moment bulk schemes,  $\mu$  and  $\nu$  are fixed parameters which do not change during a particular model simulation, so that *a* also remains fixed (only recently, authors start to make at least  $\mu$  variable in a diagnostic way for some processes, e.g., Milbrandt, 2005b; Seifert, 2008). Second, the integration limit (respectively size threshold) *m* of the process parameterization ( $m_{wg}$  in the above Eq. 5) translates into the *x*-parameter  $\lambda m^{\nu}$  of the incomplete gamma function. That means, even if *m* is a fixed parameter, *x* is not fixed because  $\lambda$  is variable. Therefore, in cloud physics either *a* is fixed and *x* varies during a model simulation (which will be of importance later) or both *a* and *x* vary; the case of a variable *a* and fixed *x* has not yet occured in literature to the best knowlegde of the author. Third, *a* attains non-integer values in most cases, depending on the choice of  $\mu$  and  $\nu$ . As it is outlined later, an integer value of a leads to an analytic expression for the incomplete gamma function, which facilitates its computation. Unfortunately, there is only a chance to get integer values of a if one assumes exponential particle size- or mass-distributions, which is a serious and perhaps in many cases unphysical restriction. "

# So it should become clear that the case of a fixed x does not occur, even if the integration limit (size- or mass threshold) is fixed.

In contrast to the referee, the author does not feel that much more discussion about the implications of a "non-fixed" *a* for the efficiency of computing incomplete gamma functions is needed, because it is already discussed at several places during the manuscript (end of section 1, end of section 2, conclusions). The implication is simply more computational burden, leaving only a moderate improvement over the method of Press.

However, regarding the treatment of the lookup tables near the end of section 1, the sentence

"In case that *a* is not fixed, the lookup table would have to be two-dimensional, requiring two-dimensional interpolation techniques (bilinear in the most simple case)." has been added.

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### Response to minor comments:

 Good point. As requested, the analytic formulas for integer values of *a* are now mentioned in the middle of page 456 (refering to the original manuscript), along with a cross reference to the later given equations. Unfortunately, one only has a good chance to get integer values of *a*, if exponential particle size- or mass distributions are assumed, which gets more and more unpopular these days. This is now mentioned als point "Third, ..." in the new paragraph cited in the above answer to the major comment #1.

Moreover, in many bulk schemes the user can choose the shape parameters  $\mu$  and  $\nu$  for each hydrometeor species, so that the programmer does not know beforehand if the resulting *a* will be an integer or not.

2. For the cited example, it involved fitting a piecewise linear function to the incomplete gamma function w.r.t. x for a fixed and non-integer value of a. This has been added:

"In some cases where incomplete gamma functions have been used, simple analytical approximations for very special **and fixed** values of a were employed, e.g., **a piecewise linear "ramp" function of** x **for non-integer** a **in Cotton et al. (1986).**"

- Integrals similar to the one in Eq. 5. This is now clarified:
  "Or, as in Farley et al. (1989), finite integrals similar to the one in Eq. 5, (otherwise resulting in incomplete gamma functions) have been ...."
- 4. No, that is not the case. See my response to major comment #1.
- 5. That is true. Therefore, the restriction to "equidistant" tables has been generalized to "regular" tables, where the term regular is meant in the sense that the indices of needed table values can be calculated to avoid a grid point search.

Equidistant tables are retained as the perhaps most simple example of a regular table. The text passage about lookup tables in the introduction has been rewritten accordingly.

Additionally, he referee's comment has stimulated to mention the application of higher-order interpolation schemes with reduced  $\Delta x$  compared to the presented simple linear interpolation.

- 6. The functional forms have been chosen simply by guessing and experimentation. This is now mentioned in the paper right below the fit functions  $\hat{c}_x$ .
- 7. No, that is not the case. See my response to major comment #1.
- 8. No, that is not the case. See my response to major comment #1.

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#### **Technical corrections:**

- 1. corrected.
- 2. corrected.
- 3. corrected.

Interactive comment on Geosci. Model Dev. Discuss., 3, 451, 2010.