

Supplement of Geosci. Model Dev., 9, 223–245, 2016
<http://www.geosci-model-dev.net/9/223/2016/>
doi:10.5194/gmd-9-223-2016-supplement
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Supplement of

A multi-layer land surface energy budget model for implicit coupling with global atmospheric simulations

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Supplementary material to: ‘A multi-layer land surface energy budget for implicit coupling with global atmospheric simulations’

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S1 Notes

- **Objective:** This document expands on the model description present in the main text. We aimed at presenting a single documentation of the model. Therefore, we have duplicated parts of the methods here, to avoid multiple references to the main document.
- **Symbolic notation:** The symbolic notation used here is listed in full in Table 1 of the main text.
- **Numbering of equations:** the three ‘key’ equations (and variations as a result of substitution) are labelled a), b) and c) on the left hand side throughout the document. The assumptions are labelled in Roman numerals on the left hand side. All equations are numbered conventionally on the right hand side for ease of reference.
- **Potential enthalpy:** the present ORCHIDEE uses the term ‘surface static energy’ as the potential for calculating sensible heat flux. This is defined in the model (for the surface layer) as:

$$ps_{surf} = C_p^{air} T_{surf}$$

where ps_{surf} is the surface static energy, C_p^{air} is the mass specific heat capacity of air and T_{surf} the surface temperature.

Now the enthalpy of a system (H) is defined $H = U + pV$, but over the height of a surface model (< 30m approx), change in p and V is negligible, so:

$$\begin{aligned}\delta H &= \delta U + p\delta V + V\delta p \\ &= (\delta Q + \delta W + \delta W') + p\delta V + V\delta p\end{aligned}$$

(where we apply the first law of thermodynamics: $dU = \delta Q - \delta W$)

now $\delta W = -p\delta V$, so we can say:

$$\begin{aligned}\delta H &= \delta Q + \delta W' + V\delta p \\ &= Q + W' + \int_{p_0}^p V\delta p \approx Q \\ &\approx C_p^{air} T\end{aligned}$$

So here we can also assume a proportional relationship between enthalpy and temperature over the vertical range of the model.

- **Sign convention:** For latent and sensible heat fluxes, an upward flux is positive (so a positive flux from the ground is cooling the ground)

S2 Key equations and schemes

The structure of the derivation here outlined is based on that of the LMDz transport scheme (Dufresne & Ghattas, 2009), but extended to include interactions with the vegetation layer at each level.

S2.1 Leaf vapour pressure assumption

The air within leaf level cavities is assumed completely saturated. This means that the vapour pressure of the leaf can be calculated as the saturated vapour pressure at that leaf temperature. Therefore the change in pressure within the leaf is assumed proportional to the difference in temperature between the present timestep and next timestep, multiplied by the rate of change in saturated pressure against temperature.

$$q_0 \equiv q_{L,i}^{t+1} = q_{sat}^{T_{L,i}^t} + \frac{\partial q_{sat}}{\partial T} \Big|_{T_{L,i}^t} (T_{L,i}^{t+1} - T_{L,i}^t) \quad (\text{S2.1})$$

$$= \frac{\partial q_{sat}}{\partial T} \Big|_{T_{L,i}^t} (T_{L,i}^{t+1}) + \left(q_{sat}^{T_{L,i}^t} - T_{L,i}^t \frac{\partial q_{sat}}{\partial T} \Big|_{T_{L,i}^t} \right) \quad (\text{S2.2})$$

$$= \alpha_i T_{L,i}^{t+1} + \beta_i \quad (\text{S2.3})$$

where α_i and β_i are regarded as constants for each particular level and timestep so $\alpha_i = \frac{\delta q_{sat}}{\delta T} \Big|_{T_{L,i}^t}$ and $\beta_i = \left(q_{sat}^{T_{L,i}^t} - T_{L,i}^t \frac{\partial q_{sat}}{\partial T} \Big|_{T_{L,i}^t} \right)$

But to find a solution we still need to find an expression for the terms $q_{sat}^{T_{L,i}^t}$ and $\frac{\partial q_{sat}}{\partial T} \Big|_{T_{L,i}^t}$ in α_i and β_i above.

Using the empirical approximation of Tetens (e.g. as in Monteith and Unsworth (2008)) and the specific humidity vapour pressure relationship we can describe the saturation vapour pressure to within 1 Pa up to a temperature of about 35 °C.

S2.2 Physical and biophysical parameters:

We here concentrate on the formulation of an implicit solution that assumes a parameterisation for R_i (the resistance to sensible heat flux at each level), R'_i (resistance to latent heat flux at each level) and k_i (transport coefficients at each level). The derivation of these coefficients, based on literature study, will be described in a separate document.

S2.3 The leaf energy balance equation for each layer

Now at the leaf level, we assume the energy balance for each layer. It is assumed that (for a leaf layer of volume ΔV_i , area ΔA_i and thickness Δh_i):

$$\Delta V_i \theta_i \rho_v \frac{dT_{leaf,i}}{dt} = (-H_i - \lambda E_i + R_{SW,i} + R_{LW,i}) \Delta A_i \quad (\text{S2.4})$$

Dividing (S2.4) by ΔV_i :

$$\theta_i \rho_v \frac{dT_{leaf,i}}{dt} = (-H_i - \lambda E_i + R_{SW,i} + R_{LW,i}) \left(\frac{1}{\Delta h_i} \right) \quad (\text{S2.5})$$

The source sensible heat flux from the leaf at level i is the difference between the leaf temperature and that above it, divided by R_i which is the leaf resistance to sensible heat flux (a combination of stomatal and boundary layer resistance)). Similarly, the source latent heat flux from the leaf at level i is the difference between the leaf temperature and that above it, divided by R'_i which is the leaf resistance to sensible heat flux. So the terms of (S2.5) are defined (in units W/m^2):

$$H_i = C_p^{air} \rho_a \frac{(T_{leaf,i} - T_{a,i})}{R_i} \quad (\text{S2.6})$$

$$\lambda E_i = \lambda \rho_a \frac{(q_{leaf,i} - q_{a,i})}{R'_i} \quad (\text{S2.7})$$

- R_{LW} is the sum total of longwave radiation - that is: downwelling LW radiation from above the canopy, the LW radiation emitted from vegetation layer i and the LW radiation reflected from the vegetation layers $i + 1$ and $i - 1$.

- R_{SW} is the sum of short radiation, that is to say radiation downwelling on a level, but also that which is forward- of back- reflected from one level to another, or from the soil surface. So we express the sensible and latent heat fluxes between the leaf and the atmosphere respectively as:

$$a) \quad \theta_i \rho_v \frac{dT_{leaf,i}}{dt} = \left(-C_p^{air} \rho_a \frac{(T_{leaf,i} - T_{a,i})}{R_i} - \lambda \rho_a \frac{(q_{leaf,i} - q_{a,i})}{R'_i} + R_{SW,i} + R_{LW(tot),i} \right) \left(\frac{1}{\Delta h_i} \right) \quad (\text{S2.8})$$

S2.4 Sensible heat transport between each atmospheric layer

The sensible heat flux profile is not constant over the height of the canopy. The rate of change of $T_{a,i}$ (the temperature of the atmosphere surrounding the leaf at level i)

is proportional to the rate of change of sensible heat flux with height and the source sensible heat flux from the leaf at that level as in (S2.6) above:

$$b) \quad C_p^{air} \rho_a \frac{dT_{a,i}}{dt} \Delta V_i = -\frac{\partial H_{a,i}}{\partial z} \Delta V_i + \left(\frac{T_{leaf,i} - T_{a,i}}{R_i} \right) \left(\frac{C_p^{air} \rho_a}{\Delta h_i} \right) \Delta V_i \quad (\text{S2.9})$$

now $H_{a,i} = -(\rho_a C_p^{air}) k_i \frac{\partial T_{a,i}}{\partial z}$ (if the flux-gradient relation is assumed) so we can say:

$$b) \quad \boxed{\frac{dT_{a,i}}{dt} \Delta V_i = \frac{\partial}{\partial z} \left(k_i \frac{\partial T_{a,i}}{\partial z} \right) \Delta V_i + \left(\frac{T_{leaf,i} - T_{a,i}}{R_i} \right) \left(\frac{1}{\Delta h_i} \right) \Delta V_i} \quad (\text{S2.10})$$

S2.5 Latent heat transport between each atmospheric layer

The latent heat flux profile is also not constant over the height of the canopy. The rate of change of $q_{a,i}$ (the specific humidity of the atmosphere surrounding the leaf at level i) is proportional to the rate of change of latent heat flux with height and the source latent heat flux from the leaf as in (S2.7):

$$c) \quad \lambda \rho_a \frac{dq_{a,i}}{dt} \Delta V_i = -\frac{\partial (\lambda E)_{a,i}}{\partial z} \Delta V_i + \left(\frac{q_{leaf,i} - q_{a,i}}{R'_i} \right) \left(\frac{\lambda \rho_a}{\Delta h_i} \right) \Delta V_i \quad (\text{S2.11})$$

$$= -\frac{\partial (\lambda E)_{a,i}}{\partial z} \Delta V_i + \left(\frac{(\alpha T_{leaf,i} + \beta_i) - q_{a,i}}{R'_i} \right) \left(\frac{\lambda \rho_a}{\Delta h_i} \right) \Delta V_i \quad (\text{S2.12})$$

now $(\lambda E)_{a,i} = -(\lambda \rho_a) k_i \frac{dq_{a,i}}{dz}$ (again assuming the flux-gradient relation) so:

$$c) \quad \boxed{\frac{dq_{a,i}}{dt} \Delta V_i = \frac{\partial}{\partial z} \left(k_i \frac{\partial q_{a,i}}{\partial z} \right) \Delta V_i + \left(\frac{(\alpha T_{leaf,i} + \beta_i) - q_{a,i}}{R'_i} \right) \left(\frac{1}{\Delta h_i} \right) \Delta V_i} \quad (\text{S2.13})$$

S2.6 The ‘zero-leaf’ scenario

Canopy layers that do not contain foliage may be accounted for at a level by assuming that $R_i = R'_i = \infty$ for that level (i.e. an open circuit), and that the various coefficients

that relate to the leaf interactions at that level (R_{SW} , R_{LW} , $C_{T,i}$, $C_{T,i+1}$, $C_{q,i}$, $C_{q,i+1}$, D_i , E_i , F_i , D_{i+1} , E_{i+1} , F_{i+1}) are zero.

S2.7 Radiation scheme

The radiation approach is the application of the longwave Radiation Transfer Matrix (LRTM) (Gu (1988); Gu et al. (1999)), as applied in Ogée et al. (2003). This approach separates the calculation of the radiation distribution completely from the implicit expression. Instead a single source term for the longwave radiation is added at each level. This means that the distribution of radiation is no longer completely explicit (i.e. an explicit scheme makes use of information only from the ‘present’ and not the ‘next’ time step). However, an advantage of the approach is that it accounts for a higher order of reflections from adjacent levels than the single order that is assumed in the process above.

The components for longwave radiation are abbreviated as:

$$R_{LW,i} = \eta_{1,i} T_{leaf,i}^{t+1} + \eta_{2,i} \quad (\text{S2.14})$$

The shortwave radiation component is abbreviated as:

$$R_{SW,i} = \eta_{3,i} R_{SW}^{down} \quad (\text{S2.15})$$

where $\eta_{1,i}$, $\eta_{2,i}$ and $\eta_{3,i}$ are components of the radiation scheme. $\eta_{1,i}$ accounts for the components relating to emission and absorption of LW radiation from the vegetation at level i (i.e. the implicit parts of the longwave scheme relating to the level i) and $\eta_{2,i}$ the components relating to radiation from vegetation at all other levels incident on the vegetation at level i (i.e. the non-implicit part of the longwave scheme), as well as explicit parts from the level i .

$\eta_{3,i}$ is the component of the SW radiation scheme - it describes the fraction of the total downwelling shortwave light that is absorbed at each layer, including over multiple forward- and back-reflections, as simulated by the multilayer albedo scheme (McGrath et al., 2015). The fraction of original downwelling SW radiation that is ultimately reflected from the surface and from the vegetation cover back to the canopy can then be calculated using this information.

S2.7.1 Longwave radiation

We applied a version of the Longwave Radiation Transfer Scheme of Gu (1988, 1999), with some modifications that are summarised here. The method assumes that scattering coefficients for longwave radiation are very small (of the order of 0.05), and can thus be

ignored. The basics of the scheme can be described by the matrix equation for a canopy of m levels:

$$\begin{pmatrix} \Delta \mathcal{N}_{surf} \\ \Delta \mathcal{N}_1 \\ \cdot \\ \cdot \\ \cdot \\ \Delta \mathcal{N}_m \\ \Delta \mathcal{N}_{above} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0}^{LW} & \alpha_{0,1}^{LW} & \cdot & \cdot & \cdot & \alpha_{0,m}^{LW} & \alpha_{0,m+1}^{LW} \\ \alpha_{1,0}^{LW} & \alpha_{1,1}^{LW} & \cdot & \cdot & \cdot & \alpha_{1,m}^{LW} & \alpha_{1,m+1}^{LW} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha_{m,0}^{LW} & \alpha_{m,1}^{LW} & \cdot & \cdot & \cdot & \alpha_{m,m}^{LW} & \alpha_{m,m+1}^{LW} \\ \alpha_{m+1,0}^{LW} & \alpha_{m+1,1}^{LW} & \cdot & \cdot & \cdot & \alpha_{m+1,m}^{LW} & \alpha_{m+1,m+1}^{LW} \end{pmatrix} \begin{pmatrix} \sigma(T_{surf}^t)^4 \\ \sigma(T_{leaf,1}^t)^4 \\ \cdot \\ \cdot \\ \cdot \\ \sigma(T_{leaf,m}^t)^4 \\ R_{LW} \end{pmatrix}$$

for which each element $\alpha_{i,j}^{LW}$ is defined as:

$$\alpha_{i,j} = \begin{cases} -1, & i = j = 0. \\ \mathfrak{S}(\ell_t - \ell_{j-1}) - \mathfrak{S}(\ell_t - \ell_j), & i=0, j=1, 2, \dots, m \\ \mathfrak{S}(\ell_t), & i=0, j=m+1 \\ \mathfrak{S}(\ell_j - \ell_{i-1}) - \mathfrak{S}(\ell_{j-1} - \ell_{i-1}) - \mathfrak{S}(\ell_j - \ell_i) + \mathfrak{S}(\ell_{j-1} - \ell_i), & i=1, 2, \dots, m, j=1, 2, \dots, i-1 \\ 2\Delta\mathfrak{S}(\ell_i) - 2, & i=1, 2, \dots, m, j=i \\ \mathfrak{S}(\ell_i - \ell_{j-1}) - \mathfrak{S}(\ell_i - \ell_j) - \mathfrak{S}(\ell_{i-1} - \ell_{j-1}) + \mathfrak{S}(\ell_{i-1} - \ell_j), & i=1, 2, \dots, m, j=i+1, i+2, \dots, m \\ \mathfrak{S}(\ell_t), & i=m+1, j=0 \\ \mathfrak{S}(\ell_j) - \mathfrak{S}(\ell_{j-1}), & i=m+1, j=1, 2, \dots, m \\ -1, & i = m+1, j=m+1. \end{cases} \quad (\text{S2.16})$$

Now, the column on the left hand side of the expression $\Delta \mathcal{N}_i$ represents the net longwave radiation that is absorbed at each level vegetation i , as well as the soil surface layer (\mathcal{N}_{surf}) and the atmosphere directly above the canopy (\mathcal{N}_{above}). T_i , is the temperature of each layer, and R_{LW} represents the downwelling longwave radiation from above the canopy. Here ℓ_i represents the cumulative leaf area index when working up to level i from the ground, that is to say calculated as:

$$\ell_i = \sum_1^i LAI_i \quad (\text{S2.17})$$

The function $\mathfrak{S}(\ell)$ simulates the effect of canopy structure on the passage of longwave radiation, and is defined as:

$$\mathfrak{S}(\ell) = 2 \int_0^1 e^{-\frac{\ell G_{leaf}(\mu)}{\mu}} \mu d\mu \quad (\text{S2.18})$$

$G_{leaf}(\mu)$ is a function that represents the orientation of the leaves. $\mathfrak{S}(\ell)$ is then solved from integrations. So multiplying out the terms, we have the an expression for $\Delta\aleph$ at each level:

$$\Delta\aleph = \alpha_{i,0}^{LW} \sigma(T_{surf}^t)^4 + \alpha_{i,1}^{LW} \sigma(T_{leaf,1}^t)^4 \dots, \dots + \alpha_{i,i}^{LW} \sigma(T_{leaf,i}^t)^4 \\ \dots, \dots + \alpha_{i,m}^{LW} \sigma(T_{leaf,m}^t)^4 + \alpha_{LW}^{i,m+1} R_{LW} \quad (\text{S2.19})$$

This part of the energy budget model is explicit, relying on temperature at the last time step. However, for the level i in each case we can make the expression semi-implicit, by expressing partly in terms of the leaf temperature at the next time step, through use a truncated Taylor expansion, such that:

$$\alpha_{i,i}^{LW} \sigma(T_{leaf,i}^{t+1})^4 \approx \alpha_{i,i}^{LW} \sigma((T_{leaf,i}^t)^4 + 4(T_{leaf,i}^t)^3(T_{leaf,i}^{t+1} - T_{leaf,i}^t)) \quad (\text{S2.20})$$

$$= \alpha_{i,i}^{LW} \sigma(4(T_{leaf,i}^t)^3 T_{leaf,i}^{t+1} - 3(T_{leaf,i}^t)^4) \quad (\text{S2.21})$$

so, in effect, (S2.19) can be expressed as:

$$\Delta\aleph = \alpha_{i,0}^{LW} \sigma(T_{surf}^t)^4 + \alpha_{i,1}^{LW} \sigma(T_{leaf,1}^t)^4 \dots, \dots + \alpha_{i,i}^{LW} \sigma(4(T_{leaf,i}^t)^3 T_{leaf,i}^{t+1} - 3(T_{leaf,i}^t)^4) \\ \dots, \dots + \alpha_{i,m}^{LW} \sigma(T_{leaf,m}^t)^4 + \alpha_{LW}^{i,m+1} R_{LW} \quad (\text{S2.22})$$

and so we calculate the matrix (44) above with the central diagonal for which $i = j$ set to zero and designate the coefficients (S2.14) as:

$$\eta_{1,i} = \alpha_{i,i}^{LW} \sigma(T_{leaf,i}^t)^3 \quad (\text{S2.23})$$

$$\eta_{2,i} = \aleph_i - 3\alpha_{i,i}^{LW} \sigma(T_{leaf,i}^t)^4 \quad (\text{S2.24})$$

S2.7.2 Shortwave radiation

We implement the scheme from McGrath et al. (2015), which is a development of Pinty et al. (2006). The scheme accounts for three-dimensional canopies through use of a domain-averaged structure factor (the effective Leaf Area Index). To summarise, in this

approach the SW radiation is divided into several terms at each level expressed as a fraction of the total SW downwelling radiation, as listed below.

We use the notation ψ to denote the fraction of the above canopy SW radiation that is absorbed (ψ_i^{abs}), is incoming to each level i either by direct transmission (uncollided) or by reflection (collided). The symbol ‘ \downarrow ’ refers to the sum of all downwelling shortwave radiation (i.e. directly transmitted radiation, and second order reflected radiation), whilst ‘ \uparrow ’ refers to the sum of all upwelling shortwave radiation (i.e. sum of first-order and second-order reflected radiation from all levels).

- $\psi_{i,\downarrow,out}^{uncollided}$ - fraction corresponding to uncollided light transmitted through level i without striking any element. This is also described as ‘unscattered, collimated radiation’.
- $\psi_{i,\downarrow}^{collided}$ - fraction corresponding to collided, transmitted light transmitted through level i after striking vegetation one or more times. This is also described as ‘forward scattered isotropic radiation’.
- $\psi_{i,\uparrow}^{collided}$ - fraction corresponding to collided light reflected upwards after striking vegetation one or more times. This is also described as ‘back scattered isotropic radiation’.

Now, using these probabilities of the fate of the light, the equations of Pinty et al. (2006) are applied to each layer of the canopy in turn, initially for the top layer, with the assumption of a black background underneath. Some of the flux is reflected back into the atmosphere, some absorbed, and some transmitted or forward scattered into the level below. The nature of the light (collimated or isotropic) determines how it interacts with the canopy, so these two types of light are accounted for separately in the model. The calculations are repeated for this lower level, with this fraction of the light. Calculations through all of the levels are continued as an iterative process until all light is accounted for through either reflection (or back scatter) back to the atmosphere or absorption by the vegetation or by the soil. McGrath et al. (2015) provides full details of the iteration scheme, which produces as output ψ_i^{abs} , which is the fraction of the incident SW at the top of the canopy that is absorbed at each level at the end of the convergence loop. Tests in that work demonstrate that results from a multi-level case are in acceptable agreement with those from the single level.

Over the canopy vegetation levels, we can now define the coefficient $\eta_{3,i}$ in equation (S2.15):

$$\eta_{3,i} = \psi_i^{abs} \tag{S2.25}$$

$$\eta_{3,surf} = \psi_{surface}^{abs} \tag{S2.26}$$

S3 Derivation of implicit solution

S3.1 Write equations in implicit format

To maintain the implicit coupling between the atmospheric model (i.e. LMDZ) and the land surface model (i.e. ORCHIDEE) we need to express the relationships that are outlined above in terms of a linear relationship between the 'present' timestep t and the 'next' timestep $t + 1$.

We therefore re-cast equations a), b) and c) in implicit form (i.e. in terms of the 'next' timestep, which is $t + 1$, as below.

S3.2 Implicit form of the energy balance equation

We substitute the expressions (S2.14) and (S2.15) to the energy balance equation (S2.5), which we rewrite in implicit form:

$$a) \quad \theta_i \rho_v \frac{(T_{leaf,i}^{t+1} - T_{leaf,i}^t)}{\Delta t} = \left(\frac{1}{\Delta h_i} \right) \left(-C_p^{air} \rho_a \frac{(T_{leaf,i}^{t+1} - T_{a,i}^{t+1})}{R_i} - \lambda \rho_a \frac{(\alpha_i T_{leaf,i}^{t+1} + \beta_i - q_{a,i}^{t+1})}{R'_i} + \eta_1 T_{leaf,i}^{t+1} + \eta_2 + \eta_3 R_{SW}^{down} \right) \quad (S3.1)$$

Rearranging to isolate the state variables terms (temperature and specific humidity) at the 'next' timestep:

$$a) \quad T_{leaf,i}^{t+1} - T_{leaf,i}^t = \frac{\lambda \rho_a \Delta t \beta_i}{(\rho_v \Delta h_i) R'_i \theta_i} + \frac{\eta_4 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_i) \theta_i} + \frac{\eta_1 R_{LW}^{down} \Delta t}{(\rho_v \Delta h_i) \theta_i} + \frac{\eta_3 \Delta t}{(\rho_v \Delta h_i) \theta_i} + T_{leaf,i}^{t+1} \left(-C_p^{air} \rho_a \frac{\Delta t}{(\rho_v \Delta h_i) R_i \theta_i} - \lambda \rho_a \frac{\Delta t \alpha_i}{(\rho_v \Delta h_i) R'_i \theta_i} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_i) \theta_i} \right) + T_{a,i}^{t+1} C_p^{air} \rho_a \left(\frac{\Delta t}{R_i \theta_i (\rho_v \Delta h_i)} \right) + q_{a,i}^{t+1} \lambda \rho_a \left(\frac{\Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} \right) \quad (S3.2)$$

S3.3 Implicit form of the sensible heat flux transport equation

We difference (S2.10), and divide by ΔV_i :

$$\begin{aligned}
b) \quad \frac{T_{a,i}^{t+1} - T_{a,i}^t}{\Delta t} &= k_i \left(\frac{(T_{a,i+1}^{t+1} - T_{a,i}^{t+1})}{\Delta z_i \Delta h_i} \right) - k_{i-1} \left(\frac{(T_{a,i}^{t+1} - T_{a,i-1}^{t+1})}{\Delta z_{i-1} \Delta h_i} \right) \\
&\quad + \left(\frac{1}{\Delta h_i} \right) \frac{(T_{leaf,i}^{t+1} - T_{a,i}^{t+1})}{R_i} \quad (S3.3)
\end{aligned}$$

S3.4 Implicit form of the latent heat flux transport equation

We difference (S2.13), and divide by ΔV_i :

$$\begin{aligned}
c) \quad \frac{q_{a,i}^{t+1} - q_{a,i}^t}{\Delta t} &= k_i \left(\frac{(q_{a,i+1}^{t+1} - q_{a,i}^{t+1})}{\Delta z_i \Delta h_i} \right) - k_{i-1} \left(\frac{(q_{a,i}^{t+1} - q_{a,i-1}^{t+1})}{\Delta z_{i-1} \Delta h_i} \right) \\
&\quad + \left(\frac{1}{\Delta h_i} \right) \frac{(\alpha_i T_{leaf,i}^{t+1} + \beta_i - q_{a,i}^{t+1})}{R'_i} \quad (S3.4)
\end{aligned}$$

S3.5 Solving the leaf energy balance equation by induction

We determine to solve these equations by assuming a solution of a particular form and finding the coefficients that are introduced in terms of the coefficients of the layer above. This is ‘proof by induction’. Now, for (S3.2) we want to express $T_{a,i}^{t+1}$ in terms of values further down the column, to allow the equation to be solved by ‘moving up’ the column, as in Richtmyer and Morton (1967) and Dufresne and Ghattas (2009).

We assume that:

$$i) \quad \boxed{T_{a,i}^{t+1} = A_{T,i} T_{a,i-1}^{t+1} + B_{T,i} + C_{T,i} T_{leaf,i}^{t+1} + D_{T,i} q_{a,i-1}^{t+1}} \quad (S3.5)$$

$$ii) \quad \boxed{q_{a,i}^{t+1} = A_{q,i} q_{a,i-1}^{t+1} + B_{q,i} + C_{q,i} T_{leaf,i}^{t+1} + D_{q,i} T_{a,i-1}^{t+1}} \quad (S3.6)$$

These two expressions are the equivalent of equation 11.7 (from Richtmyer and Morton (1967)) for the present system.

We also re-write these expressions in terms of the values of the next level:

$$i) \quad \boxed{T_{a,i+1}^{t+1} = A_{T,i+1}T_{a,i}^{t+1} + B_{T,i+1} + C_{T,i+1}T_{leaf,i+1}^{t+1} + D_{T,i+1}q_{a,i}^{t+1}} \quad (S3.7)$$

$$ii) \quad \boxed{q_{a,i+1}^{t+1} = A_{q,i+1}q_{a,i}^{t+1} + B_{q,i+1} + C_{q,i+1}T_{leaf,i+1}^{t+1} + D_{q,i+1}T_{a,i}^{t+1}} \quad (S3.8)$$

where $A_{T,i}$, $B_{T,i}$, $C_{T,i}$, $D_{T,i}$, $A_{q,i}$, $B_{q,i}$, $C_{q,i}$ and $D_{q,i}$ are constants for that particular level and timestep but are (as yet) unknown. We thus substitute (S3.5) and (S3.6) into (S3.2) to eliminate $T_{a,i}^{t+1}$

$$\begin{aligned} a) \quad T_{leaf,i}^{t+1} - T_{leaf,i}^t &= -\lambda\rho_a \frac{\Delta t \beta_i}{(\rho_v \Delta h_i) R'_i \theta_i} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_i) \theta_i} \\ &+ \frac{\eta_3 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_i) \theta_i} \\ &+ \frac{T_{L,i}^{t+1}}{(\rho_v \Delta h_i)} \left(-\frac{\lambda\rho_a \Delta t \alpha_i}{R'_i \theta_i} - \frac{C_p^{air} \rho_a \Delta t}{\theta_i R_i} + \frac{\eta_1 \Delta t}{\theta_i} \right) \\ &+ \left(\frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_i) R_i \theta_i} \right) (A_{T,i} T_{a,i-1}^{t+1} + B_{T,i} + C_{T,i} T_{leaf,i}^{t+1} + D_{T,i} q_{a,i-1}^{t+1}) \\ &+ \left(\frac{\lambda\rho_a \Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} \right) (A_{q,i} q_{a,i-1}^{t+1} + B_{q,i} + C_{q,i} (T_{leaf,i}^{t+1}) + D_{q,i} T_{a,i-1}^{t+1}) \end{aligned} \quad (S3.9)$$

or, to rearrange again in terms of the unknown state variables (left hand side) and the know variables (right hand side):

$$\begin{aligned} a) \quad T_{L,i}^{t+1} \left(1 - \frac{\Delta t C_p^{air}}{(\rho_v \Delta h_i) R_i \theta_i} C_{T,i} + \frac{\lambda\rho_a \alpha_i \Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} - \frac{\eta_1 \Delta t}{(\rho_v \Delta h_i) \theta_i} \right. \\ \left. + \frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_i) \theta_i R_i} - \frac{\lambda\rho_a \Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} C_{q,i} \right) = \\ T_{L,i}^t + q_{a,i-1}^{t+1} \left(\frac{\lambda\rho_a \Delta t A_{q,i}}{(\rho_v \Delta h_i) R'_i \theta_i} + \frac{C_p^{air} \rho_a \Delta t D_{T,i}}{(\rho_v \Delta h_i) R_i \theta_i} \right) \\ + T_{a,i-1}^{t+1} \left(\frac{C_p^{air} \rho_a \Delta t A_{T,i}}{(\rho_v \Delta h_i) R_i \theta_i} + \frac{\lambda\rho_a \Delta t D_{q,i}}{(\rho_v \Delta h_i) R_i \theta_i} \right) + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_i) \theta_i} \\ + \frac{\eta_3 R_{SW,i}^{down} \Delta t}{(\rho_v \Delta h_i) \theta_i} + \frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_i) R_i \theta_i} B_{T,i} + \frac{\lambda\rho_a \Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} B_{q,i} - \frac{\lambda\rho_a \Delta t}{(\rho_v \Delta h_i) R'_i \theta_i} \beta_i \end{aligned} \quad (S3.10)$$

So, to abbreviate (where E_i , F_i and G_i are known assumed constants for the level and timestep in question, (S3.10) can be written as:

$$iii) \quad \boxed{T_{leaf,i}^{t+1} = E_i q_{a,i-1}^{t+1} + F_i T_{a,i-1}^{t+1} + G_i} \quad (S3.11)$$

so we define the coefficients as:

$$E_i = \left(\frac{\Delta t A_{q,i} \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\Delta t D_{T,i} C_p^{air} \rho_a}{(\rho_v \Delta h_i) R_i \theta_i} \right) / \left(1 + \frac{\Delta t \alpha \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) \theta_i R_i} - \frac{\eta_1 \Delta t}{(\rho_v \Delta h_i) \theta_i} - \frac{\Delta t \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} C_{q,i} - \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) \theta_i R_i} C_{T,i} \right) \quad (S3.12)$$

$$F_i = \left(\frac{\Delta t A_{T,i} C_p^{air} \rho_a}{(\rho_v \Delta h_i) R_i \theta_i} + \frac{\Delta t D_{q,i} \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} \right) / \left(1 + \frac{\Delta t \alpha \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) \theta_i R_i} - \frac{\eta_1 \Delta t}{(\rho_v \Delta h_i) \theta_i} - \frac{\Delta t \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} C_{q,i} - \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) \theta_i R_i} C_{T,i} \right) \quad (S3.13)$$

$$G_i = \left(T_{leaf,i}^t + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_i) \theta_i} + \frac{\eta_3 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_i) \theta_i} - \frac{\lambda \rho_a \Delta t \beta_i}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) R_i \theta_i} B_{T,i} + \frac{\Delta t \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} B_{q,i} \right) / \left(1 + \frac{\Delta t \alpha \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} + \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) \theta_i R_i} - \frac{\eta_1 \Delta t}{(\rho_v \Delta h_i) \theta_i} - \frac{\Delta t \lambda \rho_a}{(\rho_v \Delta h_i) R_i' \theta_i} C_{q,i} - \frac{\Delta t C_p^{air} \rho_a}{(\rho_v \Delta h_i) \theta_i R_i} C_{T,i} \right) \quad (S3.14)$$

S3.6 Solving latent and sensible heat flux equations between layers by induction

To prove by induction, we must express $T_{a,i}^{t+1}$ and $q_{a,i}^{t+1}$ in terms that are identical to (S3.5) and (S3.6) We first seek to eliminate $T_{a,i+1}^{t+1}$ from b) and c) We first

substitute the assumed expressions for temperature and humidity in the layer above or, that is to say, (equations (S3.7) and (S3.8) here) We substitute for $T_{a,i+1}^{t+1}$ in b), to eliminate that term:

$$b) \quad \frac{T_{a,i}^{t+1} - T_{a,i}^t}{\Delta t} = k_i \frac{A_{T,i+1} T_{a,i}^{t+1} + B_{T,i+1} + C_{T,i+1} T_{leaf,i+1}^{t+1} + D_{T,i+1} q_{a,i}^{t+1}}{\Delta z_i \Delta h_i} - \frac{k_i T_{a,i}^{t+1}}{\Delta z_i \Delta h_i} - \frac{k_{i-1} T_{a,i}^{t+1}}{\Delta z_{i-1} \Delta h_i} + \frac{k_{i-1} T_{a,i-1}^{t+1}}{\Delta z_{i-1} \Delta h_i} + \frac{T_{leaf,i}^{t+1}}{\Delta h_i R_i} - \frac{T_{a,i}^{t+1}}{\Delta h_i R_i} \quad (S3.15)$$

$$b) \quad T_{a,i}^{t+1} \left(1 - \Delta t \left(A_{T,i+1} \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} - \frac{1}{\Delta h_i R_i} \right) \right) = T_{a,i}^t + \frac{B_{T,i+1} k_i \Delta t}{\Delta z_i \Delta h_i} + T_{a,i-1}^{t+1} \left(\frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} \right) \Delta t + q_{a,i}^{t+1} \left(\frac{k_i D_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t + T_{leaf,i}^{t+1} \left(\frac{\Delta t}{\Delta h_i R_i} \right) + T_{leaf,i+1}^{t+1} \left(\frac{k_i C_{T,i+1}}{\Delta z_i \Delta h_i} \right) \quad (S3.16)$$

Similarly, we substitute for $q_{a,i+1}^{t+1}$ in c), in order to eliminate that term:

$$c) \quad \frac{q_{a,i}^{t+1} - q_{a,i}^t}{\Delta t} = k_i \frac{(A_{q,i+1} q_{a,i}^{t+1} + B_{q,i+1} + C_{q,i+1} T_{leaf,i+1}^{t+1} + D_{q,i+1} T_{a,i}^{t+1})}{\Delta z_i \Delta h_i} - \frac{k_i q_{a,i}^{t+1}}{\Delta z_i \Delta h_i} - \frac{k_{i-1} q_{a,i}^{t+1}}{\Delta z_{i-1} \Delta h_i} + \frac{k_{i-1} q_{a,i-1}^{t+1}}{\Delta z_{i-1} \Delta h_i} + \frac{\alpha_i T_{leaf,i}^{t+1} + \beta_i - q_{a,i}^{t+1}}{\Delta h_i R'_i} \quad (S3.17)$$

$$c) \quad q_{a,i}^{t+1} \left(1 - \Delta t \left(A_{q,i+1} \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} - \frac{1}{\Delta h_i R'_i} \right) \right) = q_{a,i}^t + \left(\frac{B_{q,i+1} k_i \Delta t}{\Delta z_i \Delta h_i} + \frac{\beta_i \Delta t}{\Delta h_i R'_i} \right) + q_{a,i-1}^{t+1} \left(\frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} \right) \Delta t + T_{a,i}^{t+1} \left(\frac{k_i D_{q,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t + (T_{leaf,i+1}^{t+1}) \left(\frac{k_i}{\Delta z_i \Delta h_i} \right) C_{q,i+1} \Delta t + T_{leaf,i}^{t+1} \left(\frac{\alpha_i}{\Delta h_i R'_i} \right) \Delta t \quad (S3.18)$$

Now, we substitute expression iii) for the leaf temperature in the layer above (S3.11). This step is in order to eliminate the term $T_{L,i+1}^{t+1}$ from both expressions:

$$\begin{aligned}
b) \quad T_{a,i}^{t+1} \left(1 - \Delta t \left(A_{T,i+1} \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} - \frac{1}{\Delta h_i R_i} \right) \right) = \\
T_{a,i}^t + \frac{B_{T,i+1} k_i \Delta t}{\Delta z_i \Delta h_i} + T_{a,i-1}^{t+1} \left(\frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} \right) \Delta t + q_{a,i}^{t+1} \left(\frac{k_i D_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t + T_{leaf,i}^{t+1} \left(\frac{\Delta t}{\Delta h_i R_i} \right) \\
+ (E_{i+1} q_{a,i}^{t+1} + F_{i+1} T_{a,i}^{t+1} + G_{i+1}) \left(\frac{k_i C_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t \quad (S3.19)
\end{aligned}$$

$$\begin{aligned}
c) \quad q_{a,i}^{t+1} \left(1 - \Delta t \left(A_{q,i+1} \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} - \frac{1}{\Delta h_i R'_i} \right) \right) = \\
q_{a,i}^t + \left(\frac{B_{q,i+1} k_i \Delta t}{\Delta z_i \Delta h_i} + \frac{\beta \Delta t}{\Delta h_i R'_i} \right) + q_{a,i-1}^{t+1} \left(\frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} \Delta t \right) \\
T_{a,i}^{t+1} \left(\frac{k_i D_{q,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t + (E_{i+1} q_{a,i}^{t+1} + F_{i+1} T_{a,i}^{t+1} + G_{i+1}) \left(\frac{k_i}{\Delta z_i \Delta h_i} \right) C_{q,i+1} \Delta t \\
+ T_{leaf,i}^{t+1} \left(\frac{\alpha}{\Delta h_i R'_i} \right) \Delta t \quad (S3.20)
\end{aligned}$$

We now abbreviate equation a) as:

$$b) \quad T_{a,i}^{t+1} X_{1,i} = X_{2,i} + X_{3,i} T_{a,i-1}^{t+1} + X_{4,i} q_{a,i}^{t+1} + X_{5,i} T_{leaf,i}^{t+1} \quad (S3.21)$$

and abbreviate equation b) as:

$$c) \quad q_{a,i}^{t+1} Y_{1,i} = Y_{2,i} + Y_{3,i} q_{a,i-1}^{t+1} + Y_{4,i} T_{a,i}^{t+1} + Y_{5,i} T_{leaf,i}^{t+1} \quad (S3.22)$$

where:

$$\begin{aligned}
X_{1,i} = 1 - \Delta t \left(A_{T,i+1} \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} - \frac{1}{\Delta h_i R_i} \right) \\
- F_{i+1} \left(\frac{k_i C_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t \quad (S3.23)
\end{aligned}$$

$$X_{2,i} = T_{a,i}^t + \frac{B_{T,i+1} k_i \Delta t}{\Delta z_i \Delta h_i} + G_{i+1} \left(\frac{k_i C_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t \quad (S3.24)$$

$$X_{3,i} = \left(\frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} \right) \Delta t \quad (S3.25)$$

$$X_{4,i} = \left(\frac{k_i D_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t + E_{i+1} \left(\frac{k_i C_{T,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t \quad (\text{S3.26})$$

$$X_{5,i} = \left(\frac{\Delta t}{\Delta h_i R_i} \right) \quad (\text{S3.27})$$

$$Y_{1,i} = 1 - \Delta t \left(A_{q,i+1} \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_i}{\Delta z_i \Delta h_i} - \frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} - \frac{1}{\Delta h_i R'_i} \right) - E_{i+1} \left(\frac{k_i}{\Delta z_i \Delta h_i} \right) C_{q,i+1} \Delta t \quad (\text{S3.28})$$

$$Y_{2,i} = q_{a,i}^t + \left(\frac{B_{q,i+1} k_i \Delta t}{\Delta z_i \Delta h_i} + \frac{\beta_i \Delta t}{\Delta h_i R'_i} \right) + G_{i+1} \left(\frac{k_i}{\Delta z_i \Delta h_i} \right) C_{q,i+1} \Delta t \quad (\text{S3.29})$$

$$Y_{3,i} = \left(\frac{k_{i-1}}{\Delta z_{i-1} \Delta h_i} \Delta t \right) \quad (\text{S3.30})$$

$$Y_{4,i} = \left(\frac{k_i D_{q,i+1}}{\Delta z_i \Delta h_i} \right) \Delta t + F_{i+1} \left(\frac{k_i}{\Delta z_i \Delta h_i} C_{q,i+1} \right) \Delta t \quad (\text{S3.31})$$

$$Y_{5,i} = \left(\frac{\alpha_i}{\Delta h_i R'_i} \right) \Delta t \quad (\text{S3.32})$$

We then cross-substitute for $q_{a,i}^{t+1}$ from c) to b), to eliminate that term:

$$\begin{aligned} \text{b) } T_{a,i}^{t+1} X_{1,i} &= X_{2,i} + X_{3,i} T_{a,i-1}^{t+1} + X_{4,i} \left(\frac{Y_{2,i}}{Y_{1,i}} + \frac{Y_{3,i}}{Y_{1,i}} q_{a,i-1}^{t+1} + \frac{Y_{4,i}}{Y_{1,i}} T_{a,i}^{t+1} + \frac{Y_{5,i}}{Y_{1,i}} T_{leaf,i}^{t+1} \right) \\ &\quad + X_{5,i} T_{leaf,i}^{t+1} \quad (\text{S3.33}) \end{aligned}$$

$$\begin{aligned} \text{b) } T_{a,i}^{t+1} \left(X_{1,i} - X_{4,i} \frac{Y_{4,i}}{Y_{1,i}} \right) &= T_{a,i-1}^{t+1} X_{3,i} + \left(X_{2,i} + X_{4,i} \frac{Y_{2,i}}{Y_{1,i}} \right) \\ &\quad + T_{L,i}^{t+1} \left(X_{4,i} \frac{Y_{5,i}}{Y_{1,i}} + X_{5,i} \right) + q_{a,i-1}^{t+1} \left(X_{4,i} \frac{Y_{3,i}}{Y_{1,i}} \right) \quad (\text{S3.34}) \end{aligned}$$

similarly, we cross-substitute for $T_{a,i}^{t+1}$ from b) to a), to eliminate that term:

$$\begin{aligned} \text{c) } q_{a,i}^{t+1} Y_{1,i} &= Y_{2,i} + Y_{3,i} q_{a,i-1}^{t+1} + Y_{4,i} \left(\frac{X_{2,i}}{X_{1,i}} + \frac{X_{3,i}}{X_{1,i}} T_{a,i-1}^{t+1} + \frac{X_{4,i}}{X_{1,i}} q_{a,i}^{t+1} + \frac{X_{5,i}}{X_{1,i}} T_{leaf,i}^{t+1} \right) \\ &\quad + Y_{5,i} T_{leaf,i}^{t+1} \quad (\text{S3.35}) \end{aligned}$$

$$\begin{aligned}
c) \quad q_{a,i}^{t+1} \left(Y_{1,i} - Y_{4,i} \frac{X_{4,i}}{X_{1,i}} \right) &= q_{a,i-1}^{t+1} Y_{3,i} + \left(Y_{2,i} + Y_{4,i} \frac{X_{2,i}}{X_{1,i}} \right) \\
&+ T_{leaf,i}^{t+1} \left(Y_{4,i} \frac{X_{5,i}}{X_{1,i}} + Y_{5,i} \right) + T_{a,i-1}^{t+1} \left(Y_{4,i} \frac{X_{3,i}}{X_{1,i}} \right) \quad (S3.36)
\end{aligned}$$

So this demonstrates the expressions b) and c) can be described in terms of the respective original substitutions (S3.5) and (S3.6). The respective coefficients from (S3.5) and (S3.6) may be described as:

$$A_{T,i} = \frac{X_{3,i}}{X_{1,i} - X_{4,i} \left(\frac{Y_{4,i}}{Y_{1,i}} \right)} \quad (S3.37)$$

$$B_{T,i} = \frac{X_{2,i} + X_{4,i} \left(\frac{Y_{2,i}}{Y_{1,i}} \right)}{X_{1,i} - X_{4,i} \left(\frac{Y_{4,i}}{Y_{1,i}} \right)} \quad (S3.38)$$

$$C_{T,i} = \frac{\left(X_{4,i} \left(\frac{Y_{5,i}}{Y_{1,i}} \right) + X_{5,i} \right)}{X_{1,i} - X_{4,i} \left(\frac{Y_{4,i}}{Y_{1,i}} \right)} \quad (S3.39)$$

$$D_{T,i} = \frac{X_{4,i} \left(\frac{Y_{3,i}}{Y_{1,i}} \right)}{X_{1,i} - X_{4,i} \left(\frac{Y_{4,i}}{Y_{1,i}} \right)} \quad (S3.40)$$

and:

$$A_{q,i} = \frac{Y_{3,i}}{Y_{1,i} - Y_{4,i} \left(\frac{X_{4,i}}{X_{1,i}} \right)} \quad (S3.41)$$

$$B_{q,i} = \frac{Y_{2,i} + Y_{4,i} \left(\frac{X_{2,i}}{X_{1,i}} \right)}{Y_{1,i} - Y_{4,i} \left(\frac{X_{4,i}}{X_{1,i}} \right)} \quad (S3.42)$$

$$C_{q,i} = \frac{\left(Y_{4,i} \left(\frac{X_{5,i}}{X_{1,i}} \right) + Y_{5,i} \right)}{Y_{1,i} - Y_{4,i} \left(\frac{X_{4,i}}{X_{1,i}} \right)} \quad (S3.43)$$

$$D_{q,i} = \frac{Y_{4,i} \left(\frac{X_{3,i}}{X_{1,i}} \right)}{Y_{1,i} - Y_{4,i} \left(\frac{X_{4,i}}{X_{1,i}} \right)} \quad (S3.44)$$

Now, all of the coefficients $X_{1,i}$, $X_{2,i}$, $X_{3,i}$, $X_{4,i}$, $X_{5,i}$, $Y_{1,i}$, $Y_{2,i}$, $Y_{3,i}$, $Y_{4,i}$ and $Y_{5,i}$ and, in turn, the coefficients $A_{T,i}$, $B_{T,i}$, $C_{T,i}$, $D_{T,i}$, $A_{q,i}$, $B_{q,i}$, $C_{q,i}$ and $D_{q,i}$ can be described

in terms of the coefficients from the level above and the potentials (i.e. T and q) at the previous timestep.

So we have a set of coefficients that may be determined for each time-step, and we have the means to determine T_{surf} (and q_{surf} via the saturation assumption). We thus have a process to calculate the temperature and humidity profiles for each timestep by systematically calculating each of the coefficients from the top of the column (the ‘downwards sweep’) then calculating the ‘initial value’ (the surface temperature and humidity) and finally calculating each T_a , q_a and T_{leaf} by working up the column (the ‘upwards sweep’).

The term $T_{leaf,i+1}^{t+1}$ can also be described in terms of the variables at the level below by using equation iii) and its terms E_i , F_i and G_i . We can therefore describe the changes in the canopy between the present timestep t and the next timestep $t + 1$ by ‘working down’ the column from the interaction with the LMDZ atmospheric model to determine the coefficients A_T , B_T , C_T etc. and then ‘working up’ the column to determine the potentials T and q.

S3.7 Alternative derivation

A more direct method of derivation is to assume solutions of the form:

$$\vec{u}_i^{t+1} = \mathbf{E}'_i \vec{u}_{i+1}^{t+1} + \mathbf{F}'_i \quad (\text{S3.45})$$

where we define the vector \vec{u}_i^{t+1} such that:

$$\vec{u}_i^{t+1} = \begin{pmatrix} T_{a,i}^{t+1} \\ q_{a,i}^{t+1} \end{pmatrix}$$

Now, the original expression for the leaf temperature is:

$$\begin{aligned} \text{a) } \theta_i \rho_v \frac{(T_{leaf,i}^{t+1} - T_{leaf,i}^t)}{\Delta t} &= \left(\frac{1}{\Delta h_i} \right) \left(-C_p^{air} \rho_a \frac{(T_{leaf,i}^{t+1} - T_{a,i}^{t+1})}{R_i} \right. \\ &\quad \left. - \lambda \rho_a \frac{(\alpha_i T_{leaf,i}^{t+1} + \beta_i - q_{a,i}^{t+1})}{R'_i} + \eta_1 T_{leaf,i}^{t+1} + \eta_2 + \eta_3 R_{SW}^{down} \right) \end{aligned} \quad (\text{S3.46})$$

which we can abbreviate (with a change of variable label here, to reduce confusion) as:

$$\text{a) } T_{L,i}^{t+1} = X_i q_{a,i}^{t+1} + Y_i T_{a,i}^{t+1} + Z_i \quad (\text{S3.47})$$

The original expression for the temperature column is:

$$\begin{aligned} \text{b) } \frac{T_{a,i}^{t+1} - T_{a,i}^t}{\Delta t} &= k_i \left(\frac{(T_{a,i+1}^{t+1} - T_{a,i}^{t+1})}{\Delta z_i \Delta h_i} \right) - k_{i-1} \left(\frac{(T_{a,i}^{t+1} - T_{a,i-1}^{t+1})}{\Delta z_{i-1} \Delta h_i} \right) \\ &\quad + \left(\frac{1}{\Delta h_i} \right) \frac{(T_{leaf,i}^{t+1} - T_{a,i}^{t+1})}{R_i} \end{aligned} \quad (\text{S3.48})$$

The original expression for the specific humidity column is:

$$\begin{aligned} \text{c) } \frac{q_{a,i}^{t+1} - q_{a,i}^t}{\Delta t} &= k_i \left(\frac{(q_{a,i+1}^{t+1} - q_{a,i}^{t+1})}{\Delta z_i \Delta h_i} \right) - k_{i-1} \left(\frac{(q_{a,i}^{t+1} - q_{a,i-1}^{t+1})}{\Delta z_{i-1} \Delta h_i} \right) \\ &\quad + \left(\frac{1}{\Delta h_i} \right) \frac{(\alpha_i T_{leaf,i}^{t+1} + \beta_i - q_{a,i}^{t+1})}{R'_i} \end{aligned} \quad (\text{S3.49})$$

which we abbreviate to the form of Richtmyer and Morton (1967), equation 11.7, substituting for $T_{L,i}^{t+1}$ from a), above:

$$\text{b) } -\mathbf{A}'_i \vec{u}_{i+1}^{t+1} + \mathbf{B}'_i \vec{u}_i^{t+1} - \mathbf{C}'_i \vec{u}_{i-1}^{t+1} = \mathbf{D}'_i \quad (\text{S3.50})$$

Now, directly from Richtmyer and Morton (1967), we can substitute for $u_{i-1}^{\vec{t}+1} = \mathbf{E}'_{i-1} \vec{u}_i^{t+1} + \mathbf{F}'_{i-1}$. This results in a pair of expressions of the form:

$$\vec{u}_{a,i}^{t+1} = (\mathbf{B}'_i - \mathbf{C}'_i \mathbf{E}'_{i-1})^{-1} \mathbf{A}'_i \vec{u}_{a,i+1}^{t+1} + \mathbf{B}'_i - \mathbf{C}'_i \mathbf{E}'_{i-1})^{-1} \mathbf{D}'_i + \mathbf{C}'_i \mathbf{F}'_{i-1} \quad (\text{S3.51})$$

So we can describe the variables for (S3.45) as:

$$\mathbf{E}'_i = (\mathbf{B}'_i - \mathbf{C}'_i \mathbf{E}'_{i-1})^{-1} \mathbf{A}'_i \quad (\text{S3.52})$$

$$\mathbf{F}'_i = (\mathbf{B}'_i - \mathbf{C}'_i \mathbf{E}'_{i-1})^{-1} (\mathbf{D}'_i + \mathbf{C}'_i \mathbf{F}'_{i-1}) \quad (\text{S3.53})$$

Now \mathbf{A}'_i , \mathbf{B}'_i , \mathbf{C}'_i and \mathbf{D}'_i may be described from substitution of the equations (S3.47), (S3.48) and (S3.49).

We thus have a set of two equations to solve simultaneously, which is possible starting from the upper boundary conditions, as laid out in section S4.1. However, we still need to describe \mathbf{E}'_i and \mathbf{F}'_i which can be achieved by rearranging equations (S4.3), (S4.4), (S4.7), (S4.30) and (S4.31), so that we have two expressions of the form:

$$\vec{u}_{surf}^{t+1} = \mathbf{E}'_0 u_1^{t+1} + \mathbf{F}'_0 \quad (\text{S3.54})$$

So if we can describe \mathbf{E}'_0 and \mathbf{F}'_0 from use of (S3.52) and (S3.53), we can then derive the value of these variables working up the column.

Table 1: *Input coefficients at the top layer of the model, where $A_{T,n}, B_{T,n}...$ etc are the respective coefficients at the top of the surface model and $A_{T,atmos}, B_{T,atmos}$ are the coefficients at the lowest level of the atmospheric model.*

stand-alone model	coupled model
$A_{T,n} = 0$	$A_{T,n} = A_{T,atmos}$
$B_{T,n} = B_{T,input}$	$B_{T,n} = B_{T,atmos}$
$C_{T,n} = 0$	$C_{T,n} = 0$
$D_{T,n} = 0$	$D_{T,n} = 0$
$A_{q,n} = 0$	$A_{q,n} = A_{q,atmos}$
$B_{q,n} = B_{q,input}$	$B_{q,n} = B_{q,atmos}$
$C_{q,n} = 0$	$C_{q,n} = 0$
$D_{q,n} = 0$	$D_{q,n} = 0$

S4 The boundary conditions

S4.1 The upper boundary conditions

In stand-alone simulations, the top level variables $A_{T,n}, C_{T,n}, D_{T,n}$ and $A_{q,n}, C_{q,n}, D_{q,n}$, are set to zero and $B_{T,n}$ and $B_{q,n}$ set to the input temperature and specific humidity respectively for the relevant time step (as in Best et al. (2004)) In coupled simulations, $A_{T,n}, B_{T,n}$ and $A_{q,n}, B_{q,n}$ are taken from the respective values at lowest level of the atmospheric model. Table 1 summarises the boundary conditions for both the coupled and un-coupled simulations.

S4.2 The lower boundary condition

We need to solve the lowest level transport equations separately:

$$b) \quad \frac{T_{a,1}^{t+1} - T_{a,1}^t}{\Delta t} = k_1 \left(\frac{T_{a,2}^{t+1} - T_{a,1}^{t+1}}{\Delta z_1 \Delta h_1} \right) - \left(\frac{1}{\rho_a C_p^{air}} \right) \frac{\phi_H^{t+1}}{\Delta h_1} + \left(\frac{1}{\Delta h_1} \right) \left(\frac{T_{L,1}^{t+1} - T_{a,1}^{t+1}}{R_1} \right) \quad (S4.1)$$

$$c) \quad \frac{q_{a,1}^{t+1} - q_{a,1}^t}{\Delta t} = k_1 \left(\frac{q_{a,2}^{t+1} - q_{a,1}^{t+1}}{\Delta z_1 \Delta h_1} \right) - \left(\frac{1}{\rho_a \lambda} \right) \frac{\phi_{\lambda E}^{t+1}}{\Delta h_1} + \left(\frac{1}{\Delta h_1} \right) \left(\frac{\alpha_i T_{L,1}^{t+1} + \beta_1 - q_{a,1}^{t+1}}{R'_1} \right) \quad (S4.2)$$

We substitute to the above to eliminate $T_{a,2}^{t+1}$ from b) and $q_{a,2}^{t+1}$ from c):

$$T_{a,1}^{t+1} = A_{T,1} \phi_H^{t+1} + B_{T,1} + C_{T,1} T_{L,1}^{t+1} + D_{T,1} \phi_{\lambda E}^{t+1} \quad (S4.3)$$

and:

$$q_{a,1}^{t+1} = A_{q,1} \phi_{\lambda E}^{t+1} + B_{q,1} + C_{q,1} T_{L,1}^{t+1} + D_{q,1} \phi_H^{t+1} \quad (S4.4)$$

Now for the leaf at level 1, just above the ground level:

$$a) \quad T_{leaf,1}^{t+1} - T_{leaf,1}^t = \frac{\lambda \rho_a \Delta t \beta_1}{(\rho_v \Delta h_1) R'_1 \theta_1} + \frac{\eta_3 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_1) \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} \\ + T_{leaf,1}^{t+1} \left(C_p^{air} \rho_a \frac{\Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \lambda \rho_a \frac{\Delta t \alpha}{(\rho_v \Delta h_1) R'_1 \theta_1} + \frac{\eta_1 \Delta t}{(\rho_v \Delta h_1) \theta_1} \right) \\ - T_{a,1}^{t+1} C_p^{air} \rho_a \left(\frac{\Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} \right) - q_{a,1}^{t+1} \lambda \rho_a \left(\frac{\Delta t}{(\rho_v \Delta h_1) R'_1 \theta_1} \right) \quad (S4.5)$$

and substitute for $T_{a,1}^{t+1}$ and $q_{a,1}^{t+1}$:

$$a) \quad T_{leaf,1}^{t+1} - T_{leaf,1}^t = \frac{\lambda \rho_a \Delta t \beta_i}{(\rho_v \Delta h_1) R'_1 \theta_1} + \frac{\eta_3 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_1) \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} \\ + T_{leaf,1}^{t+1} \left(C_p^{air} \rho_a \frac{\Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \lambda \rho_a \frac{\Delta t \alpha}{(\rho_v \Delta h_1) R'_1 \theta_1} + \frac{\eta_1 \Delta t}{(\rho_v \Delta h_1) \theta_1} \right) \\ - \frac{C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} (A_{T,1} \phi_H^{t+1} + B_{T,1} + C_{T,1} T_{leaf,1}^{t+1} + D_{T,1} \phi_{\lambda E}^{t+1}) \\ - \frac{\lambda \rho_a \Delta t}{R'_1 \theta_1 (\rho_v \Delta h_1)} (A_{q,1} \phi_{\lambda E}^{t+1} + B_{q,1} + C_{q,1} T_{leaf,1}^{t+1} + D_{q,1} \phi_H^{t+1}) \quad (S4.6)$$

In a similar approach to the previous section, this should be reduced to the form:

$$T_{leaf,1}^{t+1} = E_1 \phi_{\lambda E}^{t+1} + F_1 \phi_H^{t+1} + G_1 \quad (S4.7)$$

and the expression re-arranged to isolate the factors E_1 , F_1 and G_1 :

$$\begin{aligned} a) \quad T_{leaf,1}^{t+1} & \left(1 - \left(\frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} \right) + \frac{C_{T,1} C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} \right. \\ & \left. + \frac{C_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} \right) = T_{leaf,1}^t + \phi_{\lambda E}^{t+1} \left(- \frac{C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} D_{T,1} - \frac{\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} A_{q,1} \right) \\ & + \phi_H^{t+1} \left(- \frac{C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} A_{T,1} - \frac{\lambda r h o_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} D_{q,1} \right) \\ & + \left(\frac{\lambda \rho_a \Delta t \beta_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_3 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_1) \theta_1} \right. \\ & \left. + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} - \frac{B_{T,1} C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} - \frac{B_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} \right) \end{aligned} \quad (S4.8)$$

Now, substituting for $T_{a,2}^{t+1}$ in expression b):

$$\begin{aligned} b) \quad \frac{T_{a,1}^{t+1} - T_{a,1}^t}{\Delta t} & = \\ & \frac{(A_{T,2} T_{a,1}^{t+1} + B_{T,2} + C_{T,2} (E_2 q_{a,1}^{t+1} + F_2 T_{a,1}^{t+1} + G_2) + D_{T,2} q_{a,1}^{t+1} + T_{a,1}^{t+1})}{k_1 \Delta z_1 \Delta h_1} \\ & - \left(\frac{1}{\rho_a C_p^{air}} \right) \frac{\phi_H^{t+1}}{\Delta h_1} + \frac{T_{leaf,1}^{t+1} - T_{a,1}^{t+1}}{R_1 \Delta h_1} \end{aligned} \quad (S4.9)$$

$$\begin{aligned} b) \quad T_{a,1}^{t+1} & \left(\frac{1}{\Delta t} - \frac{k_1 A_{T,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{T,2} F_2}{\Delta z_1 \Delta h_1} + \frac{k_1}{\Delta z_1 \Delta h_1} + \frac{1}{\Delta h_1 R_1} \right) = \\ & \frac{T_{a,1}^t}{\Delta t} + q_{a,1}^{t+1} \left(\frac{k_1 C_{T,2} E_2 + k_1 D_{T,2}}{\Delta z_1 \Delta h_1} \right) + \\ & T_{leaf,1}^{t+1} \left(\frac{1}{\Delta h_1 R_1} \right) + \left(\frac{k_1 B_{T,2}}{\Delta z_1 \Delta h_1} + \frac{k_1 C_{T,2} G_2}{\Delta z_1 \Delta h_1} \right) - \left(\frac{1}{\rho_a C_p^{air}} \right) \phi_H^{t+1} \left(\frac{1}{\Delta h_1} \right) \end{aligned} \quad (S4.10)$$

and for $q_{a,2}^{t+1}$ in expression c):

$$c) \quad \frac{q_{a,1}^{t+1} - q_{a,1}^t}{\Delta t} = k_1 \frac{(A_{q,2}q_{a,1}^{t+1} + B_{q,2} + C_{q,2}T_{leaf,2}^{t+1} + D_{q,2}T_{a,1}^{t+1} - q_{a,1}^{t+1})}{\Delta z_1 \Delta h_1} =$$

$$- \left(\frac{1}{\rho_a \lambda} \right) \frac{\phi_{\lambda E}^{t+1}}{\Delta h_1} + \frac{1}{\Delta h_1} \frac{\alpha T_{leaf,1}^{t+1} + \beta_1 - q_{a,1}^{t+1}}{R'_1} \quad (S4.11)$$

$$c) \quad \frac{q_{a,1}^{t+1} - q_{a,1}^t}{\Delta t} = k_1 \frac{(A_{q,2}q_{a,1}^{t+1} + B_{q,2} + C_{q,2}(E_2q_{a,1}^{t+1} + F_2T_{a,1}^{t+1} + G_2) + D_{q,2}T_{a,1}^{t+1} - q_{a,1}^{t+1})}{\Delta z_1 \Delta h_1}$$

$$- \left(\frac{1}{\rho_a \lambda} \right) \frac{\phi_{\lambda E}^{t+1}}{\Delta h_1} + \frac{1}{\Delta h_1} \frac{(\alpha T_{leaf,1}^{t+1} + \beta_1 - q_{a,1}^{t+1})}{R'_1} \quad (S4.12)$$

$$c) \quad q_{a,1}^{t+1} \left(\frac{1}{\Delta t} - \frac{k_1 A_{q,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{q,2} E_2}{\Delta z_1 \Delta h_1} + \frac{k_1}{\Delta z_1 \Delta h_1} \right) =$$

$$\frac{q_{a,1}^t}{\Delta t} + T_{a,1}^{t+1} \left(\frac{C_{q,2} F_2}{\Delta z_1 \Delta h_1} + \frac{D_{q,2}}{\Delta z_1 \Delta h_1} \right) + T_{L,1}^{t+1} \left(\frac{\alpha}{\Delta h_1 R'_1} \right)$$

$$+ \left(\frac{k_1 B_{q,2}}{\Delta z_1 \Delta h_1} + \frac{C_{q,2} G_2}{\Delta z_1 \Delta h_1} + \frac{\beta_1}{\Delta h_1 R'_1} \right) - \phi_{\lambda E}^{t+1} \left(\frac{1}{\Delta h_1} \right) \quad (S4.13)$$

We now isolate the terms in (S4.8):

$$a) \quad T_{L,1}^{t+1} = E_1 \phi_{\lambda E}^{t+1} + F_1 \phi_H^{t+1} + G_1 \quad (S4.14)$$

so we have:

$$E_1 = \left(-\frac{C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} D_{T,1} - \frac{\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R'_1 \theta_1} A_{q,1} \right) /$$

$$\left(1 - \left(\frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha_i}{(\rho_v \Delta h_1) R'_1 \theta_1} + \frac{\eta_1 \Delta t}{(\rho_v \Delta h_1) \theta_1} \right) + \frac{C_{T,1} C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} + \frac{C_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R'_1 \theta_1} \right) \quad (S4.15)$$

$$F_1 = \left(-\frac{C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} A_{T,1} - \frac{\lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R'_1 \theta_1} D_{q,1} \right) /$$

$$\left(1 - \left(\frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha_i}{(\rho_v \Delta h_1) R'_1 \theta_1} + \frac{\eta_1 \Delta t}{(r h o_v \Delta h_1) \theta_1} \right) + \frac{C_{T,1} C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} + \frac{C_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R'_1 \theta_1} \right) \quad (S4.16)$$

and:

$$G_1 = \left(T_{L,1}^t + \frac{\lambda \rho_a \Delta t \beta_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_3 R_{SW}^{down} \Delta t}{(\rho_v \Delta h_1) \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} - \frac{B_{T,1} C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} - \frac{B_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} \right) / \left(1 - \left(\frac{C_p^{air} \rho_a \Delta t}{(\rho_v \Delta h_1) R_1 \theta_1} + \frac{\lambda \rho_a \Delta t \alpha_i}{(\rho_v \Delta h_1) R_1' \theta_1} + \frac{\eta_2 \Delta t}{(\rho_v \Delta h_1) \theta_1} \right) + \frac{C_{T,1} C_p^{air} \rho_a \Delta t}{R_1 \theta_1 (\rho_v \Delta h_1)} + \frac{C_{q,1} \lambda \rho_a \Delta t}{(\rho_v \Delta h_1) R_1' \theta_1} \right) \quad (S4.17)$$

We now seek to rearrange b) and c) into expressions of the form:

$$i) \quad T_{a,1}^{t+1} X_1 = X_2 + \phi_H^{t+1} X_3 + q_{a,1}^{t+1} X_4 + T_{leaf,1}^{t+1} X_5 \quad (S4.18)$$

and:

$$ii) \quad q_{a,1}^{t+1} Y_1 = Y_2 + \phi_{\lambda E}^{t+1} Y_3 + T_{a,1}^{t+1} Y_4 + T_{leaf,1}^{t+1} Y_5 \quad (S4.19)$$

The same process as in the previous section means that we can assign $A_{T,1}$, $B_{T,1}$, $C_{T,1}$, $D_{T,1}$, $A_{q,1}$, $B_{q,1}$, $C_{q,1}$, $D_{q,1}$ exactly as previously (expressions (S3.37) to (S3.44)), and define X_1 to Y_5 as follows:

$$X_1 = 1 - \Delta t \left(\frac{k_1 A_{T,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{T,2} F_2}{\Delta z_1 \Delta h_1} + \frac{k_1}{\Delta z_1 \Delta h_1} + \frac{1}{\Delta h_1 R_1} \right) \quad (S4.20)$$

$$X_2 = T_{a,1}^t + \Delta t \left(\frac{k_1 B_{T,2}}{\Delta z_1 \Delta h_1} + \frac{k_1 C_{T,2} G_2}{\Delta z_1 \Delta h_1} \right) \quad (S4.21)$$

$$X_3 = -\Delta t \left(\frac{1}{\Delta h} \right) \left(\frac{1}{\rho_a C_p^{air}} \right) \quad (S4.22)$$

$$X_4 = \Delta t \left(\frac{k_1 C_{T,2} E_2 + k_1 D_{T,2}}{\Delta z_1 \Delta h_1} \right) \quad (S4.23)$$

$$X_5 = \Delta t \left(\frac{1}{\Delta h_1 R_1} \right) \quad (S4.24)$$

$$Y_1 = 1 - \Delta t \left(\frac{k_1 A_{q,2}}{\Delta z_1 \Delta h_1} - \frac{k_1 C_{q,2} E_2}{\Delta z_1 \Delta h_1} + \frac{1}{\Delta h_1 R'} + \frac{k_1}{\Delta z_1 \Delta h_1} \right) \quad (S4.25)$$

$$Y_2 = q_{a,1}^t + \Delta t \left(\frac{k_1 B_{q,2}}{\Delta z_1 \Delta h_1} + \frac{k_1 C_{q,2} G_2}{\Delta z_1 \Delta h_1} + \frac{\beta_1}{\Delta h_1 R_1'} \right) \quad (S4.26)$$

$$Y_3 = -\Delta t \left(\frac{1}{\Delta h_1} \right) \left(\frac{1}{\rho_a \lambda} \right) \quad (\text{S4.27})$$

$$Y_4 = \Delta t \left(\frac{k_1 C_{q,2} F_2}{\Delta z_1 \Delta h_1} + \frac{k_1 D_{q,2}}{\Delta z_1 \Delta h_1} \right) \quad (\text{S4.28})$$

$$Y_5 = \Delta t \left(\frac{\alpha_1}{\Delta h_1 R'_1} \right) \quad (\text{S4.29})$$

Now, for the lower boundary condition we consider the interaction between the lowest atmospheric level (level 1) and the infinitesimal surface layer (level S). Fluxes of the sensible and latent heat from this layer are given, respectively, by:

$$i) \quad \phi_H^{t+1} = -(\rho_a C_p^{air}) k_{surf} \frac{T_{a,1}^{t+1} - T_{surf}^{t+1}}{\Delta z_{surf}} \quad (\text{S4.30})$$

$$ii) \quad \phi_{\lambda E}^{t+1} = -(\rho_a \lambda) k_{surf} \frac{q_{a,1}^{t+1} - q_{surf}^{t+1}}{\Delta z_{surf}} \quad (\text{S4.31})$$

$$i) \quad \phi_H^{t+1} = \frac{\rho_a C_p^{air} k_{surf}}{\Delta z_{surf}} (A_{T,1} \phi_H^{t+1} + B_{T,1} + C_{T,1} T_{L,1}^{t+1} + D_{T,1} \phi_{\lambda E}^{t+1} - T_{surf}^{t+1}) \quad (\text{S4.32})$$

$$ii) \quad \phi_{\lambda E}^{t+1} = \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (A_{q,1} \phi_{\lambda E}^{t+1} + B_{q,1} + C_{q,1} T_{L,1}^{t+1} + D_{q,1} \phi_H^{t+1} - q_{surf}^{t+1}) \quad (\text{S4.33})$$

We use a substitution for the leaf temperature:

$$T_{L,1}^{t+1} = E_1 \phi_{\lambda E}^{t+1} + F_1 \phi_H^{t+1} + G_1 \quad (\text{S4.34})$$

$$i) \quad \phi_H^{t+1} = -\frac{(\rho_a C_p^{air}) k_{surf}}{\Delta z_{surf}} (A_{T,1} \phi_H^{t+1} + B_{T,1} + C_{T,1} (E_1 \phi_{\lambda E}^{t+1} + F_1 \phi_H^{t+1} + G_1) + D_{T,1} \phi_{\lambda E}^{t+1} - T_{surf}^{t+1}) \quad (\text{S4.35})$$

$$ii) \quad \phi_{\lambda E}^{t+1} = -\frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (A_{q,1} \phi_{\lambda E}^{t+1} + B_{q,1} + C_{q,1} (E_1 \phi_{\lambda E}^{t+1} + F_1 \phi_H^{t+1} + G_1) + D_{q,1} \phi_H^{t+1} - q_{surf}^{t+1}) \quad (\text{S4.36})$$

$$\begin{aligned}
i) \quad \phi_H^{t+1} \left(1 + \frac{(\rho_a C_p^{air}) k_{surf}}{\Delta z_{surf}} (A_{T,1} + C_{T,1} F_1) \right) &= - \frac{(\rho_a C_p^{air}) k_{surf}}{\Delta z_{surf}} (B_{T,1} + C_{T,1} G_1 - T_{surf}^{t+1}) \\
&\quad - \frac{(\rho_a C_p^{air}) k_{surf}}{\Delta z_{surf}} (\phi_{\lambda E}^{t+1} (C_{T,1} E_1 + D_{T,1})) \quad (S4.37)
\end{aligned}$$

$$\begin{aligned}
ii) \quad \phi_{\lambda E}^{t+1} \left(1 + \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (A_{q,1} + C_{q,1} E_1) \right) &= \\
&\quad - \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (B_{q,1} + C_{q,1} G_1 - q_{surf}^{t+1}) \\
&\quad - \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (\phi_H^{t+1} (C_{q,1} F_1 + D_{q,1})) \quad (S4.38)
\end{aligned}$$

$$\begin{aligned}
ii) \quad \phi_{\lambda E}^{t+1} \left(1 + \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (A_{q,1} + C_{q,1} E_1) \right) &= \\
&\quad - \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (B_{q,1} + C_{q,1} G_1 - (\alpha_{surf} T_{surf}^{t+1} + \beta_{surf})) \\
&\quad - \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (\phi_H^{t+1} (C_{q,1} F_1 + D_{q,1})) \quad (S4.39)
\end{aligned}$$

and abbreviate to:

$$i) \quad \Omega_1 \phi_H^{t+1} = \Omega_2 + \Omega_3 T_{surf}^{t+1} + \Omega_4 \phi_{\lambda E}^{t+1} \quad (S4.40)$$

$$ii) \quad \Omega_5 \phi_{\lambda E}^{t+1} = \Omega_6 + \Omega_7 T_{surf}^{t+1} + \Omega_8 \phi_H^{t+1} \quad (S4.41)$$

where:

$$\Omega_1 = 1 + \frac{(\rho_a C_p^{air}) k_{surf}}{\Delta z_{surf}} (A_{T,1} + C_{T,1} F_1) \quad (S4.42)$$

$$\Omega_2 = - \frac{(\rho_a C_p^{air}) k_{surf}}{\Delta z_{surf}} (B_{T,1} + C_{T,1} G_1) \quad (S4.43)$$

$$\Omega_3 = \frac{\rho_a C_p^{air} k_{surf}}{\Delta z_{surf}} \quad (S4.44)$$

$$\Omega_4 = - \frac{(\rho_a C_p^{air} k_{surf})}{\Delta z_{surf}} (C_{T,1} E_1 + D_{T,1}) \quad (S4.45)$$

$$\Omega_5 = 1 + \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (A_{q,1} + C_{q,1} E_1) \quad (S4.46)$$

$$\Omega_6 = - \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (B_{q,1} + C_{q,1} G_1 - \beta_{surf}) \quad (S4.47)$$

$$\Omega_7 = \frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} \alpha_{surf} \quad (S4.48)$$

$$\Omega_8 = -\frac{(\rho_a \lambda) k_{surf}}{\Delta z_{surf}} (C_{q,1} F_1 + D_{q,1}) \quad (S4.49)$$

cross substitute:

$$\begin{aligned} i) \quad \Omega_1 \phi_H^{t+1} &= \Omega_2 + \Omega_3 T_{surf}^{t+1} + \frac{\Omega_4}{\Omega_5} (\Omega_6 + \Omega_7 T_{surf}^{t+1} + \Omega_8 \phi_H^{t+1}) \\ i) \quad \phi_H^{t+1} \left(\Omega_1 - \frac{\Omega_4}{\Omega_5} \Omega_8 \right) &= \left(\Omega_2 + \frac{\Omega_4}{\Omega_5} \Omega_6 \right) + T_{surf}^{t+1} \left(\Omega_3 + \frac{\Omega_4 \Omega_7}{\Omega_5} \right) \end{aligned} \quad (S4.50)$$

and:

$$\begin{aligned} ii) \quad \Omega_5 \phi_{\lambda E}^{t+1} &= \Omega_6 + \Omega_7 T_{surf}^{t+1} + \frac{\Omega_8}{\Omega_1} (\Omega_2 + \Omega_3 T_{surf}^{t+1} + \Omega_4 \phi_{\lambda E}^{t+1}) \\ ii) \quad \phi_{\lambda E}^{t+1} \left(\Omega_5 - \frac{\Omega_8}{\Omega_1} \Omega_4 \right) &= \left(\Omega_6 + \frac{\Omega_8}{\Omega_1} \Omega_2 \right) + T_{surf}^{t+1} \left(\Omega_7 + \frac{\Omega_8 \Omega_3}{\Omega_1} \right) \end{aligned} \quad (S4.51)$$

Now we abbreviate Ω by substituting for ξ :

$$\xi_1 = \frac{\Omega_2 + \frac{\Omega_4}{\Omega_5} \Omega_6}{\Omega_1 - \frac{\Omega_4}{\Omega_5} \Omega_8} \quad (S4.52)$$

$$\xi_2 = \frac{\Omega_3 + \frac{\Omega_4 \Omega_7}{\Omega_5}}{\Omega_1 - \frac{\Omega_4}{\Omega_5} \Omega_8} \quad (S4.53)$$

$$\xi_3 = \frac{\Omega_6 + \frac{\Omega_8 \Omega_2}{\Omega_1}}{\Omega_5 - \frac{\Omega_8 \Omega_4}{\Omega_1}} \quad (S4.54)$$

$$\xi_4 = \frac{\Omega_7 + \frac{\Omega_8 \Omega_3}{\Omega_1}}{\Omega_5 - \frac{\Omega_8 \Omega_4}{\Omega_1}} \quad (S4.55)$$

And we can equations *i)* and *ii)* in terms of ξ :

$$i) \quad \phi_H^{t+1} = \xi_1 + \xi_2 T_{surf}^{t+1} \quad (S4.56)$$

$$ii) \quad \phi_{\lambda E}^{t+1} = \xi_3 + \xi_4 T_{surf}^{t+1} \quad (S4.57)$$

$$T_{surf}^{t+1} = T_{surf}^t + \frac{\Delta t}{\theta_0} ((R_{LW,surf} + R_{SW,surf} + \xi_1 + \xi_2 T_{surf}^{t+1} + \xi_3 + \xi_4 T_{surf}^{t+1}) - J_{soil}) \quad (S4.58)$$

$$T_{surf}^{t+1} \left(1 - \xi_2 \frac{\Delta t}{\theta_0} - \xi_4 \frac{\Delta t}{\theta_0} \right) = T_{surf}^t + \frac{\Delta t}{\theta_0} (R_{LW,surf} + R_{SW,surf} + \xi_1 + \xi_3 - J_{soil}) \quad (\text{S4.59})$$

and so:

$$T_{surf}^{t+1} = \frac{T_{surf}^t + \frac{\Delta t}{\theta_0} (R_{LW,surf} + R_{SW,surf} + \xi_1 + \xi_3 - J_{soil})}{\left(1 - \xi_2 \frac{\Delta t}{\theta_0} - \xi_4 \frac{\Delta t}{\theta_0} \right)} \quad (\text{S4.60})$$

We therefore have an expression for the surface temperature T_{surf}^{t+1} , in terms of the downwelling radiation that is incident on the surface (R_{LW} and R_{SW}), the heat capacity of the infinitesimal surface layer (θ_0), the vegetation layer directly above the surface (ξ_1 , ξ_2 , ξ_3 and ξ_4) and the heat from the soil system (J_{soil}).

The radiation that is received by the lowermost level is provided by the radiation scheme.

So to re-write the above equation including the factors $\eta_{1,S}$, $\eta_{2,S}$ and $\eta_{3,S}$:

$$T_{surf}^{t+1} = \frac{T_{surf}^t + \frac{\Delta t}{\theta_0} (\eta_{2,S} + \eta_{3,S} R_{SW}^{down} + \xi_1 + \xi_3) - J_{soil}}{\left(1 - \frac{\Delta t}{\theta_0} (\xi_2 + \xi_4 + \eta_{1,S}) \right)} \quad (\text{S4.61})$$

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Supplementary material

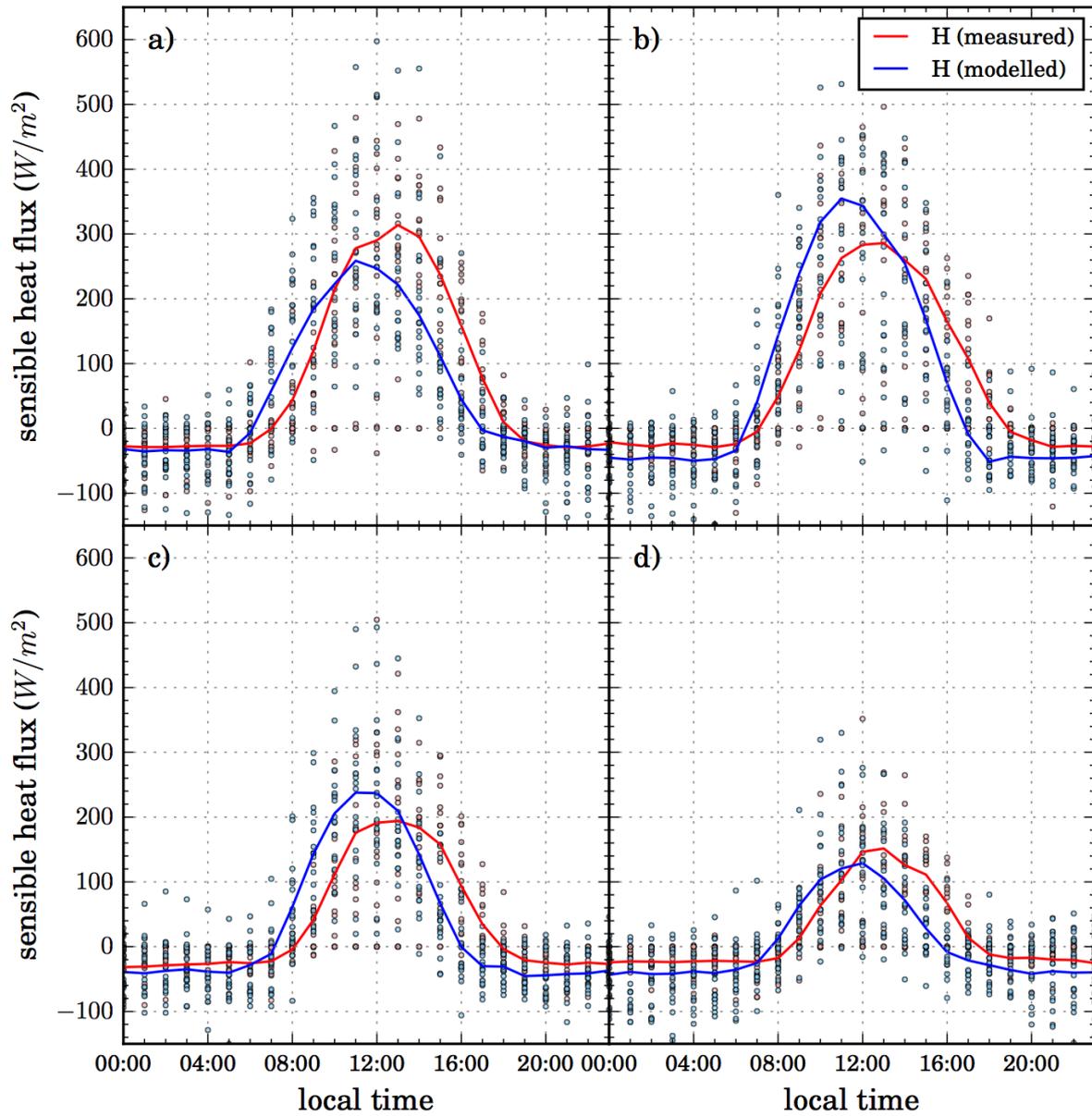


Figure S1: The analysis of Figure 3 is repeated for the four seasons of the year (every 5th measurement is shown in the background in a lighter colour). Hourly average sensible heat flux (annual average): a) spring; b) summer; c) autumn; d) winter

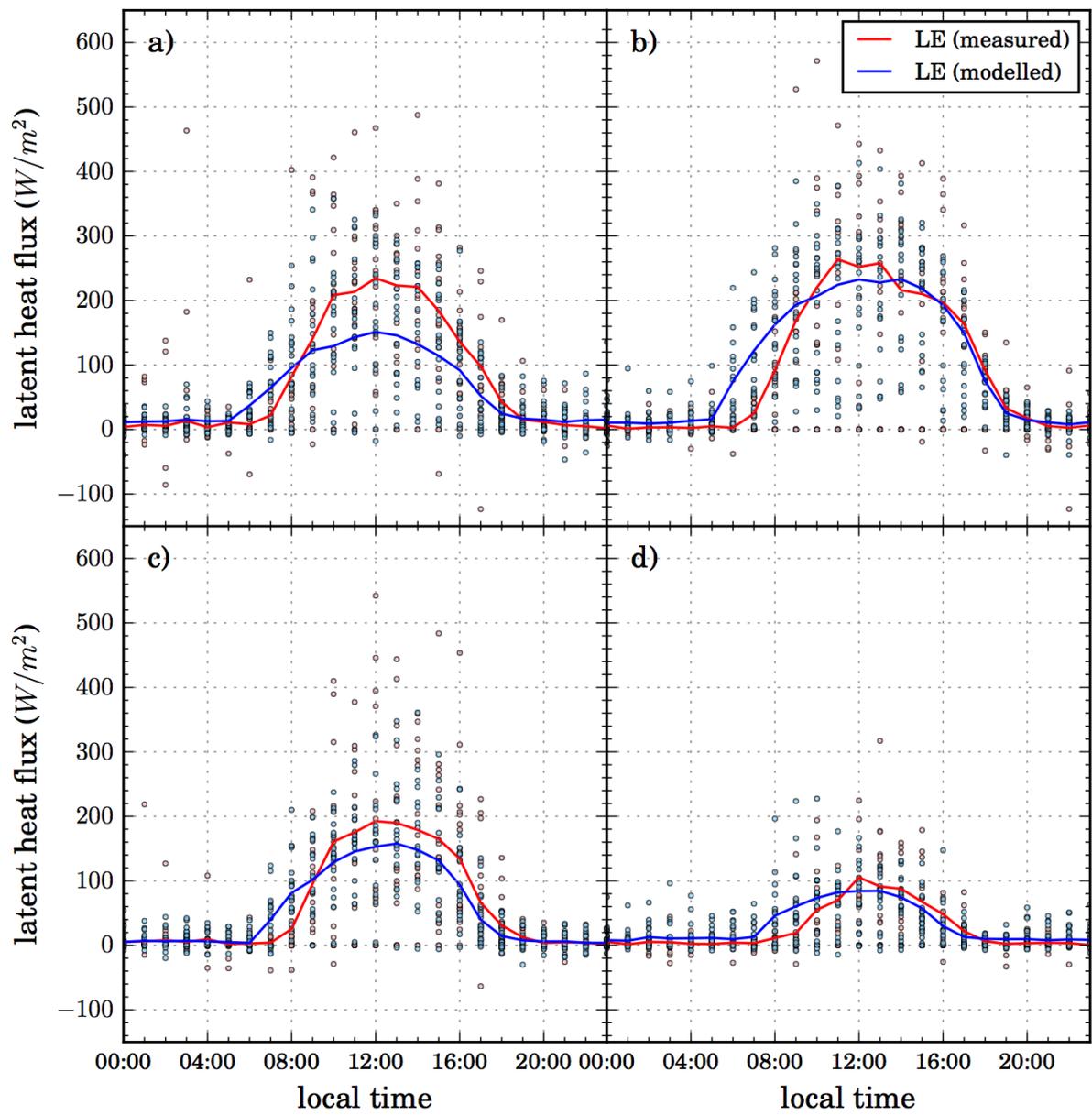


Figure S2: The analysis of Figure 3 repeated for the four seasons of the year (every 5th measurement is shown in the background in a lighter colour). hourly latent sensible heat flux (annual average): a) spring; b) summer; c) autumn; d) winter