

Supplement of Geosci. Model Dev., 9, 2031–2053, 2016
<http://www.geosci-model-dev.net/9/2031/2016/>
doi:10.5194/gmd-9-2031-2016-supplement
© Author(s) 2016. CC Attribution 3.0 License.



Supplement of

A new subgrid-scale representation of hydrometeor fields using a multivariate PDF

Brian M. Griffin and Vincent E. Larson

Correspondence to: Brian M. Griffin (bmg2@uwm.edu)

The copyright of individual parts of the supplement might differ from the CC-BY 3.0 licence.

S1 Modified Analytically Upscaled Microphysics Equations

The analytically upscaled (to grid-box size) form of the Khairoutdinov and Kogan (2000, hereafter KK) microphysics equations were first derived in Larson and Griffin (2013). KK is a warm scheme that predicts r_r and N_r . It contains equations for the warm process rates of accretion, autoconversion, and evaporation, as well rain drop mean volume radius, that are written as power laws of two-or-three variables. The modifications to the PDF in Section 2 of the associated article require modifications to the analytically upscaled microphysics equations. The upscaled microphysics calculates the grid-box mean values of microphysics process rates by integrating over the product of the microphysics function and the PDF.

S1.1 Accretion rate

The KK rate of production of r_r over time, t , due to the process of accretion is of the form

$$\left. \frac{\partial r_r}{\partial t} \right|_{\text{accr}} = C_{\text{accr}} r_c^\alpha r_r^\beta, \quad (\text{S1})$$

where $C_{\text{accr}} = 67$, $\alpha = 1.15$, and $\beta = 1.15$. Upscaling is accomplished by integrating over Eq. (S1), and in the process, using $r_c = \chi H(\chi)$ (Eq. (3) of the associated article) as a substitution. This produces the following equation for mean accretion rate

$$\begin{aligned} \overline{\left. \frac{\partial r_r}{\partial t} \right|_{\text{accr}}} &= C_{\text{accr}} \sum_{i=1}^n \xi_{(i)} \text{ACCR}_{(i)} \\ &= C_{\text{accr}} \sum_{i=1}^n \xi_{(i)} \int_{-\infty}^{\infty} \int_0^{\infty} \chi^\alpha (H(\chi))^\alpha r_r^\beta P_{(i)}(\chi, r_r) dr_r d\chi, \end{aligned} \quad (\text{S2})$$

where $P_{(i)}(\chi, r_r)$ is the bivariate marginal PDF of χ and r_r in the i th PDF component.

Since $\alpha > 0$, $\text{ACCR}_{(i)}$ can be rewritten

$$\text{ACCR}_{(i)} = \int_0^{\infty} \int_0^{\infty} \chi^\alpha r_r^\beta (f_{p(i)} P_{NL(i)}(\chi, r_r) + (1 - f_{p(i)}) P_{N(i)}(\chi) \delta(r_r)) dr_r d\chi, \quad (\text{S3})$$

where $P_{NL(i)}(\chi, r_r)$ is the i th component bivariate PDF involving one normal variate and one lognormal variate, and where $P_{N(i)}(\chi)$ is a normal distribution in the i th PDF component. This equation is integrated, solving for $\text{ACCR}_{(i)}$,

$$\text{ACCR}_{(i)} = f_{p(i)} \frac{1}{\sqrt{2\pi}} \sigma_{\chi(i)}^\alpha \exp \left\{ \tilde{\mu}_{r_r(i)} \beta + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \beta^2 - \frac{1}{4} \varsigma^2 \right\} \Gamma(\alpha + 1) D_{-(\alpha+1)}(-\varsigma), \quad (\text{S4})$$

where $D_\nu(x)$ is the parabolic cylinder function of order ν , and where ς is given by

$$\varsigma = \frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} + \tilde{\rho}_{\chi, r_r(i)} \tilde{\sigma}_{r_r(i)} \beta.$$

The in-precipitation mean of $\ln r_r$ in the i th PDF component is $\tilde{\mu}_{r_r(i)}$, and it is given by

$$\tilde{\mu}_{r_r(i)} = \ln \left(\mu_{r_r(i)} \left(1 + \frac{\sigma_{r_r(i)}^2}{\mu_{r_r(i)}^2} \right)^{-\frac{1}{2}} \right), \quad (\text{S5})$$

where $\mu_{r_r(i)}$ and $\sigma_{r_r(i)}$ are the in-precipitation mean and in-precipitation standard deviation, respectively, of r_r in the i th PDF component. The in-precipitation standard deviation of $\ln r_r$ in the i th PDF component is $\tilde{\sigma}_{r_r(i)}$, and it is given by

$$\tilde{\sigma}_{r_r(i)} = \sqrt{\ln \left(1 + \frac{\sigma_{r_r(i)}^2}{\mu_{r_r(i)}^2} \right)}. \quad (\text{S6})$$

The in-precipitation correlation of χ and $\ln r_r$ in the i th PDF component is $\tilde{\rho}_{\chi, r_r(i)}$, and it is given by

$$\tilde{\rho}_{\chi, r_r(i)} = \frac{\rho_{\chi, r_r(i)} \sigma_{r_r(i)}}{\tilde{\sigma}_{r_r(i)} \mu_{r_r(i)}}, \quad (\text{S7})$$

where $\rho_{\chi, r_r(i)}$ is the in-precipitation correlation of χ and r_r in the i th PDF component.

The evaluated integral for $\text{ACCR}_{(i)}$ given in Eq. (S4) is for a fully-varying PDF in the i th component ($\sigma_{\chi(i)} > 0$ and $\sigma_{r_r(i)} > 0$). There are times when a variable may have a constant value in a PDF sub-component. When this happens, the PDF of the constant variable is a delta function at the i th PDF sub-component mean. When $\sigma_{\chi(i)} > 0$ and $\sigma_{r_r(i)} = 0$, χ varies in i th component but r_r is constant within precipitation. The PDF $P_{NL(i)}(\chi, r_r)$ becomes $P_{N(i)}(\chi) \delta(r_r - \mu_{r_r(i)})$. The integral is solved and the equation for $\text{ACCR}_{(i)}$ becomes

$$\text{ACCR}_{(i)} = f_{p(i)} \frac{1}{\sqrt{2\pi}} \sigma_{\chi(i)}^\alpha \mu_{r_r(i)}^\beta \exp \left\{ -\frac{1}{4} \frac{\mu_{\chi(i)}^2}{\sigma_{\chi(i)}^2} \right\} \Gamma(\alpha + 1) D_{-(\alpha+1)} \left(-\frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} \right). \quad (\text{S8})$$

For the remaining forms of $\text{ACCR}_{(i)}$, $\sigma_{\chi(i)} = 0$. When $\mu_{\chi(i)} \geq 0$, the air is entirely saturated and accretion occurs. In this scenario, when $\sigma_{r_r(i)} > 0$,

$$\text{ACCR}_{(i)} = f_{p(i)} \mu_{\chi(i)}^\alpha \exp \left\{ \tilde{\mu}_{r_r(i)} \beta + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \beta^2 \right\}, \quad (\text{S9})$$

and when $\sigma_{r_r(i)} = 0$,

$$\text{ACCR}_{(i)} = f_{p(i)} \mu_{\chi(i)}^\alpha \mu_{r_r(i)}^\beta. \quad (\text{S10})$$

Otherwise, when $\sigma_{\chi(i)} = 0$ and $\mu_{\chi(i)} < 0$, the air is entirely subsaturated, accretion does not occur, and $\text{ACCR}_{(i)} = 0$.

S1.2 Autoconversion rate

The KK autoconversion rate of r_r is of the form

$$\left. \frac{\partial r_r}{\partial t} \right|_{\text{auto}} = C_{\text{auto}} r_c^\alpha N_c^\beta, \quad (\text{S11})$$

where constant $C_{\text{auto}} = 1350 (10^{-6} \rho_d)^\beta$, and where ρ_d is the density of dry air. Additionally, $\alpha = 2.47$ and $\beta = -1.79$. In the manner similar to accretion rate, upscaling is accomplished by integrating over Eq. (S11), and in the process, using $r_c = \chi H(\chi)$ and $N_c = N_{cn} H(\chi)$ (Eq. (3) and Eq. (4) of the associated article) as substitutions. This produces the following equation for mean autoconversion rate

$$\begin{aligned} \overline{\frac{\partial r_r}{\partial t}} \Big|_{\text{auto}} &= C_{\text{auto}} \sum_{i=1}^n \xi_{(i)} \text{AUTO}_{(i)} \\ &= C_{\text{auto}} \sum_{i=1}^n \xi_{(i)} \int_{-\infty}^{\infty} \int_0^{\infty} \chi^\alpha N_{cn}^\beta (H(\chi))^{\alpha+\beta} P_{(i)}(\chi, N_{cn}) dN_{cn} d\chi. \end{aligned} \quad (\text{S12})$$

Since $\alpha + \beta > 0$, $\text{AUTO}_{(i)}$ can be rewritten

$$\text{AUTO}_{(i)} = \int_0^{\infty} \int_0^{\infty} \chi^\alpha N_{cn}^\beta P_{NL(i)}(\chi, N_{cn}) dN_{cn} d\chi. \quad (\text{S13})$$

This equation is integrated, solving for $\text{AUTO}_{(i)}$ in the scenario of a fully-varying PDF ($\sigma_{\chi(i)} > 0$ and $\sigma_{N_{cn}(i)} > 0$),

$$\text{AUTO}_{(i)} = \frac{1}{\sqrt{2\pi}} \sigma_{\chi(i)}^\alpha \exp \left\{ \tilde{\mu}_{N_{cn}(i)} \beta + \frac{1}{2} \tilde{\sigma}_{N_{cn}(i)}^2 \beta^2 - \frac{1}{4} \varsigma^2 \right\} \Gamma(\alpha + 1) D_{-(\alpha+1)}(-\varsigma), \quad (\text{S14})$$

where ς is given by

$$\varsigma = \frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} + \tilde{\rho}_{\chi, N_{cn}(i)} \tilde{\sigma}_{N_{cn}(i)} \beta.$$

The values of $\tilde{\mu}_{N_{cn}(i)}$, $\tilde{\sigma}_{N_{cn}(i)}$, and $\tilde{\rho}_{\chi, N_{cn}(i)}$ are calculated analogously to the same variables for r_r in Eq. (S5), Eq. (S6), and Eq. (S7), respectively.

There are many case specifications that require a constant cloud droplet concentration within cloud, N_{c0} . The RICO, DYCOMS-II RF02, and LBA cases described in Section 4 of the associated article all use a constant cloud droplet concentration within cloud. In CLUBB's PDF, this is easily accomplished by setting $\overline{N_{cn}'} = 0$, which causes $\sigma_{N_{cn}(1)} = 0$ and $\sigma_{N_{cn}(2)} = 0$. Additionally, $\mu_{N_{cn}(1)} = \mu_{N_{cn}(2)} = \overline{N_{cn}} = N_{c0}$ (where N_{c0} has units of kg^{-1}) in this scenario.

When $\sigma_{\chi(i)} > 0$ and $\sigma_{N_{cn}(i)} = 0$, χ varies in i th component but N_{cn} is constant. The integral is solved and the equation for $\text{AUTO}_{(i)}$ becomes

$$\text{AUTO}_{(i)} = \frac{1}{\sqrt{2\pi}} \sigma_{\chi(i)}^\alpha \mu_{N_{cn}(i)}^\beta \exp \left\{ -\frac{1}{4} \frac{\mu_{\chi(i)}^2}{\sigma_{\chi(i)}^2} \right\} \Gamma(\alpha + 1) D_{-(\alpha+1)} \left(-\frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} \right). \quad (\text{S15})$$

For the remaining forms of $\text{AUTO}_{(i)}$, $\sigma_{\chi(i)} = 0$. When $\mu_{\chi(i)} \geq 0$, the air is entirely saturated and autoconversion occurs. In this scenario, when $\sigma_{N_{cn}(i)} > 0$,

$$\text{AUTO}_{(i)} = \mu_{\chi(i)}^\alpha \exp \left\{ \tilde{\mu}_{N_{cn}(i)} \beta + \frac{1}{2} \tilde{\sigma}_{N_{cn}(i)}^2 \beta^2 \right\}, \quad (\text{S16})$$

and when $\sigma_{N_{cn}(i)} = 0$,

$$\text{AUTO}_{(i)} = \mu_{\chi(i)}^\alpha \mu_{N_{cn}(i)}^\beta. \quad (\text{S17})$$

Otherwise, when $\sigma_{\chi(i)} = 0$ and $\mu_{\chi(i)} < 0$, the air is entirely subsaturated, autoconversion does not occur, and $\text{AUTO}_{(i)} = 0$.

The mean KK autoconversion rate of N_r is found by dividing the mean KK autoconversion rate of r_r by a constant. The constant is $(4\pi\rho_l/3)r_0^3$, where r_0 is the assumed initial size of rain drops and is set to its recommended value of 25×10^{-6} m.

S1.3 Evaporation rate

The KK equation set contains an equation for condensation or evaporation of r_r . CLUBB treats all liquid water in excess of saturation as cloud water and does not allow rain water to increase by condensational growth. The KK equation for evaporation of r_r is of the form

$$\left. \frac{\partial r_r}{\partial t} \right|_{\text{evap}} = 3 c_{\text{evap}*} G(T, p) \left(\frac{4}{3} \pi \rho_l \right)^\gamma (S H(-S))^\alpha r_r^\beta N_r^\gamma, \quad (\text{S18})$$

where $\alpha = 1$, $\beta = 1/3$, and $\gamma = 1 - \beta = 2/3$, and where T is temperature, p is pressure, ρ_l is the density of liquid water, and the function $G(T, p)$ is the coefficient in the drop radius growth equation (Rogers and Yau, 1989, Eq. 7.17). The constant $c_{\text{evap}*}$ is the ratio of raindrop mean geometric radius to raindrop mean volume radius, and is set by KK to a value of 0.86. Supersaturation, S , is positive when the air is saturated and negative when the air is subsaturated, and $S + 1$ is the ratio of water vapor pressure to saturation vapor pressure with respect to liquid water.

Upscaling is accomplished by integrating over Eq. (S18). This requires a substitution that relates S to χ (Larson and Griffin, 2013, Eq. 49). Additionally, $G(T, p)$ is approximated as $G(\bar{T}_l, p)$, where \bar{T}_l is liquid water temperature and is given by

$$T_l = \theta_l \left(\frac{p}{p_0} \right)^{\frac{R_d}{C_{pd}}}, \quad (\text{S19})$$

and where R_d is the gas constant for dry air, C_{pd} is the specific heat of dry air at constant pressure, and p_0 is a reference pressure of 1×10^5 Pa. This is a good approximation because $T = T_l$ when the air is subsaturated and $G(T, p)$ is slowly-varying with regards to temperature. The resulting $G(\bar{T}_l, p)$ is a constant and can be pulled outside the integral.

This produces the following equation for mean evaporation rate

$$\begin{aligned} \overline{\left. \frac{\partial r_r}{\partial t} \right|_{\text{evap}}} &= C_{\text{evap}} \sum_{i=1}^n \xi_{(i)} \text{EVAP}_{(i)} \\ &= C_{\text{evap}} \sum_{i=1}^n \xi_{(i)} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} \chi^\alpha (H(-\chi))^\alpha r_r^\beta N_r^\gamma P_{(i)}(\chi, r_r, N_r) dN_r dr_r d\chi, \end{aligned} \quad (\text{S20})$$

where $P_{(i)}(\chi, r_r, N_r)$ is the trivariate marginal PDF of χ, r_r, N_r in the i th PDF component. The constant C_{evap} is given by

$$C_{\text{evap}} = 3 c_{\text{evap}*} G(\bar{T}_l, p) \left(\frac{4}{3} \pi \rho_l \right)^\gamma \left(\frac{1 + \Lambda(\bar{T}_l) r_{sw}(\bar{T}_l, p)}{r_{sw}(\bar{T}_l, p)} \right)^\alpha, \quad (\text{S21})$$

where $r_{sw}(\bar{T}_l, p)$ is the saturation mixing ratio with respect to liquid water. Additionally,

$$\Lambda(\bar{T}_l) = \frac{R_d}{R_v} \left(\frac{L_v}{R_d \bar{T}_l} \right) \left(\frac{L_v}{C_{pd} \bar{T}_l} \right), \quad (\text{S22})$$

where R_v is the gas constant for water vapor and L_v is the latent heat of vaporization.

Since $\alpha > 0$, $\text{EVAP}_{(i)}$ can be rewritten

$$\begin{aligned} \text{EVAP}_{(i)} = & \int_{-\infty}^0 \int_0^{\infty} \int_0^{\infty} \chi^\alpha r_r^\beta N_r^\gamma (f_{p(i)} P_{NLL(i)}(\chi, r_r, N_r) \\ & + (1 - f_{p(i)}) P_{N(i)}(\chi) \delta(r_r) \delta(N_r)) dN_r dr_r d\chi, \end{aligned} \quad (\text{S23})$$

where $P_{NLL(i)}(\chi, r_r, N_r)$ is the i th component trivariate PDF involving one normal variate and two lognormal variates. When the PDF is fully-varying in the i th PDF component ($\sigma_{\chi(i)} > 0$, $\sigma_{r_r(i)} > 0$, and $\sigma_{N_r(i)} > 0$), the integrated equation for $\text{EVAP}_{(i)}$ is

$$\begin{aligned} \text{EVAP}_{(i)} = & f_{p(i)} \frac{1}{\sqrt{2\pi}} (-\sigma_{\chi(i)})^\alpha \exp\{\tilde{\mu}_{r_r(i)}\beta + \tilde{\mu}_{N_r(i)}\gamma\} \\ & \times \exp\left\{\frac{1}{2}(1 - \tilde{\rho}_{\chi, r_r(i)}^2) \tilde{\sigma}_{r_r(i)}^2 \beta^2 + \frac{1}{2}(1 - \tilde{\rho}_{\chi, N_r(i)}^2) \tilde{\sigma}_{N_r(i)}^2 \gamma^2\right. \\ & \left. + (\tilde{\rho}_{r_r, N_r(i)} - \tilde{\rho}_{\chi, r_r(i)} \tilde{\rho}_{\chi, N_r(i)}) \tilde{\sigma}_{r_r(i)} \beta \tilde{\sigma}_{N_r(i)} \gamma\right\} \\ & \times \exp\left\{\frac{1}{4}\varsigma^2 - \frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} \varsigma + \frac{1}{2} \frac{\mu_{\chi(i)}^2}{\sigma_{\chi(i)}^2}\right\} \Gamma(\alpha + 1) D_{-(\alpha+1)}(\varsigma), \end{aligned} \quad (\text{S24})$$

where

$$\varsigma = \frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} + \tilde{\rho}_{\chi, r_r(i)} \tilde{\sigma}_{r_r(i)} \beta + \tilde{\rho}_{\chi, N_r(i)} \tilde{\sigma}_{N_r(i)} \gamma.$$

The values of $\tilde{\mu}_{N_r(i)}$, $\tilde{\sigma}_{N_r(i)}$, and $\tilde{\rho}_{\chi, N_r(i)}$ are calculated analogously to the same variables for r_r in Eq. (S5), Eq. (S6), and Eq. (S7), respectively. Additionally, the in-precipitation correlation of $\ln r_r$ and $\ln N_r$ in the i th PDF component is $\tilde{\rho}_{r_r, N_r(i)}$, and it is given by

$$\tilde{\rho}_{r_r, N_r(i)} = \frac{\ln\left(1 + \rho_{r_r, N_r(i)} \frac{\sigma_{r_r(i)} \sigma_{N_r(i)}}{\mu_{r_r(i)} \mu_{N_r(i)}}\right)}{\tilde{\sigma}_{r_r(i)} \tilde{\sigma}_{N_r(i)}}, \quad (\text{S25})$$

where $\rho_{r_r, N_r(i)}$ is the correlation of r_r and N_r in the i th PDF component.

Just as with accretion and autoconversion, when one of the variables is constant in the i th PDF sub-component, the equation simplifies. In the scenario when $\sigma_{\chi(i)} > 0$, $\sigma_{r_r(i)} = 0$, and $\sigma_{N_r(i)} > 0$,

$$\begin{aligned} \text{EVAP}_{(i)} = & f_{p(i)} \frac{1}{\sqrt{2\pi}} (-\sigma_{\chi(i)})^\alpha \mu_{r_r(i)}^\beta \\ & \times \exp\left\{\tilde{\mu}_{N_r(i)}\gamma + \frac{1}{2} \tilde{\sigma}_{N_r(i)}^2 \gamma^2 - \frac{1}{4} \varsigma^2\right\} \Gamma(\alpha + 1) D_{-(\alpha+1)}(\varsigma), \end{aligned} \quad (\text{S26})$$

where

$$\varsigma = \frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} + \tilde{\rho}_{\chi, N_r(i)} \tilde{\sigma}_{N_r(i)} \gamma;$$

when $\sigma_{\chi(i)} > 0$, $\sigma_{r_r(i)} > 0$, and $\sigma_{N_r(i)} = 0$,

$$\begin{aligned} \text{EVAP}_{(i)} &= f_{p(i)} \frac{1}{\sqrt{2\pi}} (-\sigma_{\chi(i)})^\alpha \mu_{N_r(i)}^\gamma \\ &\times \exp \left\{ \tilde{\mu}_{r_r(i)} \beta + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \beta^2 - \frac{1}{4} \varsigma^2 \right\} \Gamma(\alpha + 1) D_{-(\alpha+1)}(\varsigma), \end{aligned} \quad (\text{S27})$$

where

$$\varsigma = \frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} + \tilde{\rho}_{\chi, r_r(i)} \tilde{\sigma}_{r_r(i)} \beta;$$

and when $\sigma_{\chi(i)} > 0$, $\sigma_{r_r(i)} = 0$, and $\sigma_{N_r(i)} = 0$,

$$\begin{aligned} \text{EVAP}_{(i)} &= f_{p(i)} \frac{1}{\sqrt{2\pi}} (-\sigma_{\chi(i)})^\alpha \mu_{r_r(i)}^\beta \mu_{N_r(i)}^\gamma \\ &\times \exp \left\{ -\frac{1}{4} \frac{\mu_{\chi(i)}^2}{\sigma_{\chi(i)}^2} \right\} \Gamma(\alpha + 1) D_{-(\alpha+1)} \left(\frac{\mu_{\chi(i)}}{\sigma_{\chi(i)}} \right). \end{aligned} \quad (\text{S28})$$

For the remaining forms of $\text{EVAP}_{(i)}$, $\sigma_{\chi(i)} = 0$. When $\mu_{\chi(i)} \leq 0$, the air is entirely subsaturated and evaporation occurs. In this scenario, when $\sigma_{r_r(i)} > 0$ and $\sigma_{N_r(i)} > 0$,

$$\begin{aligned} \text{EVAP}_{(i)} &= f_{p(i)} \mu_{\chi(i)}^\alpha \exp \left\{ \tilde{\mu}_{r_r(i)} \beta + \tilde{\mu}_{N_r(i)} \gamma + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \beta^2 \right. \\ &\quad \left. + \frac{1}{2} \tilde{\sigma}_{N_r(i)}^2 \gamma^2 + \tilde{\rho}_{r_r, N_r(i)} \tilde{\sigma}_{r_r(i)} \beta \tilde{\sigma}_{N_r(i)} \gamma \right\}; \end{aligned} \quad (\text{S29})$$

when $\sigma_{r_r(i)} = 0$ and $\sigma_{N_r(i)} > 0$,

$$\text{EVAP}_{(i)} = f_{p(i)} \mu_{\chi(i)}^\alpha \mu_{r_r(i)}^\beta \exp \left\{ \tilde{\mu}_{N_r(i)} \gamma + \frac{1}{2} \tilde{\sigma}_{N_r(i)}^2 \gamma^2 \right\}; \quad (\text{S30})$$

when $\sigma_{r_r(i)} > 0$ and $\sigma_{N_r(i)} = 0$,

$$\text{EVAP}_{(i)} = f_{p(i)} \mu_{\chi(i)}^\alpha \mu_{N_r(i)}^\gamma \exp \left\{ \tilde{\mu}_{r_r(i)} \beta + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \beta^2 \right\}; \quad (\text{S31})$$

and when $\sigma_{r_r(i)} = 0$ and $\sigma_{N_r(i)} = 0$,

$$\text{EVAP}_{(i)} = f_{p(i)} \mu_{\chi(i)}^\alpha \mu_{r_r(i)}^\beta \mu_{N_r(i)}^\gamma. \quad (\text{S32})$$

Otherwise, when $\sigma_{\chi(i)} = 0$ and $\mu_{\chi(i)} > 0$, the air is entirely saturated, evaporation does not occur, and $\text{EVAP}_{(i)} = 0$.

The KK evaporation rate of N_r is related to the evaporation rate of r_r by

$$\frac{\Delta N_r|_{\text{evap}}}{N_r} = \left(\frac{\Delta r_r|_{\text{evap}}}{r_r} \right)^{\nu_*}, \quad (\text{S33})$$

where $\Delta N_r|_{\text{evap}}$ is the change in N_r due to evaporation, $\Delta r_r|_{\text{evap}}$ is the change in r_r due to evaporation, and ν_* is a tunable parameter in KK that is set to its recommended value of 1. CLUBB does not handle microphysics process rates in a sequential manner, but rather in a parallel manner. However, the microphysics process rates are explicit terms in the predictive equation set, so the change in a hydrometeor due to a microphysics process is related to the rate of change by

$$\Delta r_r|_{\text{evap}} = \left. \frac{\partial r_r}{\partial t} \right|_{\text{evap}} \Delta t \quad \text{and} \quad \Delta N_r|_{\text{evap}} = \left. \frac{\partial N_r}{\partial t} \right|_{\text{evap}} \Delta t, \quad (\text{S34})$$

where Δt is the duration of one model timestep. Substituting Eq. (S34) into Eq. (S33) and solving for the rate of change of N_r due to evaporation results in

$$\left. \frac{\partial N_r}{\partial t} \right|_{\text{evap}} = (\Delta t)^{\nu_*-1} \frac{N_r}{r_r^{\nu_*}} \left(\left. \frac{\partial r_r}{\partial t} \right|_{\text{evap}} \right)^{\nu_*}. \quad (\text{S35})$$

The mean N_r evaporation rate is calculated in the same way as the mean r_r evaporation rate with α replaced by $\alpha\nu_*$, β replaced by $(\beta - 1)\nu_*$, and γ replaced by $\gamma\nu_* + 1$. Additionally, the constant C_{evap} is taken to the ν_* power and the result is multiplied by $(\Delta t)^{\nu_*-1}$. When ν_* is set to its recommended value of 1, the mean N_r evaporation rate is more simply solved the same way as the mean r_r evaporation rate with β replaced by $\beta - 1$ and γ replaced by $\gamma + 1$.

S1.4 Mean volume radius of rain drops

The KK mean volume radius of rain drops (in meters), R_{vr} , is of the form

$$R_{vr} = C_{\text{mvrr}} r_r^\alpha N_r^\beta, \quad (\text{S36})$$

where $C_{\text{mvrr}} = (4\pi\rho_l/3)^\beta$, $\alpha = 1/3$, and $\beta = -\alpha = -1/3$. Upscaling is accomplished by integrating over Eq. (S36), producing the following equation for mean volume radius

$$\begin{aligned} \overline{R_{vr}} &= C_{\text{mvrr}} \sum_{i=1}^n \xi_{(i)} \text{MVRR}_{(i)} \\ &= C_{\text{mvrr}} \sum_{i=1}^n \xi_{(i)} \int_0^\infty \int_0^\infty r_r^\alpha N_r^\beta P_{(i)}(r_r, N_r) dN_r dr_r, \end{aligned} \quad (\text{S37})$$

where $P_{(i)}(r_r, N_r)$ is the bivariate marginal PDF of r_r and N_r in the i th PDF component.

Additionally, $\text{MVRR}_{(i)}$ can be rewritten

$$\text{MVRR}_{(i)} = \int_0^\infty \int_0^\infty r_r^\alpha N_r^\beta (f_{p(i)} P_{LL(i)}(r_r, N_r) + (1 - f_{p(i)}) \delta(r_r) \delta(N_r)) dN_r dr_r, \quad (\text{S38})$$

where $P_{LL(i)}(r_r, N_r)$ is the i th component bivariate PDF involving two lognormal variates. When the PDF is fully-varying in the i th PDF component ($\sigma_{r_r(i)} > 0$ and $\sigma_{N_r(i)} > 0$), the integrated equation for $MVRR_{(i)}$ is

$$MVRR_{(i)} = f_{p(i)} \exp \left\{ \tilde{\mu}_{r_r(i)} \alpha + \tilde{\mu}_{N_r(i)} \beta + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \alpha^2 + \frac{1}{2} \tilde{\sigma}_{N_r(i)}^2 \beta^2 + \tilde{\rho}_{r_r, N_r(i)} \tilde{\sigma}_{r_r(i)} \alpha \tilde{\sigma}_{N_r(i)} \beta \right\}. \quad (S39)$$

In the scenario when $\sigma_{r_r(i)} = 0$ and $\sigma_{N_r(i)} > 0$,

$$MVRR_{(i)} = f_{p(i)} \mu_{r_r(i)}^\alpha \exp \left\{ \tilde{\mu}_{N_r(i)} \beta + \frac{1}{2} \tilde{\sigma}_{N_r(i)}^2 \beta^2 \right\}, \quad (S40)$$

when $\sigma_{r_r(i)} > 0$ and $\sigma_{N_r(i)} = 0$,

$$MVRR_{(i)} = f_{p(i)} \mu_{N_r(i)}^\beta \exp \left\{ \tilde{\mu}_{r_r(i)} \alpha + \frac{1}{2} \tilde{\sigma}_{r_r(i)}^2 \alpha^2 \right\}, \quad (S41)$$

and when $\sigma_{r_r(i)} = 0$ and $\sigma_{N_r(i)} = 0$,

$$MVRR_{(i)} = f_{p(i)} \mu_{r_r(i)}^\alpha \mu_{N_r(i)}^\beta. \quad (S42)$$

The upscaled mean volume radius is used to calculate mean sedimentation velocity of r_r and N_r . The mean sedimentation velocity of r_r is $\overline{V_{r_r}} = \min(-0.012(10^6 \overline{R_{vr}}) + 0.2, 0)$, and the mean sedimentation velocity of N_r is $\overline{V_{N_r}} = \min(-0.007(10^6 \overline{R_{vr}}) + 0.1, 0)$.

References

- Khairoutdinov, M. and Kogan, Y.: A new cloud physics parameterization in a large-eddy simulation model of marine stratocumulus, *Mon. Wea. Rev.*, 128, 229–243, 2000.
- Larson, V. E. and Griffin, B. M.: Analytic upscaling of local microphysics parameterizations, Part I: Derivation, *Quart. J. Royal Met. Soc.*, 139, 46–57, 2013.
- Rogers, R. R. and Yau, M. K.: A short course in cloud physics, Butterworth-Heinemann, 3rd edn., 1989.