Text S1. Coding of the SAS method in land models

We have developed a semi-analytical solution (SAS) to accelerate spin-up of the Australian Community Atmosphere Biosphere Land Exchange (CABLE) model to steady state of all carbon and nitrogen pools and fluxes. Since the CABLE model shares a similar structure of carbon and nitrogen transfer pathways with most pool-based terrestrial biogeochemical model as described by Fig. 1, the developed SAS is applicable to many biogeochemical models. Generally, there are four aspects of program coding to implement the SAS into a pool-based model (Fig. 2): (1) setting a criterion of steady-state NPP (or plant pools) for the initial spin-up; (2) storing the mean values of the time-varying variables; (3) creating equations to calculate the analytical solutions of pool sizes; and (4) setting a steady-state criterion for the slowest C pool for the final spin-up. Here are the 4 aspects we worked with the CABLE model:

1. Setting a criterion of steady-state NPP (or plant pools) for the initial spin-up

The criterion for ecosystem carbon input to achieve steady state is that change in *NPP* between cycles is less than 10^{-4} g C m^{-2} in this study. In order to judge if the *NPP* has reached the steady state, we created a variable NPP_0 to save the values of *NPP* in the previous cycle. If the ΔNPP ($\Delta NPP = |NPP - NPP_0|$) is lower than 10^{-4} g C m^{-2}, we call the equations to calculate the analytical solutions of steady-state carbon pools. We used the NPP criterion for the initial spin-up at Harvard Forest.

For the global spin-up, we used the total size of plant carbon pools to judge the initial spin-up. Three variables were added into the model for saving the pool sizes of leaf (C_{leaf0}), wood (C_{wood0}), and root (C_{root0}) in the previous cycle. The equations of the analytical solution will be called if the plant C pools meet the following condition:

$$\mathrm{sum}\left(\left|\frac{\mathrm{C_{leaf}} - \mathrm{C_{leaf0}}}{\mathrm{C_{leaf}}}\right|, \left|\frac{\mathrm{C_{wood}} - \mathrm{C_{wood0}}}{\mathrm{C_{wood}}}\right|, \left|\frac{\mathrm{C_{root}} - \mathrm{C_{root0}}}{\mathrm{C_{root}}}\right|\right) < criterion$$

The criterion of the steady-state carbon influx is 0.01% of the total plant C pool size from the previous cycle in this study. The above comparison is performed at the end of each recycling of the meteorological forcing.

2. Storing the mean values of the time-varying variables

We created new variables to store the mean values of the time-varying variables in equation 4 in each recycling of meteorological forcing. According to the structure of CABLE model (see equation 4 in the main text), the variables include the environmental (temperature, moisture and nitrogen) scalars ($\overline{\xi_1}$, $\overline{\xi_2}$, $\overline{\xi_3}$, $\overline{\xi_4}$, $\overline{\xi_5}$, $\overline{\xi_6}$, $\overline{\xi_7}$, $\overline{\xi_8}$, $\overline{\xi_9}$), transfer coefficients between pools ($\overline{a_{41}}$, $\overline{a_{42}}$, $\overline{a_{51}}$, $\overline{a_{52}}$, $\overline{a_{74}}$, $\overline{a_{75}}$, $\overline{a_{76}}$, $\overline{a_{85}}$, $\overline{a_{86}}$, $\overline{a_{87}}$, $\overline{a_{97}}$, $\overline{a_{98}}$), partitioning coefficients ($\overline{b_1}$, $\overline{b_2}$, $\overline{b_3}$), and N/C ratios ($\overline{NC}_i$; i = 1, 2, 3, ⋯, 9) . The mean values of those variables in the last recycling were used to calculate the analytical solutions.

3. Creating equations to calculate the analytical solutions of pool sizes

Equation 4 is the matrix form of analytical solution. This matrix form is generally applicable to all biogeochemical models regardless the number of pools. When the SAS method is applied to one specific model, the analytical solutions can be calculated using arithmetical equations. When equation 4 is applied to CABLE, the steady state of a given plant carbon pool (X_i, i = 1, 2, 3) can be calculated from its steady-state carbon influx ($NPP_{ss} \times \bar{b}_i$) and efflux ($X_i \times \bar{\xi}_i \times C_i$) as:

$X_i = (NPP_{ss} \times \bar{b}_i) / (\bar{\xi}_i \times C_i)$ (i = 1, 2, 3 in the CABLE model)

Similarly, the steady states of litter and soil carbon pools in the CABLE model can be calculated according to the structure of A matrix as:

$X_4 = (X_1 \times \overline{a_{41}} + X_2 \times \overline{a_{42}}) / (\bar{\xi}_4 \times C_4)$

$X_5 = (X_1 \times \overline{a_{51}} + X_2 \times \overline{a_{52}}) / (\bar{\xi}_5 \times C_5)$

$X_6 = X_3 / (\bar{\xi}_6 \times C_6)$

$X_7 = (X_4 \times \overline{a_{74}} + X_5 \times \overline{a_{75}} + X_6 \times \overline{a_{76}}) / (\bar{\xi}_7 \times C_7)$

$X_8 = (X_5 \times \overline{a_{85}} + X_6 \times \overline{a_{86}} + X_7 \times \overline{a_{87}}) / (\bar{\xi}_8 \times C_8)$

$X_9 = (X_7 \times \overline{a_{97}} + X_8 \times \overline{a_{98}}) / (\bar{\xi}_9 \times C_9)$

After getting the analytical solution of the steady-state carbon pools, we used the mean N/C ratios at the last loop of the initial spin-up (see step 4 in the main text) for estimating the steady states of nitrogen pools as:

$N_i = X_i \times \overline{NC}_i$ (i = 1, 2, 3, ⋯, 9 in the CABLE model)

4. Setting a steady-state criterion for the slowest C pool for the final spin-up

The spin-up time is determined by the dynamic of the slowest carbon pool of the model. In CABLE, the slowest carbon pool is passive soil organic carbon pool. After getting the analytical solution of the steady-state pools, we make a final spin-up by using the analytically solved carbon and nitrogen pools as initial values until the a steady-state criterion for the slowest pool was met (e.g., 0.5 g C m^{-2} yr^{-1} in this study). A variable named $C_{pass}0$ was created to save the size of passive soil carbon pool at the end of the previous cycle. If ΔC_{pass} ($\Delta C_{pass} = |C_{pass} - C_{pass}0|$) is lower than the criterion, the steady states of all pool sizes are assumed to be reached.