

Model documentation for “Modeling the statistical distributions of cosmogenic exposure dates from moraines”

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Overview

The codes given here are numerical models that describe the influence of moraine degradation and inheritance on the statistical distributions of cosmogenic exposure dates from glacial landforms, especially moraines. These two processes are treated in separate models, so the degradation model and the inheritance model reside in distinct files.

These codes were written in MATLAB version R2008a, and were run on an Intel-based Macintosh MacBook.

These codes were written carefully, and they have been checked for obvious errors. However, no warranty of any kind is implied. These codes may not even run on your system. The output from these codes should not be trusted without testing.

Please give proper credit if using these codes in research or teaching. Derivative works based on this code should include a reference to the original paper.

Included files

Model codes

All these files are in the MATLAB .m file format. They can be read by any text editor. The freeware editor gedit will highlight the code in the same way as MATLAB's integrated editor.

<u>File name</u>	<u>Description</u>
degradation_model.m	Main file for degradation model; calls the function m_diffusion.m
m_diffusion.m	Contains the part of the code that describes how moraine profiles change over time; based on a derivation prepared by Dr. Nathan Urban, Penn State
inheritance_model.m	Main file for inheritance model

Representative output

These files are in .csv (Comma Separated Values) format. Many of these files are too large to open in Microsoft Excel. Note that the values in these files have been rounded to 5 significant figures.

In the file names, the independent variable or variables are shown in parentheses, and the dependent variables are outside the parentheses. In the files themselves, the first column contains the independent variable, and the second column contains the dependent variable. Where there is more than one independent variable, the first few columns contain their values; the order corresponds to the order of the variable names in parentheses in the file names.

Thus, in the file `naive_age(initial_depth).csv`, the first column contains the initial depth values, and the second column contains the naive age values. In the file `naive_age(pre_time, pre_depth).csv`, the first column is `pre_time`, the second column is `pre_depth`, and the third is `naive_age`.

<u>File name</u>	<u>Description</u>
Degradation model	
<code>naive_age(initial_depth).csv</code>	Cosmogenic exposure dates generated by the degradation model for representative parameter values (see below). These exposure dates are represented as a function of the boulders' initial depths within the moraine.
<code>initial_profile(distances).csv</code> <code>final_profile(distances)_5ka.csv</code> <code>final_profile(distances)_10ka.csv</code> <code>final_profile(distances)_20ka.csv</code>	Elevation of a degrading moraine's surface above its base as a function of distance from the moraine's crest, at elapsed times of 0 ka, 5 ka, 10 ka, and 20 ka.
<code>crest_height(times).csv</code>	Height of a degrading moraine's crest above its base as a function of elapsed time, up to 20 ka.
Inheritance model	
<code>naive_age(pre_time, pre_depth).csv</code>	Cosmogenic exposure dates generated by the inheritance model for representative parameter values (see below). These exposure dates are represented as a function of the boulders' predepositional exposure times and their burial depths within the landscape during that time.

The moraine degradation model (degradation_model.m)

Input parameters

The user-adjustable input parameters are contained in the same file as the model code, degradation_model.m. The values of these parameters are defined in lines 31-87.

Name	Line	Default value	Symbol in text of paper	Description
moraine_age	32	20 ka		Age of moraine (ka)
initial_height	33	50 m	h_0	Initial height of moraine (m)
initial_slope	34	34°	$\tan^{-1}(S_0)$	Initial slope of moraine flanks; note that the variable initial_slope is the slope angle, whereas S_0 in the text of the paper is the slope (rise over run). Few measurements of this parameter exist in the literature; see Hallet and Putkonen (1994), Putkonen and Swanson (2003), and Putkonen and O'Neal (2006).
k	39	$10^{-2} \text{ m}^2/\text{yr}$ (range: 10^{-4} to 10^{-1})	k	Topographic diffusivity (m^2/yr). See Hallet and Putkonen (1994), Hanks (2000), Putkonen and Swanson (2003), and Putkonen et al. (2007).
erosion_rate	42	0 mm/ka		Erosion rate of boulders when exposed at the moraine's surface (mm/ka)
boulder_height	43	1 m	h_b	Minimum sampled boulder height (m). The minimum boulder height that a field geomorphologist would sample.
rho_rock	44	2.6 g/cm^3		Density of the boulders (g/cm^3); controls the e -folding length of cosmic rays into the boulders.
rho_till	45	2.0 g/cm^3		Density of the unconsolidated sediment surrounding the boulders; controls the e -folding length of cosmic rays into the till.
P_spall	48	$4.97 \text{ atoms }^{10}\text{Be/g/yr}$	$P_{i=1}$	Production rate of cosmogenic nuclide by spallation at the earth's surface at the latitude and elevation of the study site. Get this from Balco et al. (2008), using the St scaling model.
P_mu	53	$0.133 \text{ atoms }^{10}\text{Be/g/yr}$	$\Sigma(P_{i=2,3,4})$	Production rate of cosmogenic nuclide by muons at the earth's surface at the latitude and elevation of the study site. Get this from Balco et al. (2008).
decay_const	56	$4.67 \times 10^{-7} \text{ yr}^{-1}$ (?) for ^{10}Be	λ	Decay constant of the cosmogenic nuclide. For ^{10}Be , this value is in dispute (Balco et al., 2008, and refs therein), but this parameter has little effect on ^{10}Be

				exposure dating over the time scales of interest (10^2 - 10^5 yr).
P_slhl	59	[5, 0.09, 0.02, 0.02] atoms $^{10}\text{Be}/\text{g}/\text{yr}$	$P_{i=1, 2, 3, 4}$	Surface production rates of nuclide at sea level and high latitude for various production pathways; see Granger and Muzikar (2001).
att_length	64	[160, 738, 2688, 4360] g/cm^2	material density* $\Lambda_{i=1, 2, 3, 4}$	Attenuation lengths of components of the cosmic ray flux; see Granger and Muzikar (2001). In the text, $\Lambda_{i=1, 2, 3, 4}$ are these values, divided by the density (in g/cm^3) of the material the cosmic rays are passing through.
num_boulders	71	10^5 boulders		Number of boulders to simulate; more boulders produce a more robust distribution of exposure dates, but also cause the model to run more slowly.
time_step	75	25-100 yr		Controls fineness of model discretization in time; smaller values produce more accurate results, but also cause the model to run more slowly.
plots	82	0 or 1		If 0, the code produces no plots; if 1, the plots described below are generated.
bin_width	87	1-5 ka		If plots = 1, this variable controls the widths of the bins into which the calculated exposure dates are sorted to create the histogram (see below).

Output

The degradation model places its output into the following variables.

<u>Name</u>	<u>Line</u>	<u>Symbol in text of paper</u>	<u>Description</u>
initial_profile	109	$z(x, t = 0)$	Initial height of the moraine above its base as a function of distance from the crest (m).
final_profile	109	$z(x, t)$	Final height of the moraine above its base as a function of distance from the crest (m).
crest_height	109	$z(x = 0, t)$	Height of the moraine's crest above its base as a function of time.
distances	109	x	A plotting variable that contains evenly spaced values of distance from the moraine crest (m).
times	109	t	A plotting variable that contains evenly spaced values of time from the beginning of the simulation (yr or ka).
initial_depth	116	d_0	Initial burial depth of each boulder in the moraine (m).
naive_age	171	t_{app}	Apparent exposure time yielded by each boulder (yr or ka).

If the variable plots is set to 1, the degradation model also produces plots of these variables. The plots show the initial and final profile of the moraine (initial_profile and final_profile vs. distances), the height of the moraine crest as a function of time (crest_height vs. times), and a histogram of the exposure dates simulated by the model (naive_age).

Numerical output from the degradation model is stored in the .csv files listed near the beginning of this document. These files generally assume a moraine age of 20 ka, an initial moraine height of 50 m, an initial slope of 34° , and a topographic diffusivity of $10^{-2} \text{ m}^2/\text{yr}$. The moraine profile for intermediate times (5 and 10 ka) is also given, assuming the other parameters have the same values.

The inheritance model
(inheritance_model.m)

Input parameters

Many of the inheritance model's parameters are the same as those used in the degradation model.

<u>Name</u>	<u>Line</u>	<u>Default value</u>	<u>Symbol in text of paper</u>	<u>Description</u>
moraine_age	27	20 ka		Age of moraine (ka)
max_pre_time	28	100 ka	$\max(t_{\text{pre}})$	Maximum predepositional exposure time for the boulders (ka).
max_pre_depth	30	2.0 m	$\max(d_{\text{pre}})$	Maximum predepositional burial depth for the boulders (m).
pre_slope	32	0°		Slope of surface from which preexposed boulders were derived (°).
erosion_rate		0 mm/ka		Erosion rate of boulders after being deposited on the moraine (mm/ka)
rho_rock	36	2.6 g/cm ³		Density of the boulders (g/cm ³); controls the <i>e</i> -folding length of cosmic rays into the boulders.
rho_over	37	2.6 g/cm ³		Density of material overlying the boulders during the predepositional exposure period (g/cm ³); controls the <i>e</i> -folding length of cosmic rays into the boulders during that time.
P_spall	42	4.97 atoms ¹⁰ Be/g/yr	$P_{i=1}$	Production rate of cosmogenic nuclide by spallation at the earth's surface at the latitude and elevation of the study site. Get this from Balco et al. (2008), using the St scaling model.
P_mu	47	0.133 atoms ¹⁰ Be/g/yr	$\Sigma(P_{i=2,3,4})$	Production rate of cosmogenic nuclide by muons at the earth's surface at the latitude and elevation of the study site. Get this from Balco et al. (2008).
decay_const	50	4.67* 10 ⁻⁷ yr ⁻¹ (?) for ¹⁰ Be	λ	Decay constant of the cosmogenic nuclide. For ¹⁰ Be, this value is in dispute (Balco et al., 2008, and refs therein), but this parameter has little effect on ¹⁰ Be exposure dating over the time scales of interest (10 ² -10 ⁵ yr).
P_slhl	53	[5, 0.09, 0.02, 0.02] atoms ¹⁰ Be/g/yr	$P_{i=1,2,3,4}$	Surface production rates of nuclide at sea level and high latitude for various production pathways; see Granger and Muzikar (2001).
att_length	58	[160, 738, 2688, 4360] g/cm ²	material density* $\Lambda_{i=1,2,3,4}$	Attenuation lengths of components of the cosmic ray flux; see Granger and Muzikar (2001). In the text, $\Lambda_{i=1,2,3,4}$ are

				these values, divided by the density (in g/cm^3) of the material the cosmic rays are passing through.
num_boulders	65	10^5 boulders		Number of boulders to simulate; more boulders produce a more robust distribution of exposure dates, but also cause the model to run more slowly.
plots	71	0 or 1		If 0, the code produces no plots; if 1, the plots described below are generated.
bin_width	76	2-10 ka		If plots = 1, this variable controls the widths of the bins into which the calculated exposure dates are sorted to create the histogram (see below).

Output

The inheritance model places its output into the following variables.

<u>Name</u>	<u>Line</u>	<u>Symbol in text of paper</u>	<u>Description</u>
pre_time	99	t_{pre}	Predepositional exposure time of each boulder (yr).
pre_depth	100	d_{pre}	Predepositional burial depth of each boulder (m).
naive_age	118	t_{app}	Apparent exposure time yielded by each boulder (yr or ka).

If the variable plots is set to 1, the model produces a histogram of the apparent exposure times.

Numerical output from the inheritance model is stored in the .csv file listed near the beginning of this document. This file assumes a moraine age of 20 ka, a maximum predepositional exposure time of 100 ka, and a predepositional burial depth of 2.0 m.