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Supplement of

Implementation and evaluation of sea level operators in OceanVar2.0: an open-source oceanographic three-dimensional variational data assimilation system

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S1. Verification of Code Functionality Through Idealised Experiments

This supplementary document provides additional evidence supporting the correctness of the data assimilation code used in the manuscript. To complement the realistic experiments presented in the main text, we report a set of idealised tests designed to verify the behaviour of the assimilation system under controlled conditions, with particular focus on the structure and performance of the background error covariance (BEC) and its associated multivariate operators. These experiments were set up to allow a direct assessment of the residuals when assimilating synthetic observations, and we specifically analysed the sensitivity of the residuals to the prescribed observational error.

S1.1 Experimental Framework

Synthetic observations (y) were generated at the analysis time (t_a), allowing for an exact evaluation of misfits, residuals, and increments.

If x_b denotes the model background and δx the increment produced by the assimilation, the analysis field (x_a) is defined as:

$$x_a = x_b + \delta x$$

The misfits ($\boldsymbol{d}$) and residuals ($\boldsymbol{r}$) were computed as:

$$\boldsymbol{d} = \boldsymbol{y} - \boldsymbol{H}[x_b]$$

$$\boldsymbol{r} = \boldsymbol{y} - \boldsymbol{H}[x_a]$$

where the observation operator ($\boldsymbol{H}$), consistent with our realistic experiments, is a bilinear interpolator. This setup allows us to isolate the impact of the BEC operators and assess how consistently the system reduces residuals under different prescribed observational errors.

S1.2. Test 1 — Assimilation of SLA Track

To specifically verify the SLA operators, we assimilated a synthetic SLA track representing observations after bias and tidal signal removal. Experiments were conducted using both the barotropic and dynamic height formulations for the SLA-operator, and repeated for different prescribed observational errors (3, 5, and 8 cm). The results, shown in Figure S.1, exhibit the behaviour expected from a correctly functioning assimilation system. In all cases, the analysis residuals are smaller than the initial misfit and remain below the prescribed observational error, demonstrating that the system adjusts the state consistently with the assumed observation uncertainty. The differences between the residuals produced by the barotropic and dynamic

height formulations ($\boldsymbol{r}|_{DH} - \boldsymbol{r}|_{BM}$) are small in these idealised conditions, which is fully consistent with the fact that the main distinctions between the two operators emerge only when the sea surface elevation increments are projected vertically into temperature and salinity increments, a behaviour discussed in detail in the main manuscript.

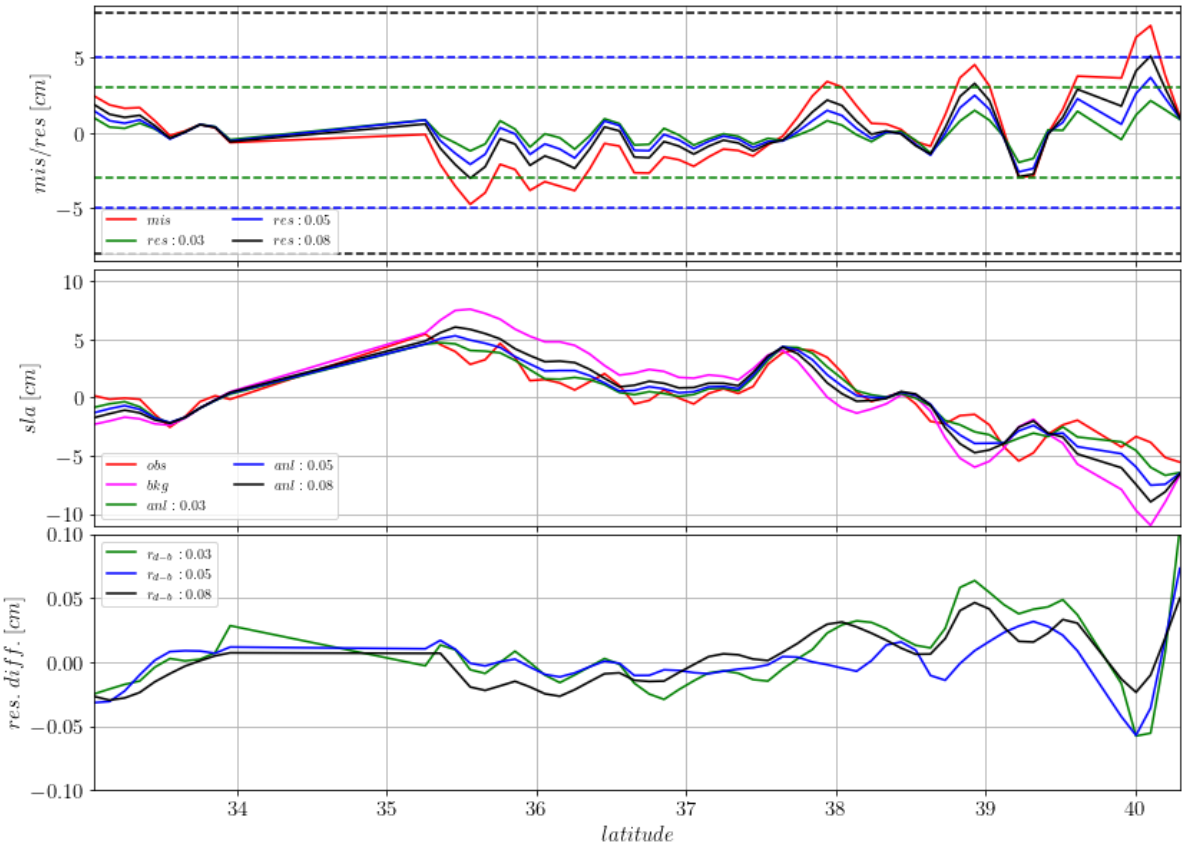

Figure S.1*: SLA Assimilation Test.* ***Top panel****: Initial misfit (red) and analysis residuals (solid-coloured lines) for experiments with varying observational errors (dashed lines).* ***Middle panel****: Synthetic "true" observation (red), the model background (magenta), and the resulting analyses (coloured lines).* ***Bottom panel****: Difference between residuals obtained using the barotropic and dynamic height operators. The numbers in the legends indicate the observational error used in the specific experiment (m).*

S1.3. Test 2: Assimilation of Temperature and Salinity Profiles

While the main manuscript does not focus on in-situ data, we conducted analogous tests for Temperature (T) and Salinity (S) profiles. Assimilating a single T or S observation is straightforward, but assimilating an entire vertical profile is more complex because OceanVar enforces multivariate vertical correlations through pre-computed Empirical Orthogonal Functions (EOFs) in the Background Error Covariance. To construct a meaningful idealised experiment, the synthetic observations were generated so that the initial misfits were consistent

with the vertical structure imposed by the BEC. This ensures a coherent vertical adjustment and allows us to isolate the impact of the prescribed observational error.

Sensitivity experiments were then performed by assimilating temperature/salinity profiles with observational errors of 0.2, 0.4, and 0.6 °C, and 0.02, 0.04, and 0.06 psu respectively. The results, presented in Figure S.2, show that the system behaves consistently with theoretical expectations. The analysis residuals decrease progressively as the observational error is reduced and remain, in all experiments, below the corresponding error threshold. The resulting analyses exhibit coherent vertical adjustments that reflect the multivariate structure imposed by the BEC, confirming that the assimilation system handles full-profile observations correctly under these controlled conditions.

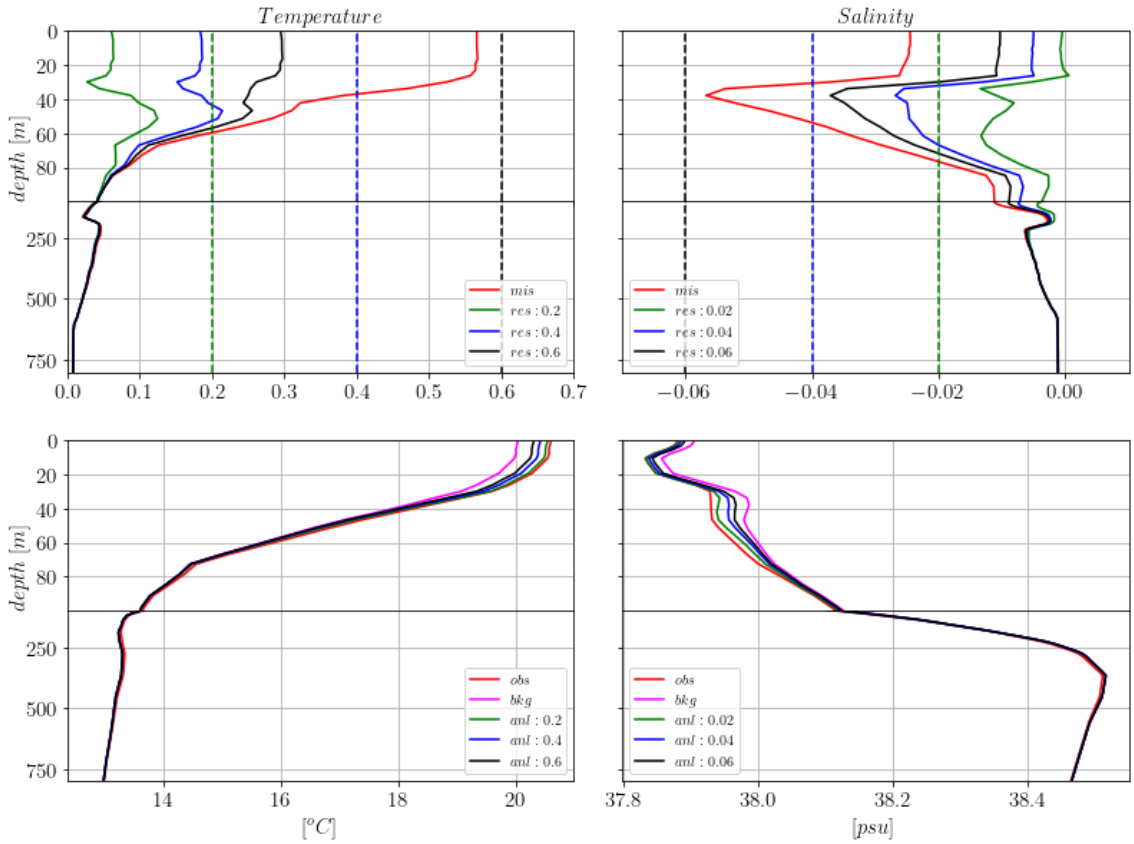

Figure S.2: *Temperature and Salinity Profile Assimilation Test.* ***Top Panels:*** *Initial misfit (red) and analysis residuals (coloured lines) for Temperature (left) and Salinity (right) for different prescribed observational errors.* ***Bottom Panels****: Synthetic "true" profiles (red), model background (magenta), and the resulting analysis profiles (coloured lines).*

S2. Conclusions

These idealised experiments confirm that the assimilation system behaves consistently with theoretical expectations and that the implementation of the BEC operators and observation

operators is functioning correctly under controlled conditions. The reduction of residuals below the prescribed observational errors, together with the coherent adjustments observed in both SLA and profile assimilation tests, provides clear evidence that the code is working as intended and that the system responds appropriately to changes in observation uncertainty. Overall, these results complement the realistic experiments presented in the main manuscript and offer an additional verification of the internal consistency and robustness of the assimilation framework.