



Supplement of

Towards viscous debris flow simulation using DualSPHysics v5.2: internal behaviour of viscous flows and mixtures

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S1 Methodology of the solver

S1.1 Time-stepping

The time scheme used is the symplectic position Verlet scheme which reads (Domínguez et al., 2021):

$$\begin{aligned}
 r_a^{n+1/2} &= r_a^n + \frac{\Delta t}{2} \underline{u}_a^n \\
 \underline{u}_a^{n+1/2} &= \underline{u}_a^n + \Delta t \left(\frac{d\underline{u}_a}{dt} \right)^n \\
 \underline{u}_a^{n+1} &= \underline{u}_a^n + \Delta t \left(\frac{d\underline{u}_a}{dt} \right)^{n+1/2} \\
 r_a^{n+1} &= r_a^{n+1/2} + \Delta t \frac{\underline{u}_a^{n+1} + \underline{u}_a^n}{2}
 \end{aligned} \tag{S.1}$$

5 and the density evolution follows:

$$\begin{aligned}
 \rho_a^{n+1/2} &= \rho_a^n + \frac{\Delta t}{2} \frac{d\rho_a}{dt}^n \\
 \rho_a^{n+1} &= \rho_a^n \frac{2 - \epsilon_a^{n+1/2}}{2 + \epsilon_a^{n+1/2}}
 \end{aligned} \tag{S.2}$$

with $\epsilon_a^{n+1/2} = - \left(\frac{d\rho_a}{dt} \right)^{n+1/2} / \rho_a^{n+1/2} \Delta t$

To achieve optimal description of the fluid and keep computational time sensible, the time-step varies dynamically :

$$dt = C_{CFL} \min(dt_1, dt_2, dt_3) \tag{S.3}$$

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$$\begin{aligned}
 dt_1 &= \sqrt{\frac{h_W}{\max_a \|d\underline{u}/dt\|}} \\
 dt_2 &= \min_a \frac{h_W}{c_s + \max_b \left| \frac{h_W \underline{u}_{ab} \cdot \underline{r}_{ab}}{r_{ab}^2 + \eta^2} \right|} \\
 dt_3 &= \frac{1}{2} \frac{h_W^2 \rho}{\nu_0 \lambda}
 \end{aligned} \tag{S.4}$$

in which, dt_1 accounts for the Courant Friedrich Lewy (CFL) condition, dt_2 accounts for both CFL condition and the numerical viscosity and dt_3 accounts for the physical viscosity, λ being a relaxation parameter of the time step. C_{CFL} is a safety factor based on the Courant Friedrich Lewy condition. In the paper, $C_{CFL} = 0.2$.

15 S1.2 Wall Boundary condition

Representing accurate velocity gradients is central to the performance of the solver to model debris-flow processes. Ensuring a true no-slip condition with the bed of the channel as well as with the solid grains allows for the fluid phase gradients and the fluid-solid interactions to be accurate. For this purpose, the *modified Dynamic Boundary Condition* (mDBC) (English et al., 2021) is used. This method imposes velocity continuity in the flow and a no-slip condition can be accurately dealt with. This
 20 no-slip condition is of high significance when dealing with highly viscous fluids.

S1.3 Shifting

Particles in the flow do not maintain homogeneous distribution. The Lagrangian nature of SPH causes voids or particle clumping to form. These voids create instabilities in the numerical solver. In order to artificially make the particle distribution more homogeneous, the shifting method, based on Fick's law of diffusion, was developed by Lind et al. (2012). This method shifts
 25 particle in high concentration areas towards areas with lower concentration. The distance by which the particle is shifted is always dependent on the gradient of particle concentration in the neighbourhood of the particle.

In this paper, the shifting implementation showed to have an influence on the macroscopic behaviour of the flow, as well as the particle arrangement within the flow (Lapillonne et al., 2022). Thus, the shifting of Lind et al. (2012) was implemented. At each time step, the particles are translated following a vector $\delta \underline{r}_a$:

$$30 \quad \delta \underline{r}_a = -\frac{A_l}{2} h_W^2 \nabla \underline{C}_a \quad (\text{S.5})$$

where A_l is a shifting coefficient, $A_l = 0.5$.

S1.4 Density diffusion term

In the governing equations, the term $hc_0 \mathcal{D}_a$ is a density diffusion term. Density diffusion is a numerical method intended to reduce density fluctuations in the flow by introducing a diffusive term. The stiffness of the equation of state does not react well to the natural disordering of SPH particles: small amplitude, high frequency oscillations appear in the pressure and density fields, leading to instabilities. Density diffusion corrects the density field by introducing a diffusion coefficient, allowing for a smoother density and pressure field.

In this work the density diffusion term introduced by Fourtakas et al. (2019) is used:

$$\begin{aligned} \mathcal{D}_a &= \delta_\phi \sum_b \psi_{ab} \cdot \nabla_a W_{ab} \frac{m_b}{\rho_b} \\ \underline{\psi}_{ab} &= 2 \left(\rho_{ab}^T - \rho_{ab}^H \right) \frac{\underline{x}_{ab}}{x_{ab}^2} \end{aligned} \quad (\text{S.6})$$

40 with ρ_{ab}^T the total density, $\rho_{ab}^H = \rho_0 \left(\left(\frac{P_{ab}^H + 1}{c_0^2 \rho_0 / \gamma} \right)^{1/\gamma} - 1 \right)$ the hydrostatic component of the density, P_{ab} being the pressure difference between a and b, and $\delta_\phi = 0.1$ a diffusion coefficient controlling the magnitude of this diffusion term.

S2 Velocity profiles of ID_2

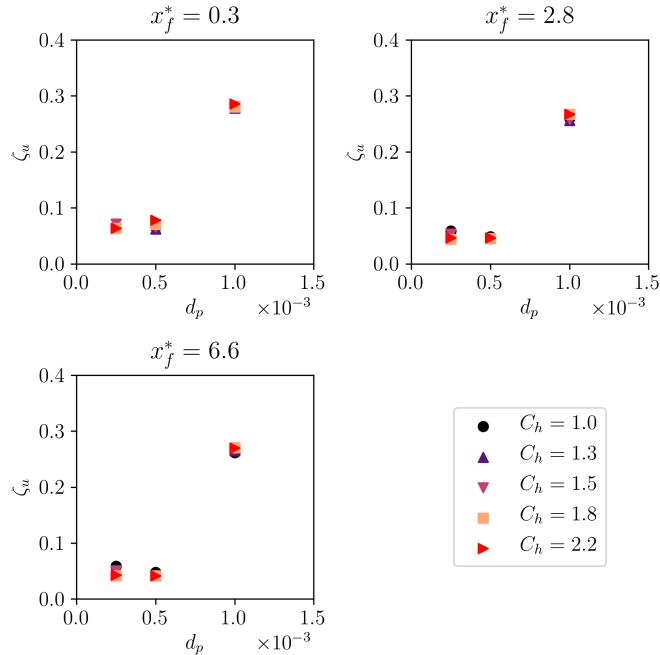


Figure S1. Root mean squared error on the velocity profiles at three positions in the flow

S3 Stationarity of viscous surges simulations

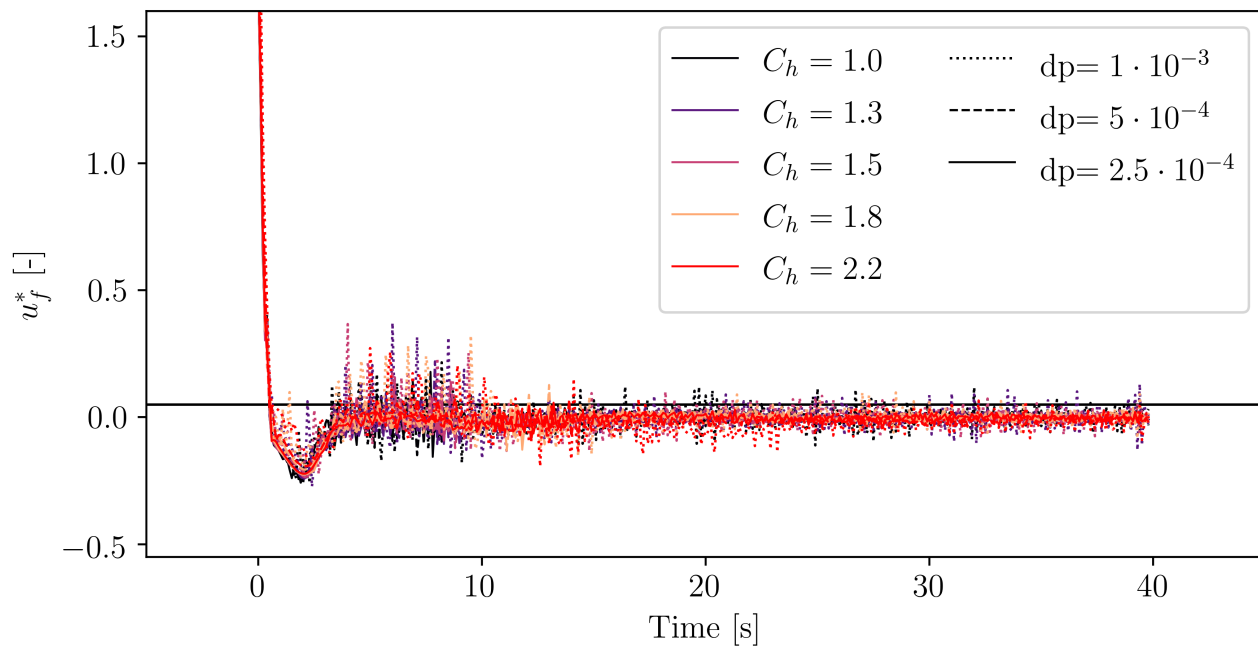


Figure S2. Normalized velocity over time of Exp. Id_1 . Horizontal black line highlights $u_f^* = 0.05$.

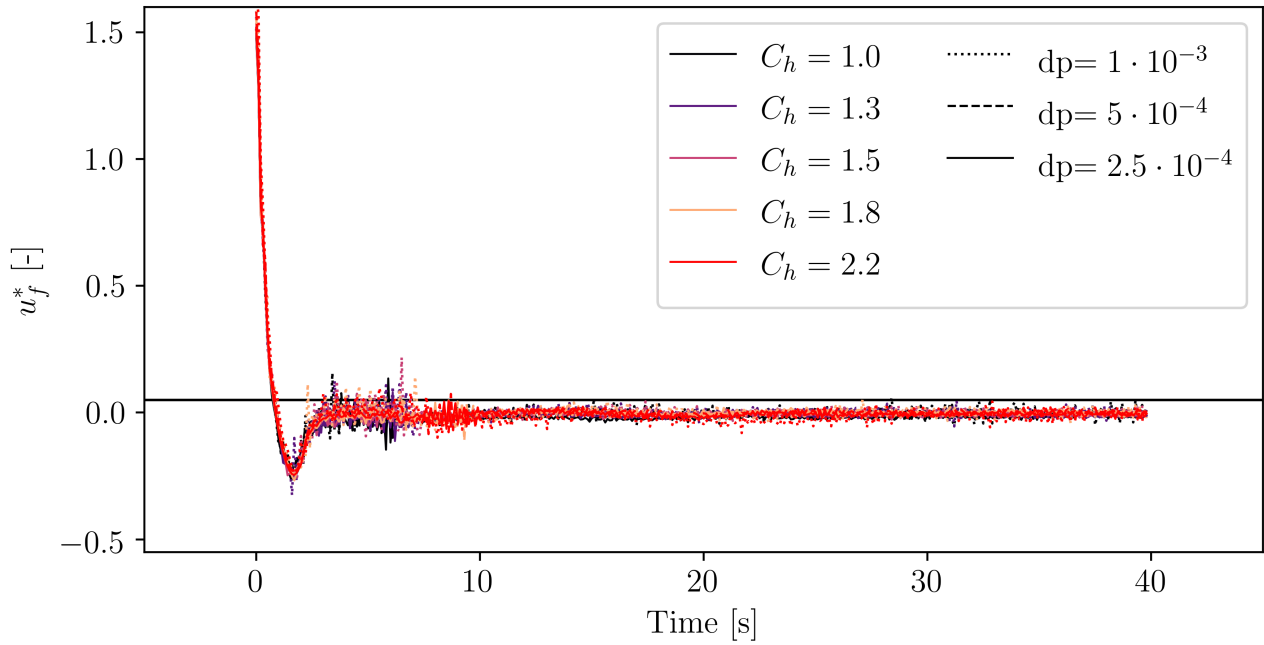


Figure S3. Normalized velocity over time of Exp. Id_2 . Horizontal black line highlights $u_f^* = 0.05$.

S4 Stationarity of mixture Poiseuille flow

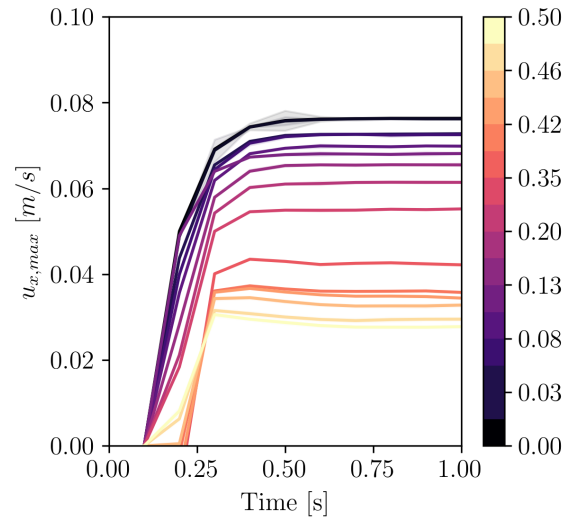


Figure S4. Maximal velocity for all concentrations. Shaded areas show quartiles within the $\pm 1\sigma$ extraction.

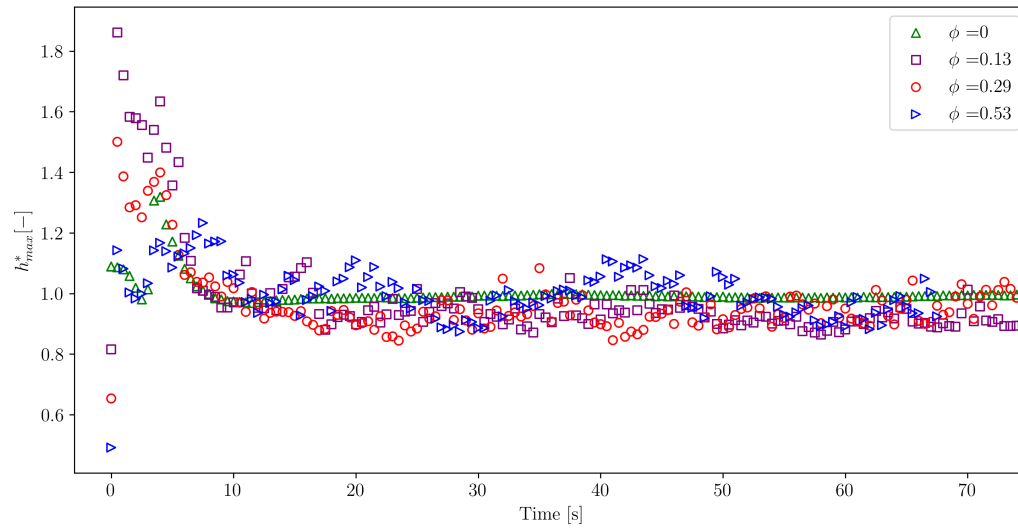


Figure S5. Normalized maximal flow height of the debris flow simulation for all concentrations. Flow height is normalized by the average flow height over the period 20s to 75s.

References

- Domínguez, J. M., Fourtakas, G., Altomare, C., Canelas, R. B., Tafuni, A., García-Feal, O., Martínez-Estévez, I., Mokos, A., Vacondio, R., Crespo, A. J. C., Rogers, B. D., Stansby, P. K., and Gómez-Gesteira, M.: DualSPHysics: from fluid dynamics to multiphysics problems, *Computational Particle Mechanics*, <https://doi.org/10.1007/s40571-021-00404-2>, 2021.
- 50 English, A., Domínguez, J., Vacondio, R., Crespo, A., Stansby, P., Lind, S., Chiapponi, L., and Gómez-Gesteira, M.: Modified dynamic boundary conditions (mDBC) for general-purpose smoothed particle hydrodynamics (SPH): application to tank sloshing, dam break and fish pass problems, *Computational Particle Mechanics*, <https://doi.org/10.1007/s40571-021-00403-3>, 2021.
- Fourtakas, G., Domínguez, J. M., Vacondio, R., and Rogers, B. D.: Local uniform stencil (LUST) boundary condition for arbitrary 3-D boundaries in parallel smoothed particle hydrodynamics (SPH) models, *Computers & Fluids*, 2019.
- 55 Lapillonne, S., Fourtakas, G., Pítou, G., Chambon, G., and Richefeu, V.: Validation of DualSPHysics for viscous Newtonian flow fronts, in: 2022 International SPHERIC Workshop, SPHERIC and University of Catania, Catane, Italy, 2022.
- Lind, S. J., Xu, R., Stansby, P. K., and Rogers, B. D.: Incompressible smoothed particle hydrodynamics for free-surface flows: A generalised diffusion-based algorithm for stability and validations for impulsive flows and propagating waves, *Journal of Computational Physics*, 231, 1499–1523, 2012.