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1 void StressUpdateHighOrder(Matrix<N,n_S>&  $S^{11}$ , Matrix<N,n_S>&  $S^{12}$ , Matrix<N,n_S>&  $S^{22}$ ,
2   const Matrix<N,n_S>&  $E^{11}$ , const Matrix<N,n_S>&  $E^{12}$ , const Matrix<N,n_S>&  $E^{22}$ ,
3   const Matrix<N,n_A>&  $H$ , const Matrix<N,n_A>&  $A$ , double  $\alpha$ ) {
4   for ( $i=0$ ;  $i < N$ ;  $++i$ ) { // in parallel
5     Vector<n_G>  $h = \max\{0, H_{i,*} \text{PSI}\langle n_A \rangle\}$ 
6     Vector<n_G>  $a = \min\{1, \max\{0, A_{i,*} \text{PSI}\langle n_A \rangle\}\}$ 
7     Vector<n_G>  $e^{11} = E_{i,*}^{11} \text{PSI}\langle n_S \rangle$ 
8     Vector<n_G>  $e^{12} = E_{i,*}^{12} \text{PSI}\langle n_S \rangle$ 
9     Vector<n_G>  $e^{22} = E_{i,*}^{22} \text{PSI}\langle n_S \rangle$ 
10
11     Vector<n_G>  $P = P^* \cdot h \cdot \exp(-20(1-a))$ 
12     Vector<n_G>  $D = \left(\Delta_{\min}^2 + \frac{5}{4}(E_{i,*}^{11} * E_{i,*}^{11} + E_{i,*}^{22} * E_{i,*}^{22}) + \frac{3}{2}E_{i,*}^{11} * E_{i,*}^{22} + E_{i,*}^{12} * E_{i,*}^{12}\right)^{\frac{1}{2}}$ 
13     Vector<n_G>  $P_D = P/D$ 
14
15      $S_{i,*}^{11} = (1 - \alpha^{-1})S_{i,*}^{11} + \alpha^{-1}M_i^{-1}(P_D * (\frac{5}{8}e^{11} + \frac{3}{8}e^{22}) - \frac{1}{2}P)$ 
16      $S_{i,*}^{12} = (1 - \alpha^{-1})S_{i,*}^{12} + \alpha^{-1}M_i^{-1}(P_D * \frac{1}{4}e^{12})$ 
17      $S_{i,*}^{22} = (1 - \alpha^{-1})S_{i,*}^{22} + \alpha^{-1}M_i^{-1}(P_D * (\frac{5}{8}e^{22} + \frac{3}{8}e^{11}) - \frac{1}{2}P)$ 
18   }
19 }

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