



Supplement of

Decision-making strategies implemented in SolFinder 1.0 to identify eco-efficient aircraft trajectories: application study in AirTraf 3.0

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SolFinder

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INTRODUCTION

1.1 Background information

SolFinder library to find a solution among a set of Pareto optimal solutions, according to the preferences of the decision-maker. This library has been developed with the aim of identifying eco-efficient aircraft trajectories. The following options are available:

- option selecting a solution closest to a target change in one of the objectives
- Gray Relational Analysis (GRA, [1])
- Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS, [1, 2])
- Viekriterijumsko Kompromisno Rangiranje (VIKOR, [1, 3]) method.

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1.2 Installation

```
pip install solfinder
```

1.3 Get started

```
import solfinder.MCDM as MCDM
import numpy as np

# Upload example dataset
with open(r'tests/Data_example/POBJ_20180101000005_0_51_28_41.dat', 'r') as f:
    data = np.loadtxt(f, unpack=True)

# Values of objective functions
soc = data[0] # Simple Operating Costs
atr = data[1] # ATR20 total

# Find Pareto optimal solutions using available options
```

```
index_target_05 = MCDM.Target.solution_found_with_target(MCDM.Target(),0.5,soc)
index_gra       = MCDM.GRA.solution_found_by_gra(MCDM.GRA(), data)
index_topsis    = MCDM.TOPSIS.solution_found_by_topsis(MCDM.TOPSIS(), data, [0.5,0.5])
set_indices_vikor, index_vikor = MCDM.VIKOR.solution_found_by_vikor(MCDM.VIKOR(),
                             data, 0.5, [0.5, 0.5])
```

CODE DOCUMENTATION

2.1 Description of solfinder.MCDM module

class solfinder.MCDM.GRA

Bases: object

A class used to identify the Pareto optimal solution using the Gray Relational Analysis (GRA).

static **dist_gra**(*v*)

Calculate distance as defined in Gray Relational Analysis (GRA)

Parameters

v (*numpy.ndarray*) – values of i^{th} objective

Returns

distances from maximum value of *v*

Return type

numpy.ndarray

static **grc**(*distances*, *n_sol*)

Calculate Gray Relational Coefficient (GRC)

Parameters

- **distances** (*numpy.ndarray*) – distances from maximum value of *v*
- **n_sol** (*int*) – number of Pareto optimal solutions

Returns

GRC for each solution

Return type

numpy.ndarray

static **norm_gra_min**(*v*)

Normalize values of objective function to be minimized

Parameters

v (*numpy.ndarray*) – values of objective function

Returns

normalized *v*

Return type

numpy.ndarray

static `pref_gra`(*grc*)

Identify solution with the largest Gray Relational Coefficient (GRC)

Parameters**grc** (*numpy.ndarray*) – values of Gray Relational Coefficient for each solution**Returns**

index of solution maximizing GRC

Return type

int

static `solution_found_by_gra`(*pobj*)

Select a single solution among the Pareto optimal solutions using Gray Relational Analysis (GRA)

Parameters**pobj** (*numpy.ndarray*) – values of objective functions**Returns**

index of identified solution

Return type

int

class `solfinder.MCDM.TOPSIS`

Bases: object

A class used to identify the Pareto optimal solution using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).

static `closeness`(*dist_nis*, *dist_pis*)

Calculate Closeness parameter

Parameters

- **dist_nis** (*numpy.ndarray*) – distances from negative ideal solution
- **dist_pis** (*numpy.ndarray*) – distances from positive ideal solution

Returns

closeness parameters for each solution

Return type*numpy.ndarray***static** `dist_nis_topsis`(*v*)

Calculate distance of each solution to the Negative Ideal Solution (NIS)

Parameters**v** (*numpy.ndarray*) – values of objective function**Returns**

distances from NIS

Return type*numpy.ndarray***static** `dist_pis_topsis`(*v*)

Calculate distance of each solution to the Positive Ideal Solution (PIS)

Parameters**v** (*numpy.ndarray*) – values of objective function

Returns

distances form PIS

Return type

numpy.ndarray

static norm_topsis(*w*, *v*)

Normalize and weight values of objective function

Parameters

- **w** (*float*) – relative weight assigned to objective *v*
- **v** (*numpy.ndarray*) – values of objective function

Returnsnormalized and weighted values of *v***Return type**

numpy.ndarray

static pref_topsis(*c*)

Identify solution with largest value of the closeness parameter

Parameters**c** (*numpy.ndarray*) – closeness parameter**Returns**

index of identified solution

Return type

int

solution_found_by_topsis(*pobj*, *weights*)

Select a single solution among the Pareto optimal solutions using Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

Parameters

- **pobj** (*numpy.ndarray*) – values of optimization objective functions
- **weights** (*list*) – relative weights of objective functions

Returns

index of solution found using TOPSIS

Return type

int

class solfinder.MCDM.Target

Bases: object

A class used to identify the Pareto optimal solution closest to a target percentage change in one of the objective functions.

static rel_change(*v*)

Calculate relative change w.r.t. minimum value of objective

Parameters**v** (*numpy.ndarray*) – values of objective function at each pareto opt. solution**Returns**relative change w.r.t. minimum value of *v*

Return type

numpy.ndarray

solution_found_with_target(*x*, *v*)

Identify solution closest to *x*% increase in objective *v*

Parameters

- *x* (*float*) – target percentage change
- *v* (*numpy.ndarray*) – values of objective function at each pareto opt. solution

Returns

index of identified solution

Return type

numpy.int64

class solfinder.MCDM.VIKOR

Bases: object

A class used to identify the Pareto optimal solution using the Viekriterijumsko Kompromisno Rangiranje (VIKOR) method.

static dist_r_vikor(*w*, *v*)

Calculate regret measure (R)

Parameters

- *w* (*list*) – relative weights of optimization objectives
- *v* (*numpy.ndarray*) – values of objective functions

Returns

values of the parameter R

Return type

numpy.ndarray

static dist_s_vikor(*w*, *v*)

Calculate utility measure (S)

Parameters

- *w* (*list*) – relative weights of optimization objectives
- *v* (*numpy.ndarray*) – values of objective functions

Returns

values of the parameter S

Return type

numpy.ndarray

static pref_vikor(*q*, *dist_s*, *dist_r*)

Rank solutions and use conditions of acceptable advantage and stability

Parameters

- *q* (*numpy.ndarray*) – values of the parameter Q
- *dist_s* (*numpy.ndarray*) – values of the parameter Q
- *dist_r* (*numpy.ndarray*) – values of the parameter Q

Returns

index/indices of recommended solution(s)

Return type

list

static `q_vikor`(*gamma*, *s*, *r*)

Calculate parameter Q, combining S and R

Parameters

- **gamma** (*float*) – relative importance of group utility
- **s** (*numpy.ndarray*) – values of the parameter S
- **r** (*numpy.ndarray*) – values of the parameter R

Returns

values of the parameter Q

Return type

numpy.ndarray

static `single_vikor`(*pobj*, *weights*, *set_sol_selected_by_vikor*)

Select a single solution among the set of solutions identified with the VIKOR method

Parameters

- **pobj** (*numpy.ndarray*) – values of objective function
- **weights** (*list*) – relative importance of optimization objectives
- **set_sol_selected_by_vikor** (*list*) – indices of solutions selected by VIKOR

Returns

index of selected solution

Return type

int

solution_found_by_vikor(*pobj*, *gamma*, *weights*)

Select a set of solutions among the Pareto optimal solutions using the Viekriterijumsko Kompromisno Rangiranje (VIKOR) method

Parameters

- **pobj** (*numpy.ndarray*) – values of objective function
- **gamma** (*float*) – relative importance of group utility
- **weights** (*list*) – relative importance of optimization objectives

Returns

set of solutions, single solution selected by VIKOR

Return type

list, int

class `solfinder.MCDM.VikorTarget`

Bases: object

A class used to identify the Pareto optimal solution using the Viekriterijumsko Kompromisno Rangiranje (VIKOR) method, while constraining the increase in one of the objectives

static solution_found_by_vikor_target(*pobj, gamma, weights, index_limited_obj, x*)

Select a solution with VIKOR. If the resulting change in one of the objective is larger than a threshold, then pick the solution closest to such threshold instead.

Parameters

- **pobj** (*numpy.ndarray*) – values of objective function
- **gamma** (*float*) – relative importance of group utility
- **weights** (*list*) – relative importance of optimization objectives
- **index_limited_obj** (*int*) – index of the objective to be constrained
- **x** (*float*) – target/threshold relative change of objective

Returns

index of selected solution

Return type

int

2.2 Example application

```
import solfinder.MCDM as MCDM
import numpy as np
import matplotlib.pyplot as plt

# Upload example dataset
with open(r'tests/Data_example/POBJ_20180101000005_0_51_28_41.dat', 'r') as f:
    data = np.loadtxt(f, unpack=True)

# Values of objective functions
soc = data[0]
atr = data[1]

# Number of Pareto optimal solutions
n_sol = len(soc)

# Find Pareto optimal solutions using available options
index_target_05 = MCDM.Target.solution_found_with_target(MCDM.Target(), 0.5, soc)
index_gra       = MCDM.GRA.solution_found_by_gra(MCDM.GRA(), data)
index_topsis    = MCDM.TOPSIS.solution_found_by_topsis(MCDM.TOPSIS(), data, [0.5, 0.5])
set_indices_vikor, index_vikor = MCDM.VIKOR.solution_found_by_vikor(MCDM.VIKOR(),
    data, 0.5, [0.5, 0.5])

# Plot of Pareto front and selected solutions
plt.scatter(100 * (atr - max(atr)) / max(atr),
    MCDM.Target.rel_change(soc), s=20, c='grey')
plt.scatter(100 * (atr[index_target_05] - max(atr)) / max(atr),
    100 * (soc[index_target_05] - min(soc)) / min(soc),
    s=40, c='red', label='Target +0.5% SOC')
plt.scatter(100 * (atr[index_gra] - max(atr)) / max(atr),
    100 * (soc[index_gra] - min(soc)) / min(soc),
    s=40, c='blue', label='GRA')
```

```

plt.scatter(100 * (atr[index_topsis] - max(atr)) / max(atr),
            100 * (soc[index_topsis] - min(soc)) / min(soc),
            s=40, c='orange', label='TOPSIS')
plt.scatter(100 * (atr[index_vikor] - max(atr)) / max(atr),
            100 * (soc[index_vikor] - min(soc)) / min(soc),
            s=40, c='green', label='VIKOR')
plt.xlabel(r'Change in ATR20 [%]', fontsize=16, labelpad=15)
plt.ylabel(r'Change in SOC [%]', fontsize=16, labelpad=15)
plt.title('Number of solutions: {}'.format(n_sol))
plt.legend()
plt.grid(True)
plt.show()

```

Output:

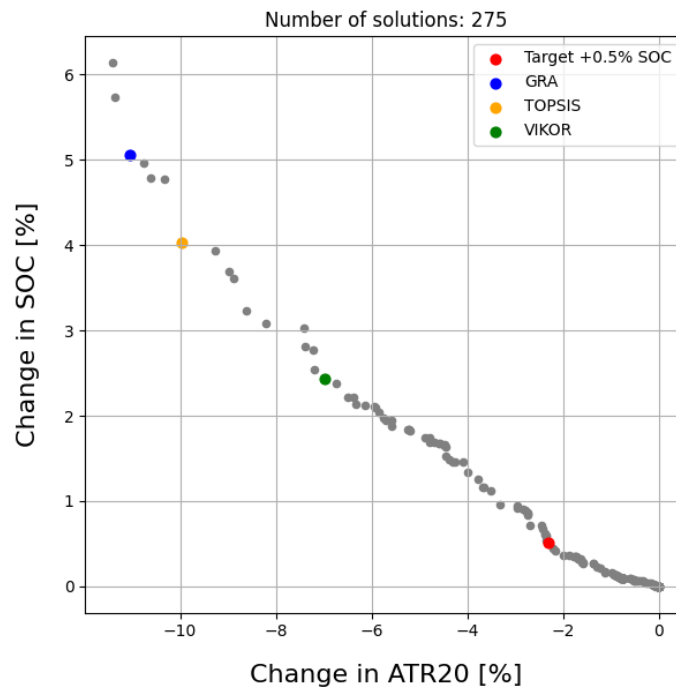


Figure S1: Output of example application code.

BIBLIOGRAPHY

- [1] Zhiyuan Wang and Gade Pandu Rangaiah. Application and Analysis of Methods for Selecting an Optimal Solution from the Pareto-Optimal Front obtained by Multiobjective Optimization. *Industrial & Engineering Chemistry Research*, 56(2):560–574, 1 2017.
- [2] Shu-Jen Chen and Ching-Lai Hwang. *Fuzzy Multiple Attribute Decision Making*, volume 375. Springer Berlin Heidelberg, Berlin, Heidelberg, 1992.
- [3] Serafim Opricovic and Gwo-Hshiung Tzeng. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2):445–455, 7 2004.

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