- Let $\{F^k\}_{k=1}^K$ denote the set of computational outputs of interest with correlation coefficients $\{\zeta_{1,k}\}_{k=2}^K$ and computational costs $\{C_k\}_{k=1}^K$ that respectively satisfy

$$\left|\zeta_{1,k-1}\right| > \left|\zeta_{1,k}\right| \quad \text{and} \quad \frac{C_{k-1}}{C_k} > \frac{\zeta_{1,k-1}^2 - \zeta_{1,k}^2}{\zeta_{1,k}^2 - \zeta_{1,k+1}^2}, \quad k = 2, \ldots, K. \tag{9}$$

  The first requirement is easily satisfied by a reordering of the surrogate outputs of interest $F^2, \ldots, F^K$. If, after that reordering, the second requirement is not satisfied for some $k \in \{2, \ldots, K\}$, then that F^k is removed from the list of surrogates.
- Let $M_{1,1}, \ldots, M_{k-1,k-1}$ be given; these are all positive by construction.
- Also let $\mathbf{M}_k^* = \{M_{k',k}^*\}_{k'=k}^K$ and $\boldsymbol{\alpha}_k^* = \{\alpha_{k',k}^*\}_{k'=k+1}^K$, and let

$$r_{k',k}^* = \frac{M_{k',k}^*}{M_{k,k}^*} = \sqrt{\frac{C_k}{C_{k'}} \left(\frac{\zeta_{1,k'}^2 - \zeta_{1,k'+1}^2}{\zeta_{1,k}^2 - \zeta_{1,k+1}^2} \right)}, \quad k' = k, \ldots, K.$$

- Then, for each $k = 1, \ldots, K$, the *unique global minimizer* $\left(\mathbf{M}_k^*, \boldsymbol{\alpha}_k^*\right)$ of the functional $\mathcal{L}^k$ is given by

$$M_{k,k}^* = \frac{B - \sum_{k'=1}^{k-1} C_{k'}}{\sum_{k'=k}^{K} C_{k'} r_{k',k}}, \tag{10}$$

  and

$$M_{k',k}^* = M_{k,k}^* r_{k',k}^* \quad \text{and} \quad \alpha_{k',k}^* = \frac{\zeta_{1,k'} \sigma_1}{\sigma_{k'}}, \quad k' = k+1, \ldots, K. \tag{11}$$