



Supplement of

A new bootstrap technique to quantify uncertainty in estimates of ground surface temperature and ground heat flux histories from geothermal data

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5 S1 Spatial degrees of freedom

Spatial correlation between surface temperature series at nearby locations may be important to determine the confidence interval of zonal and global averaged ground surface temperature histories from subsurface temperature profiles. To assess the effect of spatial correlation on global histories, we estimate the number of effective degrees of freedom of the surface air temperatures of the CRU TS 4.05 product (Harris et al., 2020). We select this product because it is based on meteorological observations and it is provided as an homogeneous, gridded dataset without gaps in the temporal series of each grid cell. In any case, GISTEMP (Lenssen et al., 2019) and HadCRU (Morice et al., 2012) products have been also tested, yielding similar results.

The degrees of freedom (dof) of two temporal series depend on the correlation coefficient (c) between them as (Fraedrich et al., 1995)

$$\text{dof} = \frac{2}{1 + c^2}. \quad (1)$$

In order to estimate the number of degrees of freedom from CRU temperatures, we apply Equation (1) to the temperature series in a given cell and the four closest neighbours, obtaining the effective degrees of freedom as the average of the four different estimates. Thereby, we can assess the level of similarity of each cell with the surrounding cells.

Figure S1 shows the degrees of freedom estimated from annual temperature series and from their 30-yr running means. Results considering the eight and twelve closest neighbours are also displayed to evaluate the effect of taking into account more distant locations. Orography seems to be the leading factor in local variability, followed by the small number of meteorological observations included in the CRU dataset for several areas, like the Arctic and Africa. That is, areas with a smaller number of observations seem to present larger spatial variability in surface temperature, leading to more degrees of freedom.

The effective number of degrees of freedom in different zones is included in the bootstrap estimates by estimating the weighted mean of the inversions in the Sampling ensemble to retrieve the corresponding member of the Bootstrapping ensemble. That is, the inversions within the Sampling ensemble are weighted by the corresponding degrees of freedom at the location of the profiles, thus inversions from temperature profiles within zones with more degrees of freedom weight more than inversions from profiles in other zones. Concretely, we consider the degrees of freedom obtained using the twelve closest neighbours and 30-yr running means, as this is the case showing higher spatial differences in Figure S1. The comparison between bootstrap inversions performed considering the effective degrees of freedom and without this consideration is shown in Figure S2. Both global averages and 95% confidence intervals with and without considering the spatial degrees of freedom present very similar results. Similar results are obtained when considering the degrees of freedom resulting from annual temperatures, and four and eight neighbours (not shown). Therefore, the effect of the different number of degrees of freedom at borehole locations is not relevant for estimating global ground surface temperature histories from subsurface temperature profiles, which is reasonable seeing the small differences between locations with high and low degrees of freedom in Figure S1.

35 S2 Additional figures

Effective degrees of freedom

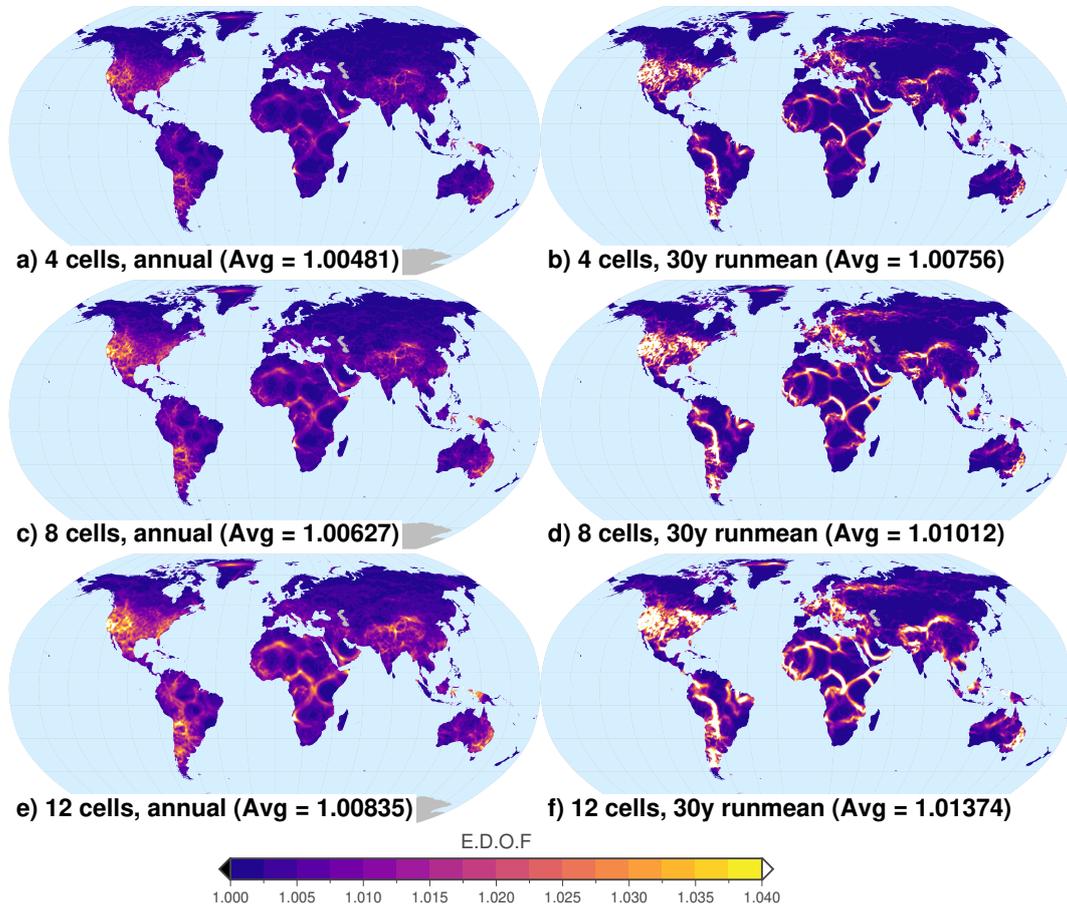


Figure S1. Number of effective degrees of freedom for CRU TS 4.05 temperatures from annual (left column) and long-term (30-yr running means, right column) series. Results considering the four (first row), eight (second row), and twelve (third row) closest grid cells are displayed.

Effect of E.D.O.F.

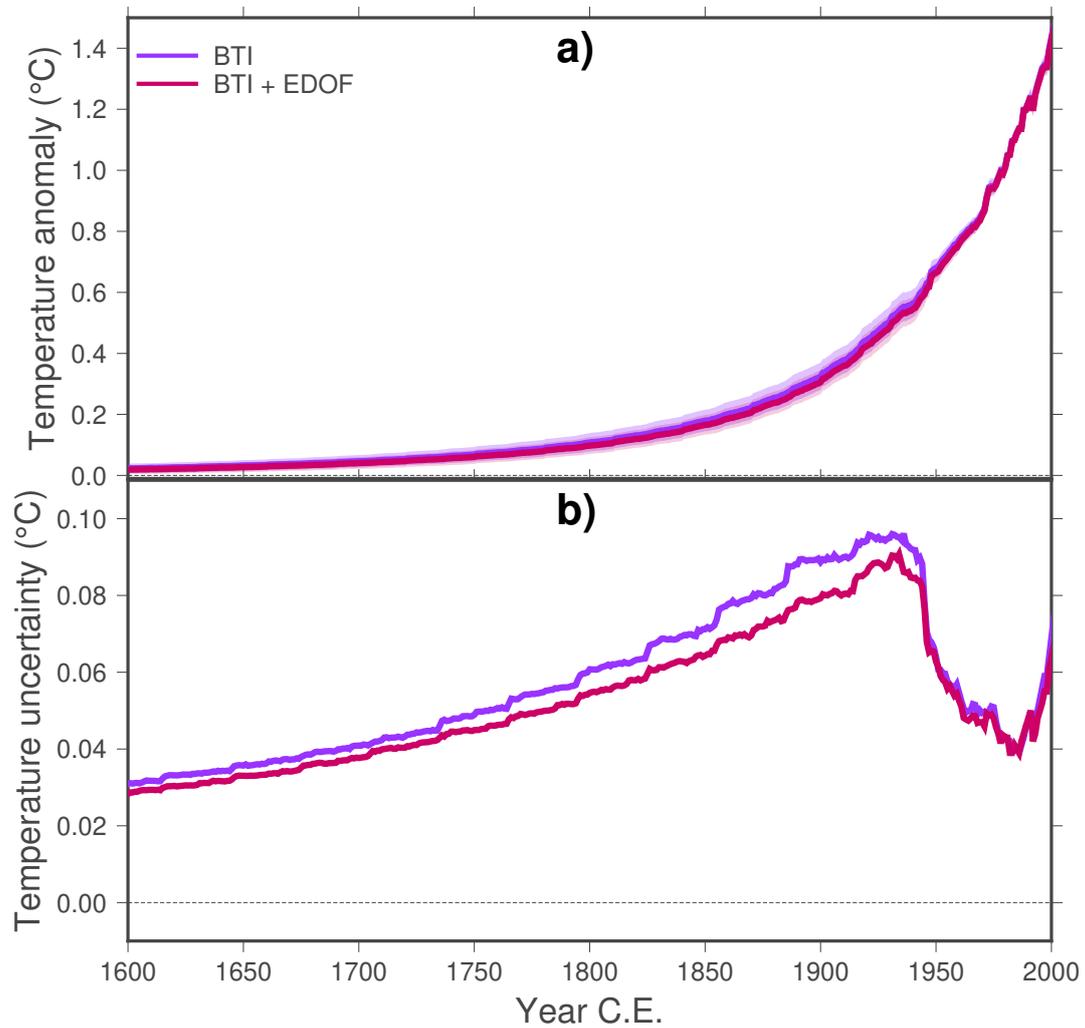


Figure S2. Estimated temperature evolution from subsurface temperature profiles. (a) Global averaged surface temperature histories considering the different effective degrees of freedom at the location of each profile (red line), and weighting all profiles equally (purple line). (b) Range of the 95% confidence interval for bootstrap inversions considering the effective degrees of freedom at the location of each profile (red line), and weighting all profiles equally (purple line).

References

- Fraedrich, K., Ziehmann, C., and Sielmann, F.: Estimates of Spatial Degrees of Freedom, *Journal of Climate*, 8, 361 – 369, [https://doi.org/10.1175/1520-0442\(1995\)008<0361:EOSDOF>2.0.CO;2](https://doi.org/10.1175/1520-0442(1995)008<0361:EOSDOF>2.0.CO;2), 1995.
- Harris, I., Osborn, T. J., Jones, P., and Lister, D.: Version 4 of the CRU TS monthly high-resolution gridded multivariate climate dataset, *Scientific Data*, 7, 109, <https://doi.org/10.1038/s41597-020-0453-3>, 2020.
- 40 Lensen, N. J. L., Schmidt, G. A., Hansen, J. E., Menne, M. J., Persin, A., Ruedy, R., and Zyss, D.: Improvements in the GISTEMP Uncertainty Model, *Journal of Geophysical Research: Atmospheres*, 124, 6307–6326, <https://doi.org/10.1029/2018JD029522>, 2019.
- Morice, C. P., Kennedy, J. J., Rayner, N. A., and Jones, P. D.: Quantifying uncertainties in global and regional temperature change using an ensemble of observational estimates: The HadCRUT4 data set, *Journal of Geophysical Research: Atmospheres*, 117, n/a–n/a,
45 <https://doi.org/10.1029/2011JD017187>, d08101, 2012.