

```

3 # Mesh Generation:
4 a, b, c, nx, ny, nz = 1.0079, 0.6283, 1.0, 20, int(0.6283/1.0 * 20), 20
5 mesh2d = RectangleMesh(nx, ny, a, b, quadrilateral=True) # Rectangular 2D mesh
4 mesh = ExtrudedMesh(mesh2d, nz)
5 bottom, top, left, right, front, back = "bottom", "top", 1, 2, 3, 4
6
7 -----
8 # Initial condition and constants:
9 Told.interpolate(0.5*(erf((1-X[2])*4)+erf(-X[2]*4)+1) + 0.2*(cos(pi*X[0]/a)+cos(pi*X[1]/b))*sin(pi*X[2]))
10 Ra = Constant(3e4) # Rayleigh number
11 k = Constant((0, 0, 1)) # Unit vector (in direction opposite to gravity).
12
13 -----
14 # Stokes Equation Solver Parameters:
15 stokes_solver_parameters = {
16     "mat_type": "matfree",
17     "snes_type": "ksponly",
18     "ksp_type": "preonly",
19     "pc_type": "fieldsplit",
20     "pc_fieldsplit_type": "schur",
21     "pc_fieldsplit_schur_type": "full",
22     "fieldsplit_0": {
23         "ksp_type": "cg",
24         "ksp_rtol": 1e-7,
25         "pc_type": "python",
26         "pc_python_type": "firedrake.AssembledPC",
27         "assembled_pc_type": "gamg",
28         "assembled_pc_gamg_threshold": 0.01,
29         "assembled_pc_gamg_square_graph": 100,
30     },
31     "fieldsplit_1": {
32         "ksp_type": "fgmres",
33         "ksp_rtol": 1e-6,
34         "pc_type": "python",
35         "pc_python_type": "firedrake.MassInvPC",
36         "Mp_ksp_rtol": 1e-5,
37         "Mp_ksp_type": "cg",
38         "Mp_pc_type": "sor",
39     }
40 }
41 # Energy Equation Solver Parameters:
42 energy_solver_parameters = {
43     "mat_type": "aij",
44     "snes_type": "ksponly",
45     "ksp_type": "gmres",
46     "ksp_rtol": 1e-7,
47     "pc_type": "sor", }
48
49 -----
50 # Set up boundary conditions:
51 bcvfb = DirichletBC(Z.sub(0).sub(1), 0, (front, back))
52 bcvfr = DirichletBC(Z.sub(0).sub(0), 0, (left, right))
53 bcvbt = DirichletBC(Z.sub(0), 0, (bot,top))
54 bctb, bctt = DirichletBC(Q, 1.0, (bot), DirichletBC(Q, 0.0, top))
55
56 -----
57 # Generating near_nullspaces for GAMG:
58 x_rotV = Function(V).interpolate(as_vector((0, X[2], -X[1])))
59 y_rotV = Function(V).interpolate(as_vector((-X[2], 0, X[0])))
60 z_rotV = Function(V).interpolate(as_vector((-X[1], X[0], 0)))
61 nns_x = Function(V).interpolate(Constant([1., 0., 0.]))
62 nns_y = Function(V).interpolate(Constant([0., 1., 0.]))
63 nns_z = Function(V).interpolate(Constant([0., 0., 1.]))
64 V_near_nullspace = VectorSpaceBasis([nns_x, nns_y, x_rotV, y_rotV, z_rotV])
65 V_near_nullspace.orthonormalize()
66 Z_near_nullspace = MixedVectorSpaceBasis(Z, [V_near_nullspace, Z.sub(1)])
67
68 -----
69 # Updated solve setup:
70 stokes_problem = NonlinearVariationalProblem(F_stokes, z, bcs=[bcvfb, bcvfr, bcvbt])
71 stokes_solver = NonlinearVariationalSolver(stokes_problem, solver_parameters=stokes_solver_parameters, apcpx={"
72     mu": mu}, nullspace=p_nullspace, transpose_nullspace=p_nullspace, near_nullspace=Z_near_nullspace)
73 energy_problem = NonlinearVariationalProblem(F_energy, Tnew, bcs=[bctb, bctt])
74 energy_solver = NonlinearVariationalSolver(energy_problem, solver_parameters=energy_solver_parameters)
75
76 -----
77 # Updated diagnostics:
78 nusselt_number_top = -1. * assemble(dot(grad(Tnew), n) * ds_t) * (1./assemble(Tnew * ds_b))

```