

```

1: from firedrake import *
2: import numpy as np
3:
4: # Mesh - use a built in meshing function:
5: mesh = UnitSquareMesh(40, 40, quadrilateral=True)
6: left, right, bottom, top = 1, 2, 3, 4 # Boundary IDs
7: n = FacetNormal(mesh) # Normals, required for Nusselt number
8: domain_volume = assemble(1.*dx(domain=mesh)) # Required for RMS velocity
9:
10: # Function spaces:
11: V = VectorFunctionSpace(mesh, family="CG", degree=2) # Velocity function space (vector)
12: W = FunctionSpace(mesh, family="CG", degree=1) # Pressure function space (scalar)
13: Q = FunctionSpace(mesh, family="CG", degree=2) # Temperature function space (scalar)
14: Z = MixedFunctionSpace([V, W]) # Mixed function space
15:
16: # Test functions and functions to hold solutions:
17: v, w = TestFunctions(Z)
18: q = TestFunction(Q)
19: z = Function(Z)
20: u, p = split(z) # Returns symbolic UFL expression for u and p
21: Told, Tnew = Function(Q, name="OldTemp"), Function(Q, name="NewTemp")
22: Ttheta = 0.5 * Tnew + 0.5 * Told # Temporal discretisation through Crank-Nicholson
23:
24: # Initialise temperature field:
25: X = SpatialCoordinate(mesh)
26: Told.interpolate(1.0 - X[1] + 0.05 * cos(pi * X[0]) * sin(pi * X[1]))
27: Tnew.assign(Told)
28:
29: # Important constants:
30: Ra, mu, kappa, delta_t = Constant(1e4), Constant(1.0), Constant(1.0), Constant(1e-6)
31: k = Constant((0, 1)) # Unit vector (in direction opposite to gravity)
32:
33: # Stokes equations in UFL form:
34: stress = 2 * mu * sym(grad(u))
35: F_stokes = inner(grad(v), stress) * dx - div(v) * p * dx - (dot(v, k) * Ra * Ttheta) * dx
36: F_stokes += -w * div(u) * dx # Continuity equation
37: # Energy equation in UFL form:
38: F_energy = q * (Tnew - Told) / delta_t * dx + q * dot(u, grad(Ttheta)) * dx + dot(grad(q), kappa * grad(Ttheta)) * dx
39:
40: # Set up boundary conditions and deal with nullspaces:
41: bcvx, bcvy = DirichletBC(Z.sub(0).sub(0), 0, sub_domain=(left, right)), DirichletBC(Z.sub(0).sub(1), 0,
42: sub_domain=(bottom, top))
43: bctb, bctt = DirichletBC(Q, 1.0, sub_domain=bottom), DirichletBC(Q, 0.0, sub_domain=top)
44: p_nullspace = MixedVectorSpaceBasis(Z, [Z.sub(0), VectorSpaceBasis(constant=True)])
45:
46: # Initialise output:
47: output_file = File('output.pvd') # Create output file
48: u_, p_ = z.split()
49: u_.rename("Velocity"), p_.rename("Pressure")
50:
51: # Solver dictionary:
52: solver_parameters = {
53:     "mat_type": "aij",
54:     "snes_type": "ksponly",
55:     "ksp_type": "preonly",
56:     "pc_type": "lu",
57:     "pc_factor_mat_solver_type": "mumps"}
58:
59: # Setup problem and solver objects so we can reuse (cache) solver setup
60: stokes_problem = NonlinearVariationalProblem(F_stokes, z, bcs=[bcvx, bcvy])
61: stokes_solver = NonlinearVariationalSolver(stokes_problem, solver_parameters=solver_parameters, nullspace=
62: p_nullspace, transpose_nullspace=p_nullspace)
63: energy_problem = NonlinearVariationalProblem(F_energy, Tnew, bcs=[bctb, bctt])
64: energy_solver = NonlinearVariationalSolver(energy_problem, solver_parameters=solver_parameters)
65:
66: # Timestepping aspects
67: no_timesteps, target_cfl_no = 2000, 1.0
68: ref_u = Function(V, name="Reference_Velocity")
69:
70: def compute_timestep(u):
71:     """Return the timestep, using CFL criterion"""
72:     tstep = (1. / np.abs(ref_u.interpolate(dot(JacobianInverse(mesh), u)).dat.data).max()) * target_cfl_no
73:     return tstep
74:
75: for timestep in range(0, no_timesteps):
76:     if timestep > 0:
77:         delta_t.assign(compute_timestep(u))
78:     if timestep % 10 == 0:
79:         output_file.write(u_, p_, Tnew)
80:     stokes_solver.solve()
81:     energy_solver.solve()
82:     vrms = sqrt(assemble(dot(u, u) * dx)) * sqrt(1./domain_volume)
83:     nu_top = -1. * assemble(dot(grad(Tnew), n) * ds(top))
84:     Told.assign(Tnew)

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