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Supplement of

Formulation of a new explicit tidal scheme in revised LICOM2.0

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Supplemental materials

1 Table for the introduction method following Sakamoto et al. (2013)

Table S1. Global mean values of the amplitudes of the eight tidal constituents during observation, Exp1 (the traditional tidal scheme) and Exp2 (the new tidal scheme), and the amplitude, phase, and total errors of the eight tidal constituents in Exp1 and Exp2. The units are cm. The better amplitude and lower errors in Exp2 relative to Exp1 are marked by bold font.

	Global mean			Amplitude Error		Phase Error		Total Error	
	Obs	Exp1	Exp2	Exp1	Exp2	Exp1	Exp2	Exp1	Exp2
M2	33.31	43.43	**38.58**	15.18	**13.11**	34.26	**21.78**	37.47	**25.43**
S2	13.35	13.70	11.46	5.30	5.44	7.71	**5.79**	9.35	**7.94**
N2	7.08	11.29	**6.91**	4.34	**1.99**	8.35	**3.03**	9.41	**3.62**
K2	3.75	7.5	**7.01**	3.92	**3.54**	7.60	**6.26**	8.56	**7.19**
K1	11.58	14.78	**10.08**	5.05	**3.78**	4.09	**3.40**	6.50	**5.08**
O1	8.34	11.65	**9.92**	3.94	**3.18**	9.22	**4.04**	10.02	**5.14**
P1	3.62	11.65	**8.25**	7.53	**4.50**	4.98	**4.12**	9.03	**6.10**
Q1	1.76	2.91	**1.80**	1.14	**0.50**	2.20	**0.51**	2.48	**0.71**

2 Detailed steps for the derivation of equation (4)

Assuming that the Earth is a rigid body, the horizontal tide-generating force is (Cartwright, 1999; Boon, 2004):

$$F_{tide,m} = \frac{GM_m}{L^2}\sin(\theta_m + r) - \frac{GM_m}{{D_m}^2}\sin\theta_m \quad (1)$$

According to analytic geometry and the law of cosines, we can obtain:

$$\sin(\theta_m + r) = \frac{D_m \sin\theta_m}{L} \quad (2)$$

$$L^2 = {D_m}^2 + a^2 - 2aD_m\cos\theta_m \quad (3)$$

Formulas (1) and (2) are combined to obtain:

$$F_{tide,m} = \frac{GM_m}{L^2}\frac{D_m \sin\theta_m}{L} - \frac{GM_m}{{D_m}^2}\sin\theta_m$$

$$F_{tide,m} = GM_m \sin\theta_m \left(\frac{D_m}{L^3} - \frac{1}{{D_m}^2}\right) \quad (4)$$

At the same time, from Formula (3), it can be obtained that:

$$L^{-1} = {D_m}^{-1}\left(1 + \frac{a^2}{{D_m}^2} - 2\frac{a}{D_m}\cos\theta_m\right)^{-\frac{1}{2}}$$

$$\frac{D_m}{L^3} = {D_m}^{-2}\left(1 + \frac{a^2}{{D_m}^2} - 2\frac{a}{D_m}\cos\theta_m\right)^{-\frac{3}{2}} \tag{5}$$

$G = g\frac{a^2}{E}$ and formulas (1) and (2) are combined to obtain:

$$F_{tide,m} = \frac{M_m}{E}\frac{a^2}{{D_m}^2} g\sin\theta_m\left[\left(1 + \frac{a^2}{{D_m}^2} - 2\frac{a}{D_m}\cos\theta_m\right)^{-\frac{3}{2}} - 1\right] \tag{6}$$

since $\frac{a}{D_m}$ equals approximately 0.017 and $\left|\frac{a^2}{{D_m}^2} - 2\frac{a}{D_m}\cos\theta_m\right| < 1$, binomial series can be used:

$$\left(1 + \frac{a^2}{{D_m}^2} - 2\frac{a}{D_m}\cos\theta_m\right)^{-\frac{3}{2}}$$
$$= 1 + 3\frac{a}{D_m}\cos\theta_m - \frac{3}{2}\frac{a^2}{{D_m}^2} + \frac{15}{8}\left(\frac{a^2}{{D_m}^2} - 2\frac{a}{D_m}\cos\theta_m\right)^2 + \cdots$$

Omit the minor term and equation (6) can be recorded as:

$$F_{tide,m} \approx \frac{M_m}{E}\frac{a^2}{{D_m}^2} g\sin\theta_m\left[1 + 3\frac{a}{D_m}\cos\theta_m - 1\right]$$
$$= \frac{3}{2}\frac{M_m}{E}\left(\frac{a}{D_m}\right)^3 g\sin 2\theta_m$$