Supplement of

Implementation of the RCIP scheme and its performance for 1-D age computations in ice-sheet models

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**S1  Comprehensive experiment sheets**

In addition to the experiments presented in the main paper, several experimental configuration combinations of parameters are examined. In this Supplement, some representative results of them are provided. All of the experiments shown are examined under non-steady surface mass balance configurations. Each page contains two figures: the upper one is the result obtained using a square-wave type surface mass balance input, while the lower is one with the same configuration except for applying a cosine-wave type input. One figure contains five sub-figures. (a) profiles of the computed age, (b,d) The computed age differences relative to the result of the RCIP+corr case, and (c,e) the annual layer thickness. The upper sub-figures (a-c) show the profiles in terms of depths, while the lower sub-figures (d-e) show them in terms of the computed ages. The surface mass balance input used in each experiment is shown in the lower left figure for only first 200 or 100 kyr. The prescribed time evolution of ice-thickness is also shown in the lower left figure for the non-steady thickness experiments. The grey and black lines in (a,c,e) correspond to ‘benchmark’ profiles which are computed using constant surface/basal mass balance and thickness.

Figures S1 to S6 are the results of a most standard experiments: constant $H = 3000$ m, $a_H, a_L = 3, 1.5$ cm yr$^{-1}$, $P_T = 100$ kyr, $P_H : P_L = 1 : 1$, and the uniform discretization of 129 levels (see Eq.[57] in the main text about definition of $a_H, a_L, P_T, P_H$ and $P_L$). Basal mass balances are set as constant $M_b = 0$ mm yr$^{-1}$ (S1,S2), $M_b = 0.3$ mm yr$^{-1}$ (S3,S4), and $M_b = 3$ mm yr$^{-1}$ (S5,S6). All of the results are the snapshots at $t = 1$ Myr.

Figures S7 to S12 are the same combination as presented above except for $a_L = 0.75$ cm yr$^{-1}$. Figures S13 to S18 are the same except for $a_L = 0$ cm yr$^{-1}$. Figures S19 to S22 are the same except for $a_L = -15$ cm yr$^{-1}$.

Two types of sensitivity experiments regarding to the shape of prescribed time-evolution of surface mass balance inputs are performed: one type obtained using a different $P_H : P_L$, and the other using a different $P_T$. Figures S23 to S26 are the results obtained using a longer $P_H$, as $P_H : P_L = 7 : 1$, and Figures S27 to S30 are the results using a shorter $P_H$, as $P_H : P_L = 1 : 7$. Figures S31 to S38 are the results obtained using $P_T = 50$ kyr, Figures S39 to S46 using $P_T = 20$ kyr, and Figures S47 to S54 using $P_T = 10$ kyr.

Two types of sensitivity experiments regarding to the shape of prescribed time evolution of ice-thickness are performed: one type with $\tau_H = 10$ kyr (Figs.S55 to S58), and the other with $\tau_H = 3$ kyr (Figs.S59 to S62). (see Eq.[61] in the main text about definition of $\tau_H$).

The series of experiments obtained using constant $H = 3000$ m and $M_b = 0$ mm yr$^{-1}$ are repeated by higher resolution configurations. Figures S63 to S70 are the results of $P_T = 100$ kyr; Figures S71 to S94 are results obtained using a uniform discretization of 513 levels. Figures S95 to S126 are the same results but using a smooth non-uniform discretization of 513 levels. Figures S127 to S158 are the same results but using a non-smooth non-uniform discretization of 477 levels. The snapshots at $t = 2$ Myr are plotted for those experiment with higher resolutions. Designs of these higher resolution discretization are shown in Fig. 14 in the main text.

Furthermore, the series of experiments obtained using constant $H = 3000$ m and $M_b = 0$ mm yr$^{-1}$ are repeated by lower resolution configurations. Figures S159 to S166 are the results of $P_T = 100$ kyr, obtained using a uniform discretization of 33 levels.
Figure S1

- Depth (m) vs. A (kyr)
- Benchmark
  - $M_s = a_h$
  - $M_s = a_l$
  - $M_s = a_m$
  - RCIP
  - UP-2
  - UP-1

- $M_s$: 3 to 1.5 cm/yr and 100 kyr
- $M_b$: 0 mm yr$^{-1}$
- $Z$: 129 levels [uniform $\Delta \zeta$]
- $t = 1$ Myr

Figure S2
Benchmark

RCIP+corr
RCIP
UP-2
UP-1

Figure S3

Figure S4
Figure S7

Figure S8
Figure S9

Figure S10
Figure S11

$M_s$: 3 to 0.75 cm/yr [square 100 kyr 1 : 1] $M_b$: 3 mm yr$^{-1}$ $Z$: 129 levels [uniform $\Delta \zeta$] $t = 1$ Myr

Figure S12

$M_s$: 3 to 0.75 cm/yr [cosine 100 kyr 1 : 1] $M_b$: 3 mm yr$^{-1}$ $Z$: 129 levels [uniform $\Delta \zeta$] $t = 1$ Myr
$M_s: 3$ to $-1.5$ cm/yr $[$cosine 100 kyr $1:1]$  $\Delta \lambda: 0$ mm yr$^{-1}$ $Z: 129$ levels $[$uniform $\Delta \zeta]$  $t = 1$ Myr

$M_s: 3$ to $-1.5$ cm/yr $[$square 100 kyr $1:1]$  $\Delta \lambda: 0$ mm yr$^{-1}$ $Z: 129$ levels $[$uniform $\Delta \zeta]$  $t = 1$ Myr

Figure S19

Figure S20
**Figure S21**

- Depth (m) vs. time (kyr)
- Depth (m) vs. ΔA (kyr)
- Benchmark and data series
- Mₘ: 3 to −1.5 cm/yr
- Mb: 0.3 mm yr⁻¹
- Z: 129 levels
- T = 1 Myr

**Figure S22**

- Depth (m) vs. time (kyr)
- Depth (m) vs. ΔA (kyr)
- Benchmark and data series
- Mₘ: 3 to −1.5 cm/yr
- Mb: 0.3 mm yr⁻¹
- Z: 129 levels
- T = 1 Myr
Figure S23

Figure S24
$M_s$: 3 to 1.5 cm/yr [square 100 kyr 1 : 7]
$M_b$: 0 mm yr$^{-1}$
$Z$: 129 levels [uniform $\Delta \zeta$]
$t = 1$ Myr

**Figure S27**

**Figure S28**
Figure S33

Figure S34
Figure S35

Figure S36
Figure S37

Figure S38
Figure S39

Figure S40
**Figure S41**

- Depth (m) vs. Age (kyr)
- Benchmark: RCIP + corr, RCIP, UP-2, UP-1
- $M_s$: 3 to 0.75 cm/yr
- $M_b$: 0 mm yr$^{-1}$
- $Z$: 129 levels
- $t = 1$ Myr

**Figure S42**

- Depth (m) vs. Age (kyr)
- Benchmark: RCIP + corr, RCIP, UP-2, UP-1
- $M_s$: 3 to 0.75 cm/yr
- $M_b$: 0 mm yr$^{-1}$
- $Z$: 129 levels
- $t = 1$ Myr
Figure S43

Figure S44
Figure S45

Figure S46
Benchmark

Ms = aH
Ms = aL
Ms = aM
RCIP+corr
RCIP
UP−2
UP−1

Ms: 3 to 0.75 cm/yr [cosine 10 kyr : 1]
ΔMs: 0 mm yr−1
Z: 129 levels [uniform Δζ]
t = 1 Myr

Figure S49

Figure S50
**Figure S51**

- **Panel a:** Depth (m) vs. \( A \) (kyr)
- **Panel b:** \( \Delta A \) (kyr) vs. \( \lambda \) (mm)
- **Panel c:** Benchmark values for different models

**Figure S52**

- **Panel a:** Depth (m) vs. \( A \) (kyr)
- **Panel b:** \( \Delta A \) (kyr) vs. \( \lambda \) (mm)
- **Panel c:** Benchmark values for different models

Additional details:
- **Benchmark Models:**
  - \( M_s = a_h \)
  - \( M_s = a_u \)
  - RCIPcorr
  - RCIP
  - UP-2
  - UP-1
- **Parameters:**
  - \( M_s \): 3 to 0 cm/yr
  - \( M_b \): 0 mm yr\(^{-1}\)
  - \( Z \): 129 levels
  - \( \Delta \zeta \)
  - \( t = 1 \) Myr
Benchmark

RCIP
RCIP+corr
UP-2
UP-1

$M_s = a_H$, $H = H_{\text{ref}}(a_H)$

$M_s = a_L$, $H = H_{\text{ref}}(a_L)$

$M_s = a_M$, $H = H(t)$

$\Delta A(kyr)$

$\Delta A(kyr)$

$\lambda (\text{mm})$

$\lambda (\text{mm})$

$M_s: 3 \text{ to } 1.5 \text{ cm/yr}$ [cosine 100 kyr : 1]

$M_b: 0 \text{ mm yr}^{-1}$

$Z: 129 \text{ levels}$ [uniform $\Delta \zeta$]

$\tau_{H}: 10 \text{ kyr}$

$t = 1 \text{ Myr}$

Figure S55

Figure S56

29
Figure S57

Figure S58
Figure S59

Figure S60
**Figure S61**

**Figure S62**
Figure S63

Figure S64
**Figure S69**

**Figure S70**
**Figure S71**

- **a)** Depth (m) vs. time (kyr)
- **b)** Depth (m) vs. ΔA (kyr)
- **c)** Depth (m) vs. λ (mm)
- **d)** Depth (m) vs. A (kyr)
- **e)** Depth (m) vs. λ (mm)

**Figure S72**

- **a)** Depth (m) vs. time (kyr)
- **b)** Depth (m) vs. ΔA (kyr)
- **c)** Depth (m) vs. λ (mm)
- **d)** Depth (m) vs. A (kyr)
- **e)** Depth (m) vs. λ (mm)

Benchmark:
- RCIP+corr
- RCIP
- UP-2
- UP-1

- **Ms**: 3 to 1.5 cm/yr (cosine 50 kyr 1:1)
- **Mb**: 0 mm yr\(^{-1}\)
- **Z**: 513 levels (uniform ∆ζ)
- **t**: 2 Myr
Figure S73

Figure S74
Figure S75

- **a** Depth vs. $A$ (kyr)
- **b** $\Delta A$ vs. kyr
- **c** $\lambda$ vs. mm
- **d** $M_s$ vs. cm yr$^{-1}$
- **e** $Z$ vs. 513 levels

**Benchmark**
- $M_s = a_H$
- $M_s = a_B$
- RCIP
- RCIP+corr
- UP-2
- UP-1

**Figure S76**

- **a** Depth vs. $A$ (kyr)
- **b** $\Delta A$ vs. kyr
- **c** $\lambda$ vs. mm
- **d** $M_s$ vs. cm yr$^{-1}$
- **e** $Z$ vs. 513 levels

**Benchmark**
- $M_s = a_H$
- $M_s = a_B$
- RCIP
- RCIP+corr
- UP-2
- UP-1

$M_s$: 3 to 0 cm/yr [cosine 50 kyr 1:1] $M_u$: 0 mm yr$^{-1}$ $Z$: 513 levels [uniform $\Delta\zeta$] $t = 2$ Myr
Figure S79

Figure S80
Figure S83

Figure S84
**Figure S87**

- **a:** Depth (m) vs. time (kyr)
- **b:** Difference in depth (ΔA) vs. time (kyr)
- **c:** λ (mm) vs. time (kyr)
- **d:** Benchmark comparison
- **e:** Additional data points

**Figure S88**

- **a:** Depth (m) vs. time (kyr)
- **b:** Difference in depth (ΔA) vs. time (kyr)
- **c:** λ (mm) vs. time (kyr)
- **d:** Benchmark comparison
- **e:** Additional data points

**Legend:**
- **Benchmark:**
  - Grey: Ms = aH
  - Black: Ms = aL
  - Orange: RCIP+corr
  - Red: RCIP
  - Pink: UP-2
  - Purple: UP-1

**Note:**
- Ms: 3 to 1.5 cm/yr [cosine 10 kyr 1:1]
- Mh: 0 mm yr^{-1}
- Z: 513 levels [uniform ∆ζ]
- t = 2 Myr
Figure S89

Figure S90
**Figure S91**

- **a** Deposition rate (km yr\(^{-1}\)) as a function of depth (m).
- **b** Change in deposition rate (km yr\(^{-1}\)) over time (kyr).
- **c** Thickness of the sediment layer (mm) as a function of time (kyr).
- **d** Sedimentation rate (cm yr\(^{-1}\)) as a function of time (kyr).
- **e** Radiation (mm) as a function of time (kyr).

**Figure S92**

- **a** Deposition rate (km yr\(^{-1}\)) as a function of depth (m).
- **b** Change in deposition rate (km yr\(^{-1}\)) over time (kyr).
- **c** Thickness of the sediment layer (mm) as a function of time (kyr).
- **d** Sedimentation rate (cm yr\(^{-1}\)) as a function of time (kyr).
- **e** Radiation (mm) as a function of time (kyr).
Figure S93

Figure S94
**Figure S95**

**Figure S96**
Figure S97

Figure S98
Figure S101

Figure S102
**Figure S103**

- **Benchmark**
  - $M_s = a_H$
  - $M_s = a_L$
  - $M_s = a_M$

**Figure S104**

- **Benchmark**
  - $M_s = a_H$
  - $M_s = a_L$
  - $M_s = a_M$

**Caption:**

- $M_s$: 3 to 1.5 cm/yr [cosine 50 kyr 1 : 1]
- $M_s$: 0 mm yr$^{-1}$
- Z: 513 levels [smooth non-uniform $\Delta \zeta$]
- $t = 2$ Myr
**Figure S105**

- **Panel a:** Depth vs. $A$ (kyr)
- **Panel b:** $\Delta A$ (kyr) vs. $\lambda$ (mm)
- **Panel c:** Benchmark
- **Panel d:** $M_s$ (cm yr$^{-1}$)
- **Panel e:** $Z$ (levels)

**Figure S106**

- **Panel a:** Depth vs. $A$ (kyr)
- **Panel b:** $\Delta A$ (kyr) vs. $\lambda$ (mm)
- **Panel c:** Benchmark
- **Panel d:** $M_s$ (cm yr$^{-1}$)
- **Panel e:** $Z$ (levels)
Figure S107

Figure S108
**Figure S109**

- Depth (m) vs. Time (kyr)
- Depth (m) vs. $\Delta A$ (kyr)
- Depth (m) vs. $\lambda$ (mm)

**Figure S110**

- Depth (m) vs. Time (kyr)
- Depth (m) vs. $\Delta A$ (kyr)
- Depth (m) vs. $\lambda$ (mm)

Benchmark:
- $M_s = a_H$
- $M_s = a_U$
- RCIP + corr
- RCIP
- UP-2
- UP-1

$M_s$: 3 to $-1.5$ cm/yr [cosine 50 kyr 1:1]
$M_b$: 0 mm yr$^{-1}$
$Z$: 513 levels [smooth non-uniform $\Delta \zeta$]
$t = 2$ Myr

$M_s$: 3 to $-1.5$ cm/yr [square 50 kyr 1:1]
$M_b$: 0 mm yr$^{-1}$
$Z$: 513 levels [smooth non-uniform $\Delta \zeta$]
$t = 2$ Myr
Figure S111

Figure S112
Figure S113

Figure S114
Figure S115

Figure S116
$M_s \, 3 \text{ to } -1.5 \text{ cm/yr}$ [cosine 20 kyr 1 : 1]  
$M_b \, 0 \text{ mm yr}^{-1}$  
$Z \, 513 \text{ levels [smooth non-uniform } \Delta \zeta \text{]}$  
$t = 2 \text{ Myr}$

Figure S117

Figure S118
Figure S119

Figure S120
Figure S121

Figure S122

$M_s$: 3 to 0.75 cm/yr [cosine 10 kyr 1 : 1]  
$M_b$: 0 mm yr$^{-1}$  
$Z$: 513 levels [smooth non-uniform $\Delta\zeta$]  
$t$ = 2 Myr

Benchmark
- $M_s = a_H$
- $M_s = a_L$
- $M_s = a_M$
- RCIP
- RCIP+corr
- UP-2
- UP-1
Figure S125

Figure S126
Benchmark

- $M_s = a_H$
- $M_s = a_L$
- $M_s = a_M$
- RCIP
- RCIP+corr
- UP-2
- UP-1

**Figure S127**

$M_s$: 3 to 1.5 cm/yr [square 100 kyr 1:1]  
$M_b$: 0 mm yr$^{-1}$  
$Z$: 477 levels [non-smooth $\Delta \xi$]  
$t = 2$ Myr

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**Figure S128**

$M_s$: 3 to 1.5 cm/yr [cosine 100 kyr 1:1]  
$M_b$: 0 mm yr$^{-1}$  
$Z$: 477 levels [non-smooth $\Delta \xi$]  
$t = 2$ Myr
Figure S129

Figure S130
Figure S131

Figure S132
Figure S133

Figure S134
Figure S135

Figure S136
Figure S137

Figure S138
Figure S139

Figure S140
**Figure S141**

- **Benchmark**: Various models are represented, including RCIP, RCIP+corr, UP-2, and UP-1.
- **M_s**: The model parameters are shown with a range of values from 3 to −1.5 cm/yr (cosine 50 kyr 1 : 1).
- **M_b**: 0 mm yr⁻¹.
- **Z**: 477 levels [non-smooth ∆ζ].
- **t**: 2 Myr.

**Figure S142**

- Similar to Figure S141, with additional details and parameters.
- **M_s**: The model parameters are shown with a range of values from 3 to −1.5 cm/yr (cosine 50 kyr 1 : 1).
- **M_b**: 0 mm yr⁻¹.
- **Z**: 477 levels [non-smooth ∆ζ].
- **t**: 2 Myr.
Figure S143

Figure S144
Figure S147

Figure S148

\( M_s: 3 \text{ to } 0 \text{ cm/yr} \) (cosine 20 kyr 1 : 1) \( M_b: 0 \text{ mm yr}^{-1} \) \( Z: 477 \text{ levels [non-smooth } \Delta \zeta \text{]} \) \( t = 2 \text{ Myr} \)
Figure S149

Figure S150
Figure S153

Figure S154
Figure S155

Figure S156
Figure S157

Figure S158
Figure S159

Figure S160
Figure S161

Figure S162
**Figure S163**

- **Depth (m)** vs. **A (kyr)**
- **Benchmark**:
  - $M_s = a_H$
  - $M_s = a_M$
  - RCIP+corr
  - RCIP
  - UP-2
  - UP-1

- **M_s (cm yr$^{-1}$)**
- **A (kyr)**
- **$\Delta A$ (kyr)**
- **$\lambda$ (mm)**

**Figure S164**

- **Depth (m)** vs. **A (kyr)**
- **Benchmark**:
  - $M_s = a_H$
  - $M_s = a_M$
  - RCIP+corr
  - RCIP
  - UP-2
  - UP-1

- **M_s (cm yr$^{-1}$)**
- **A (kyr)**
- **$\Delta A$ (kyr)**
- **$\lambda$ (mm)**

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$M_s$: 3 to 0 cm/yr [cosine 100 kyr 1 : 1]  
$M_b$: 0 mm yr$^{-1}$  
$Z$: 33 levels [uniform $\Delta \zeta$]  
$t = 1$ Myr
Benchmark

- $M_s = a_H$
- $M_s = a_M$
- RCIP + corr
- RCIP
- UP-2
- UP-1

Ms: 3 to $-1.5$ cm/yr [square 100 kyr 1 : 1]

Mb: 0 mm yr$^{-1}$

Z: 33 levels [uniform $\Delta \zeta$]

t = 1 Myr

Figure S165

Ms (cm yr$^{-1}$)

M(s cm yr$^{-1}$)

$\lambda$ (mm)

0

Figure S166