

Text S1 The approximation of A at daily to annual scales using a two-step approach

We conduct an experiment with the ASS configuration of prior uncertainty where the inversion period and domain are limited to 6 months and to the Benelux, a region with high emission density and in which the 95 emission clumps are close to each other (Fig. S1a). It is reasonable to assume that if the approximation of the posterior uncertainty of emissions from clumps within this region (because we ignore the filtering of information from different spatial overlaps of plumes on different days, see the method) is good, clumps outside this inversion domain will have very marginal impact on the results for the clumps in Benelux. In this case, the full A can be explicitly derived based on Eq. (1) in the main text. We compare this exact computation of the full A (Inv-fullA) to that obtained with the approach we proposed (Inv-2step). Figure S1b shows the posterior uncertainties in the emission budgets over individual time windows 8:30-11:30 for an exemplary clump (Antwerp) from the two computations. The results from the two computations are very close, except for very few days, and the aggregated uncertainty in emission budget for the whole period differ by less than 0.1%. This confirms that our method provides a good approximation of A at daily to annual scales for individual clumps with reasonable accuracy.

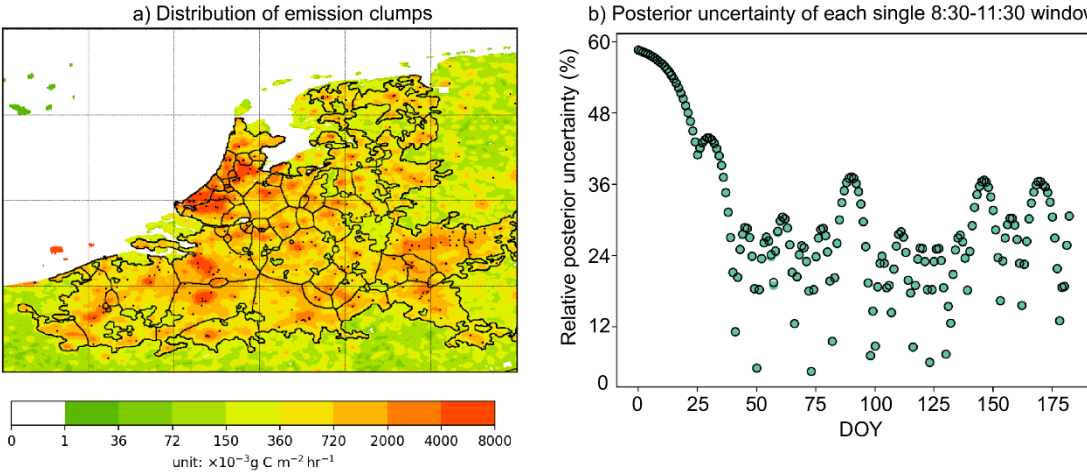


Figure S1 a) Distribution of emission clumps in the Benelux region that we account for in the Inv-fullA and Inv-2step inversions. The solid lines depict the boundaries of clumps. b) Posterior uncertainty of each single 8:30-11:30 window for Antwerp clump during the first half of the year. The green dots are the results from Inv-fullA, and the circles are the results from Inv-2step.

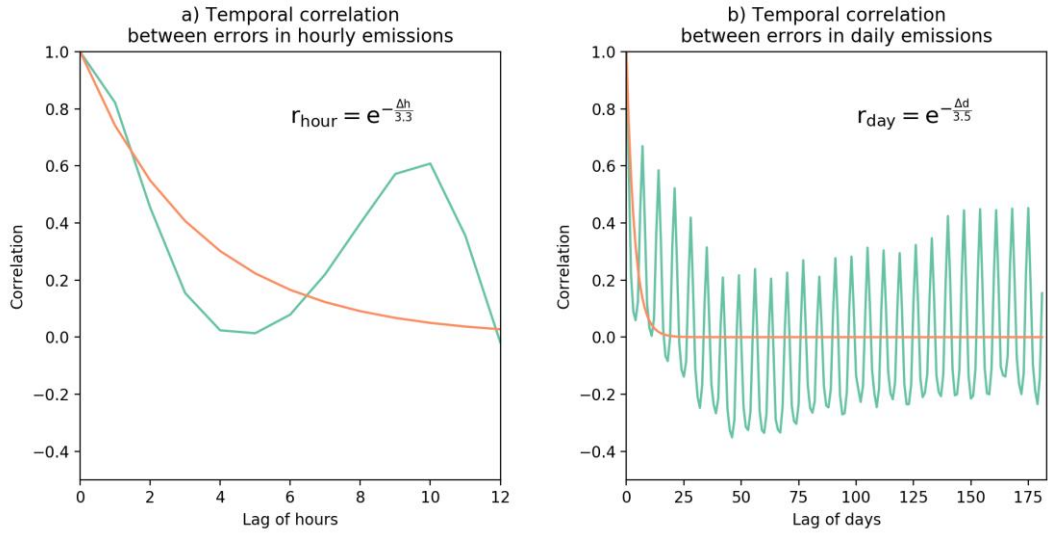


Figure S2 Temporal auto-correlation between errors in hourly emissions (a) and between errors in daily emissions (b) for the transport sector. The modelled temporal profile of the emissions from TIMES product is compared to the TomTom traffic index for Paris, assuming TomTom traffic index is a perfect proxy for the transport emissions. Green lines are the computed temporal auto-correlation, and red lines are the lines fitted with an exponential function (at the figure top).

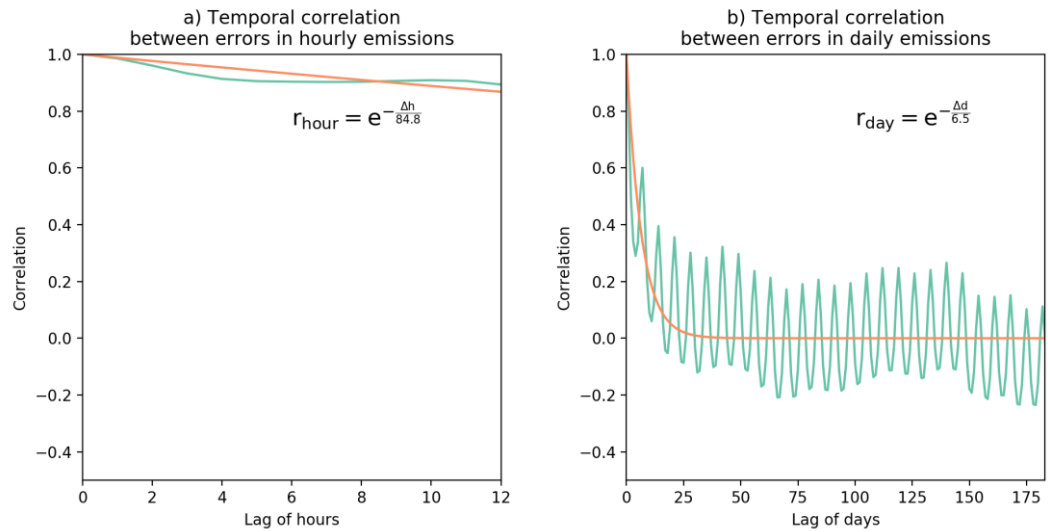


Figure S3 Temporal auto-correlation between errors in hourly emissions (a) and between errors in daily emissions (b) for the energy production. The modelled temporal profile of the emissions from TIMES product is compared to the actual CO₂ emissions from electricity production in France. Green lines are the computed temporal auto-correlation, and red lines are the lines fitted with an exponential function (at the figure top).

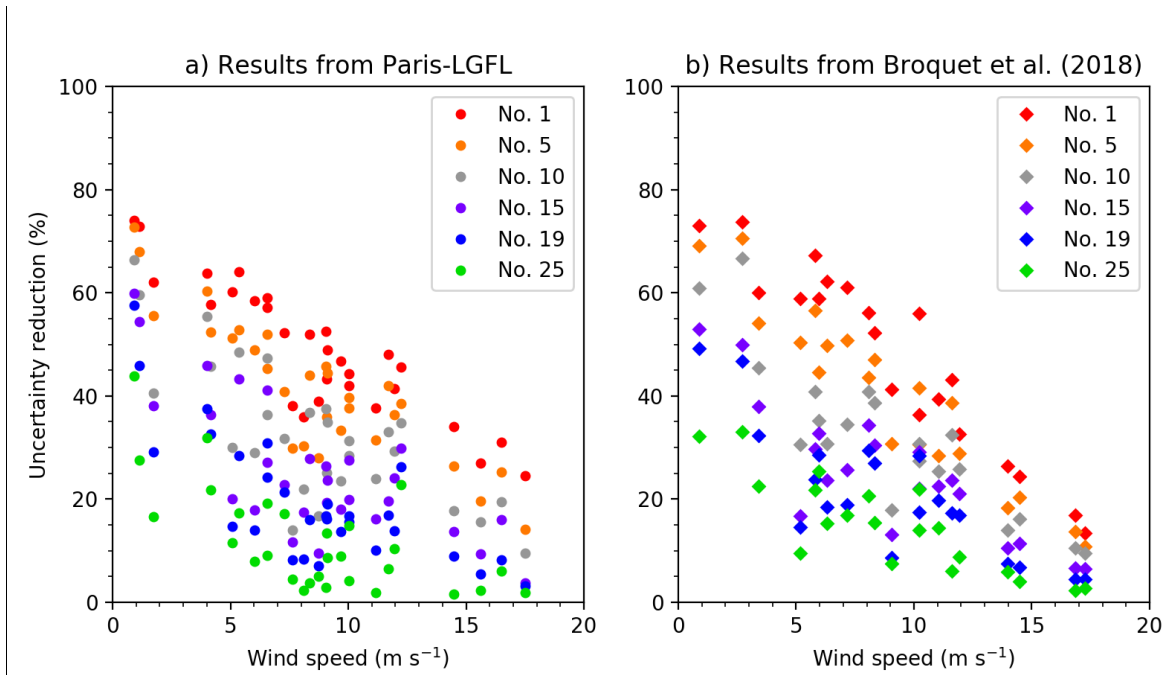


Figure S4 Theoretical uncertainty reduction for the 6 h mean emissions using the 1st (red), 5th (orange), 10th (light green), 15th (purple), 19th (blue) and 25th best observation sampling from the CarbonSat simulation. a) Results are obtained in the PMIF-Paris experiments using the PMIF system. b) Results from Broquet et al. (2018). Fig. S3b is adapted from Fig. 6 in Broquet et al. (2018), Copernicus Publications.

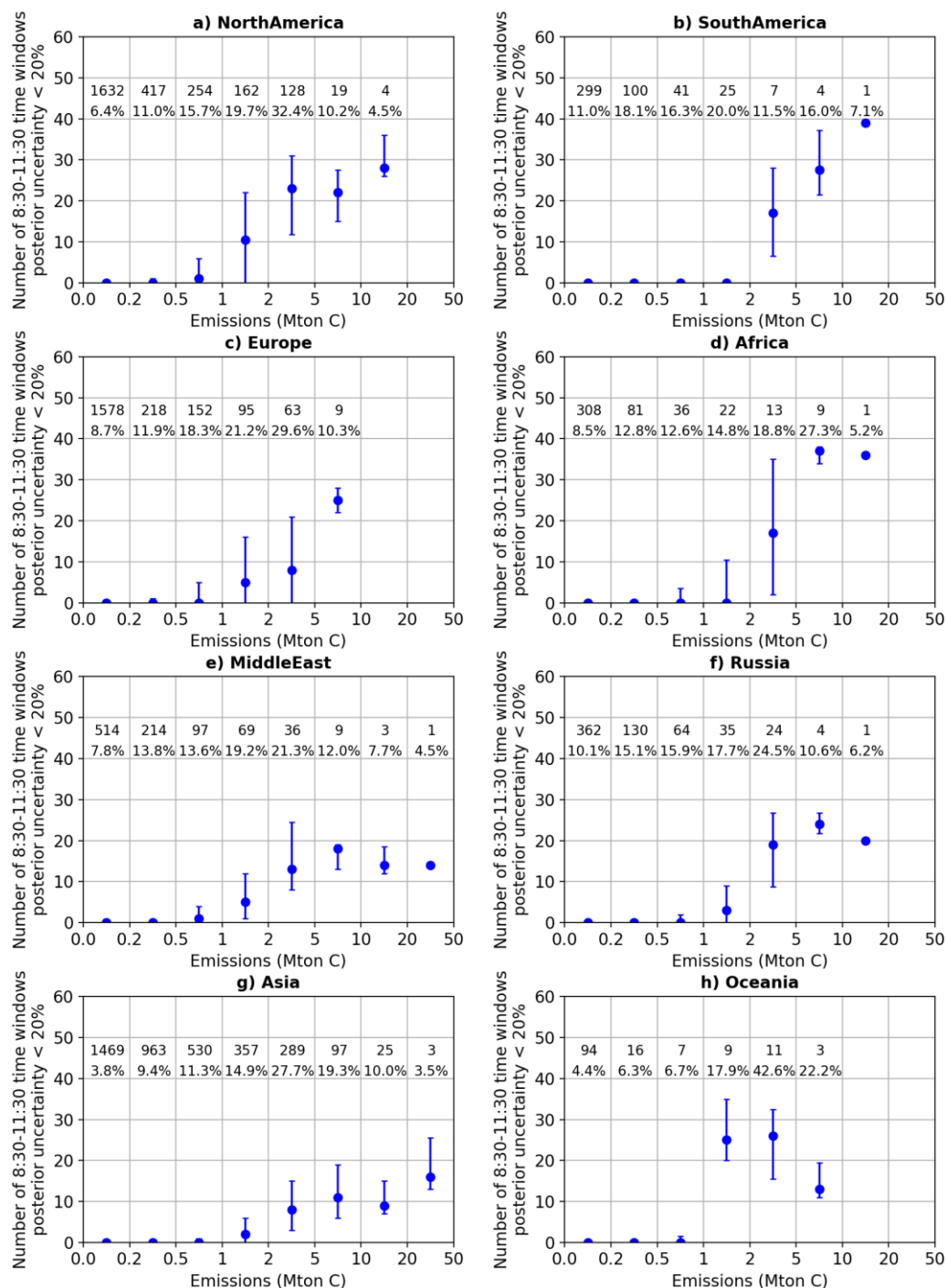


Figure S5 Same as Figure 2, but where the results are distributed per regions over the globe.

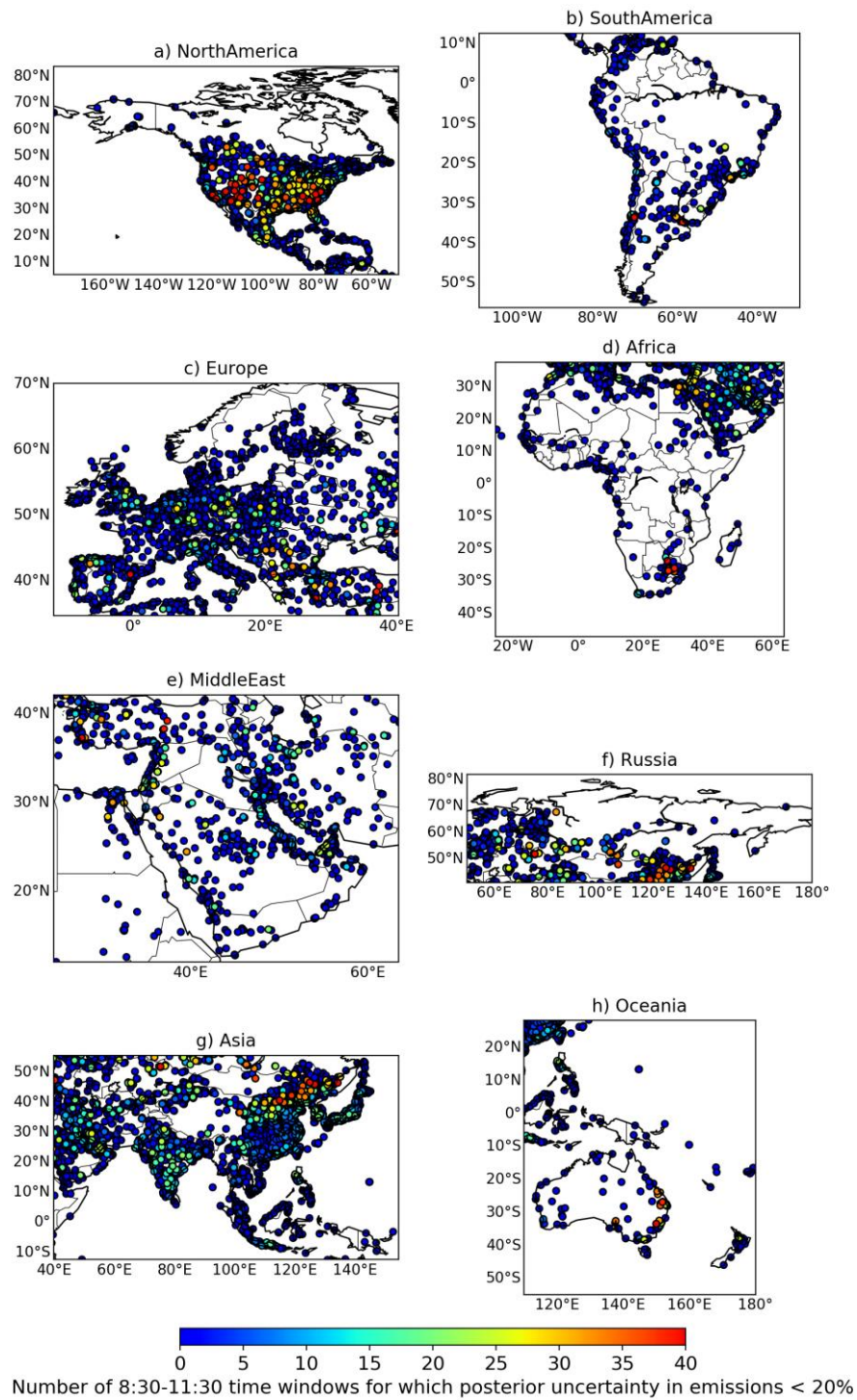


Figure S6 N20 values in different regions over the globe.

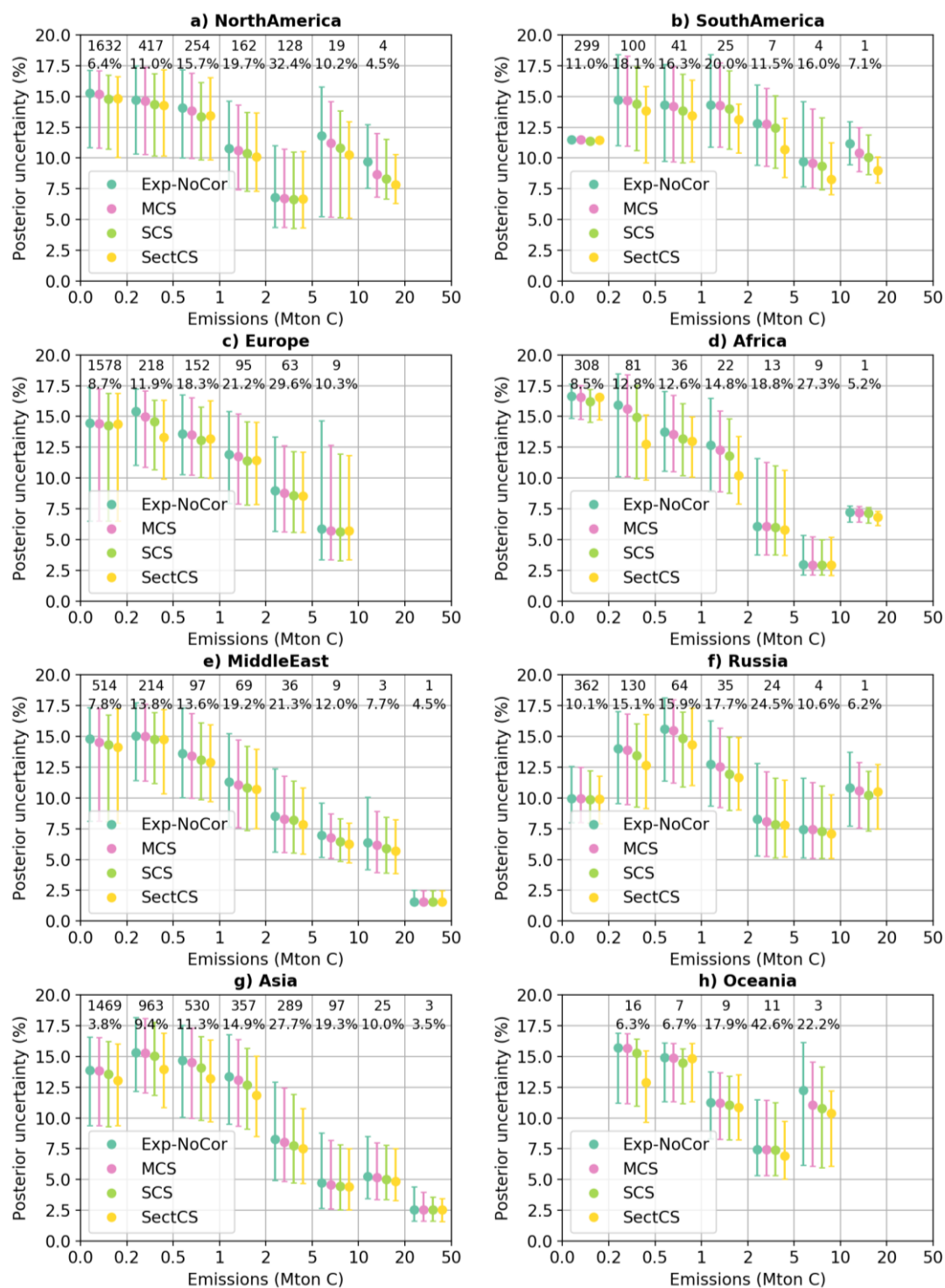


Figure S7 Same as Figure 3, but where the results are distributed per regions over the globe.

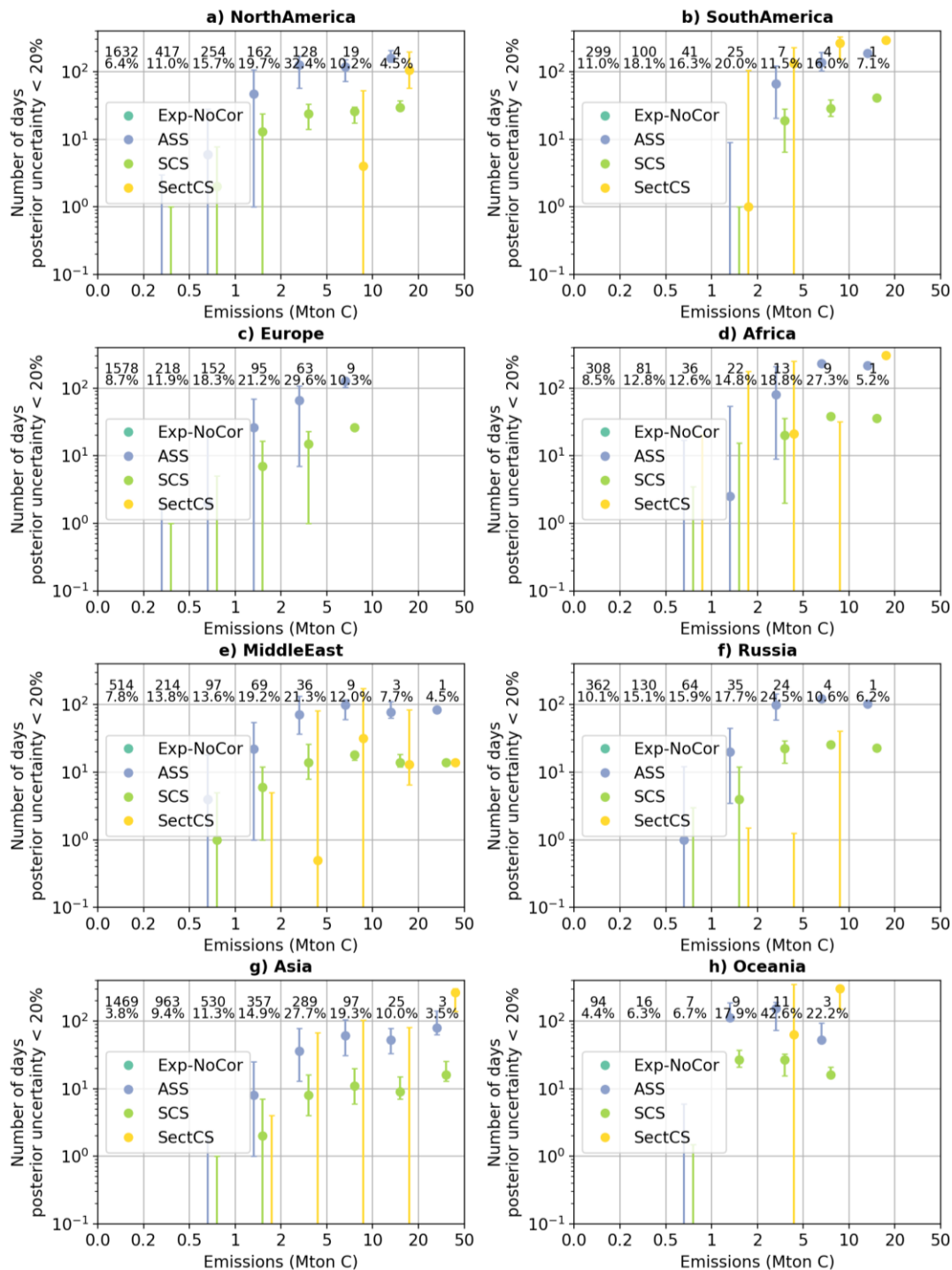


Figure S8 Same as Figure 4, but where the results are distributed per regions over the globe.