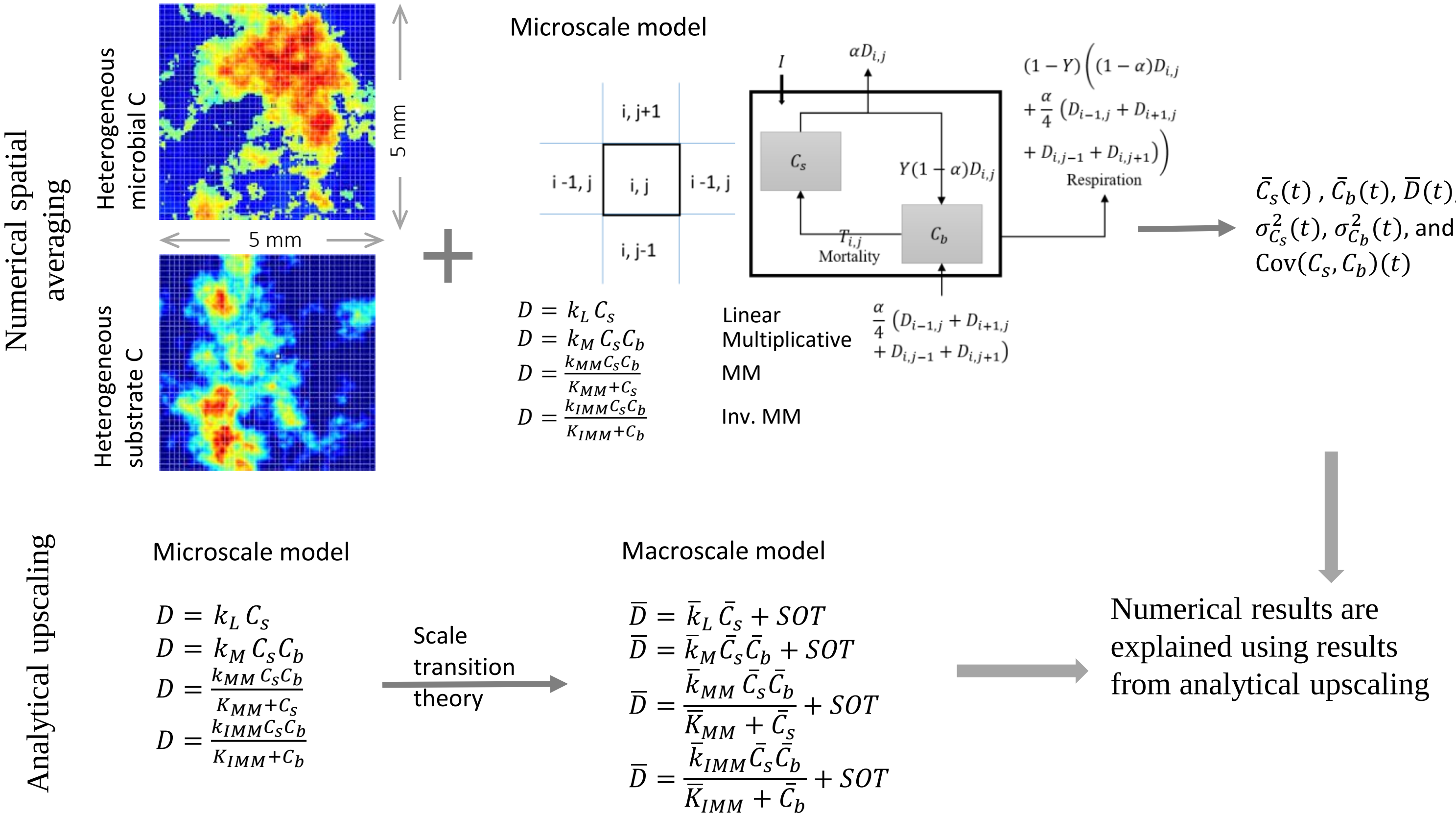

Numerical spatial averaging
Heterogeneous microbial C
Heterogeneous substrate C
5 mm
5 mm
Microscale model
i, j+1
i -1, j
i, j
i -1, j
i, j-1
I
$\alpha D_{i,j}$
C_s
C_b
$Y(1-\alpha)D_{i,j}$
$T_{i,j}$
Mortality
$(1-Y)\left((1-\alpha)D_{i,j} + \frac{\alpha}{4}\left(D_{i-1,j} + D_{i+1,j} + D_{i,j-1} + D_{i,j+1}\right)\right)$
Respiration
$\frac{\alpha}{4}\left(D_{i-1,j} + D_{i+1,j} + D_{i,j-1} + D_{i,j+1}\right)$
$D = k_L C_s$ Linear
$D = k_M C_s C_b$ Multiplicative
$D = \frac{k_{MM} C_s C_b}{K_{MM} + C_s}$ MM
$D = \frac{k_{IMM} C_s C_b}{K_{IMM} + C_b}$ Inv. MM
$\bar{C}_s(t)$, $\bar{C}_b(t)$, $\bar{D}(t)$, $\sigma^2_{C_s}(t)$, $\sigma^2_{C_b}(t)$, and $\mathrm{Cov}(C_s, C_b)(t)$
Analytical upscaling
Microscale model
$D = k_L C_s$
$D = k_M C_s C_b$
$D = \frac{k_{MM} C_s C_b}{K_{MM} + C_s}$
$D = \frac{k_{IMM} C_s C_b}{K_{IMM} + C_b}$
Scale transition theory
Macroscale model
$\bar{D} = \bar{k}_L \bar{C}_s + SOT$
$\bar{D} = \bar{k}_M \bar{C}_s \bar{C}_b + SOT$
$\bar{D} = \frac{\bar{k}_{MM} \bar{C}_s \bar{C}_b}{\bar{K}_{MM} + \bar{C}_s} + SOT$
$\bar{D} = \frac{\bar{k}_{IMM} \bar{C}_s \bar{C}_b}{\bar{K}_{IMM} + \bar{C}_b} + SOT$
Numerical results are explained using results from analytical upscaling