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Significant improvement of cloud representation in the global climate model MRI-ESM2

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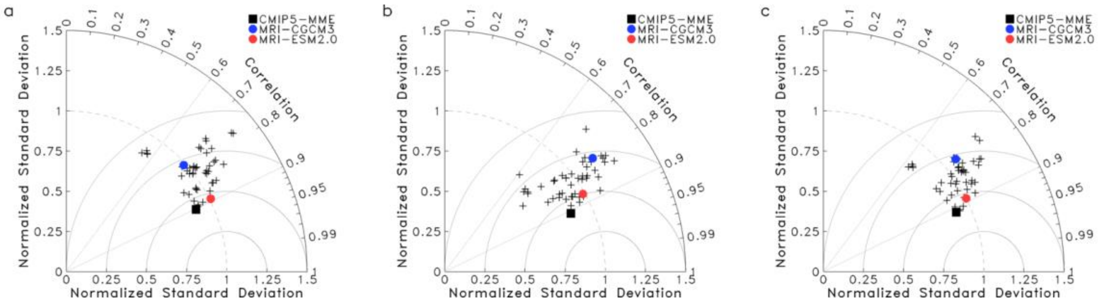

Figure S1:

Same as Fig. 2, but for cloud radiative effects: (a) shortwave, (b) longwave, and (c) net cloud radiative effects.

S. Derivation of velocities of falling cloud ice

S1 Terminal velocity of cloud ice particles and mass equivalent diameter

The mass equivalent diameter D_m (cm)[1] is defined as follows using mass M (g):

$$M = \rho_{\rm ice} \frac{\pi}{6} {D_m}^3 \tag{S1}$$

where $\rho_{\rm ice}$ (g cm^{-3}) is the density of ice and $\rho_{\rm ice}$ = 0.91 (g cm^{-3}) is used here.

On the other hand, Heymsfield and Iaquinta (2000) proposed the mass of each cloud ice particle M (g) and the terminal velocity V_t (cm s^{-1}) as functions of the maximum length D_L (cm), with $M(D_L)$ given by:

$$M(D_L) = \alpha {D_L}^{\beta} \tag{S2}$$

Values of α and β vary depending on the crystal shape of cloud ice. Columns for $D_m < 0.01$ (cm) and bullet rosettes for $D_m > 0.01$ (cm) are assumed in this study in accordance with Zurovac-Jevtic and Zhang (2003). Based on the above assumptions, the values for columns $\alpha = 1.649 \times 10^{-3}$ and $\beta = 2.20$ are used for $D_m < 0.01$ (cm), and the values for bullet rosettes $\alpha = 3.99 \times 10^{-4} n_b$ and $\beta = 2.27$ are used for $D_m > 0.01$ (cm) (Zurovac-Jevtic and Zhang 2003) where n_b is the number of bullets in a rosette and $n_b = 2$ is assumed here. Then $V_t(D_L)$ is given by:

$$V_t(D_L) = x {D_L}^{y} \tag{S3}$$

Though x and y vary depending on the shapes of cloud ice, the values for columns $x = 3086$ and $y = 1.26$ are used for $D_m < 0.01$ (cm), and the values for bullet rosettes $x = 492$ and $y = 0.70$ are used for $D_m > 0.01$ (cm) under the same assumptions as mentioned above (Zurovac-Jevtic and Zhang, 2003).

Eq. (S3) can be written as follows using Eq. (S1) and Eq. (S2):

[1] In Supplement A, the CGS system of units is used in the derivation in order to refer directly to the original papers. However, as an exception we use the unit kg m^{-3} for ice water content IWC.

$$V_t(D_m) = x\left(\frac{\pi\rho_{\mathrm{ice}}{D_m}^3}{6\alpha}\right)^{\frac{y}{\beta}} \tag{S4}$$

S2 Size-distribution function of cloud ice

McFarquhar and Heymsfield (1997; hereafter MH97) derived a number distribution function $N(D_m)$ for particles with mass-equivalent diameter D_m based on observations of cirrus. This subsection is a brief extract from MH97.

$N(D_m)$ for $D_m < 0.01$ (cm) is as follows:

$$N(D_m) = \frac{6\mathrm{IWC_{all}}{\alpha_{<100}}^5 D_m}{\pi\rho_{\mathrm{ice}}\Gamma(5)}\exp(-\alpha_{<100}D_m) \tag{S5}$$

where Γ is the gamma function and $\mathrm{IWC_{all}}$ is ice water content (kg m^{-3}) for the whole cloud ice. $\alpha_{<100}$ is a parameter that determines the shape of the distribution, and it can be represented as follows using ice water content with size smaller than 0.01cm, $\mathrm{IWC_{<100}}$ (kg m^{-3}).

$$\alpha_{<100} = b_\alpha - m_\alpha \log_{10}\left(\frac{\mathrm{IWC_{<100}}}{\mathrm{IWC_0}}\right) \tag{S6}$$

where $b_\alpha = -49.9 \pm 55.0$ (cm^{-1}), $m_\alpha = 494 \pm 29$ (cm^{-1}) and $\mathrm{IWC_0} = 1 \times 10^{-3}$ (kg m^{-3}). These equations mean that the peak of the distribution moves toward a larger value of D_m with increasing ice water content with size smaller than 0.01cm, $\mathrm{IWC_{<100}}$

For $D_m > 0.01$ (cm), the distribution is as follows:

$$N(D_m) = \frac{6\mathrm{IWC_{all}}}{\pi^{\frac{3}{2}}\rho_{\mathrm{ice}}\sqrt{2}\exp\left(3\mu_{>100}+\frac{9}{2}{\sigma_{>100}}^2\right)D_m\sigma_{>100}{D_0}^3}\exp\left[-\frac{1}{2}\left(\frac{\log\frac{D_m}{D_0}-\mu_{>100}}{\sigma_{>100}}\right)^2\right] \tag{S7}$$

where $D_0 = 1 \times 10^{-4}$ (cm), and $\mu_{>100}$ and $\sigma_{>100}$ are parameters that determine the peak and the width of the distribution, respectively. These are given as follows in terms of ice water content with size larger than 0.01cm, $\mathrm{IWC_{>100}}$ (kg m^{-3}).

$$\mu_{>100} = a_\mu(T) + b_\mu(T)\log_{10}\left(\frac{\mathrm{IWC_{>100}}}{\mathrm{IWC_0}}\right) \tag{S8}$$

$$\sigma_{>100} = a_\sigma(T) + b_\sigma(T)\log_{10}\left(\frac{\mathrm{IWC_{>100}}}{\mathrm{IWC_0}}\right) \tag{S9}$$

where $a_{\mu}(T)$, $b_{\mu}(T)$ and $a_{\sigma}(T)$, $b_{\sigma}(T)$ are constants that depend on temperature. In this study, $a_{\mu} = 5.148$, $b_{\mu} = 0.089$, $a_{\sigma} = 0.396$ and $b_{\sigma} = 0.044$ are adopted (values for −50ºC < T < −40 ºC were chosen as representative values for cirrus ice). These equations mean that the peak of the distribution moves toward a larger value of D_m and the width of the distribution expands as ice water content whose size is larger than 0.01cm, $IWC_{>100}$, becomes greater.

On the other hand, $IWC_{<100}$ can be calculated as follows:

$$\mathrm{IWC}_{<100} = \min\left[\mathrm{IWC}_{\mathrm{T}}, a\left(\frac{\mathrm{IWC}_{\mathrm{T}}}{\mathrm{IWC}_{0}}\right)^{b}\right] \quad \text{(S10)}$$

where a and b are constants taken as 2.52×10^{-4} (kg m^{-3}) and 0.837, respectively, and IWC_T is ice water content (kg m^{-3}) for the whole cloud ice. $IWC_{>100}$ can be calculated from the relation $IWC_T = IWC_{<100} + IWC_{>100}$. The term α_i in the text refers to the ratio $IWC_{<100} / IWC_T$; this α_i is shown in Fig. S2 in this supplement.

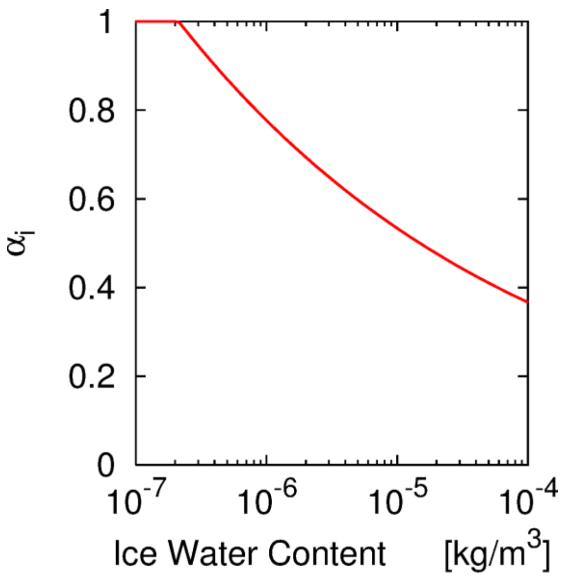

Figure S2:

Ratio of particles smaller than 100 μm to total ice water content, α_i (MH97). The abscissa shows total ice water content IWC_T (kg m^{-3}).

S3 Velocities of falling cloud ice

The procedure in the following derivation is similar to Zurovac-Jevtic and Zhang (2003). Although they derived one velocity, two separate velocities that correspond to small and large particles are derived here.

S3.1 Fall velocity for ice particles smaller than 100 μm

Bulk fall velocity for cloud ice particles smaller than 0.01cm, v_i, was derived as follows by integration using Eq. (S4) and Eq. (S5).

$$v_i = \int_0^{0.01\mathrm{cm}} V_t(D_m) N(D_m)\, \rho_{\mathrm{ice}} \frac{\pi}{6} {D_m}^3 dD_m / \mathrm{IWC}_{<100}$$

$$= \frac{\mathrm{IWC}_{\mathrm{all}}}{\Gamma(5)} x \left(\frac{\pi \rho_{\mathrm{ice}}}{6\alpha {\alpha_{<100}}^3}\right)^{\frac{y}{\beta}} \int_0^{0.01\alpha_{<100}} t^{4+\frac{3y}{\beta}} \exp(-t) dt / \mathrm{IWC}_{<100} \qquad \text{(S11)}$$

On the other hand, the relation between $\mathrm{IWC}_{<100}$ and $\mathrm{IWC}_{\mathrm{all}}$ is as follows ($t \equiv \alpha_{<100} D_m$), using Eq. (S5):

$$\mathrm{IWC}_{<100} = \int_0^{0.01\mathrm{cm}} N(D_m)\, \rho_{\mathrm{ice}} \frac{\pi}{6} {D_m}^3 dD_m$$

$$= \frac{\mathrm{IWC}_{\mathrm{all}}}{\Gamma(5)} \int_0^{0.01\alpha_{<100}} t^4 \exp(-t) dt \qquad \text{(S12)}$$

This relationship should be used because the number distribution function for D_m < 0.01 (cm) still has a non-negligible value in the region where D_m is larger than 0.01 cm. Using Eq. (S11) and Eq. (S12), v_i can be written as follows:

$$v_i = x \left(\frac{\pi \rho_{\mathrm{ice}}}{6\alpha {\alpha_{<100}}^3}\right)^{\frac{y}{\beta}} \frac{\int_0^{0.01\alpha_{<100}} t^{4+\frac{3y}{\beta}} \exp(-t) dt}{\int_0^{0.01\alpha_{<100}} t^4 \exp(-t) dt} \qquad \text{(S13)}$$

Because $\alpha_{<100}$ given by Eq. (S6) is a function of $\mathrm{IWC}_{<100}$, the ratio of the two integrations in Eq. (S13) can be derived as a function of $\mathrm{IWC}_{<100}$ numerically. It was fitted by $\mathrm{IWC}_{<100}$ as follows:

$$\frac{\int_0^{0.01\alpha_{<100}} t^{4+\frac{3y}{\beta}} \exp(-t)dt}{\int_0^{0.01\alpha_{<100}} t^4 \exp(-t)dt} = 17.86 \exp\left(-1.211 \times 10^4 \mathrm{IWC}_{<100} + 3.64 \times 10^7 {\mathrm{IWC}_{<100}}^2\right)$$

Then Eq. (S13) results in the following:

$$v_i = 17.86x \left(\frac{\pi\rho_{\mathrm{ice}}}{6\alpha {\alpha_{<100}}^3}\right)^{\frac{y}{\beta}} \exp\left(-1.211 \times 10^4 \mathrm{IWC}_{<100} + 3.64 \times 10^7 {\mathrm{IWC}_{<100}}^2\right)$$

The final form was derived by fitting the above equation by $\mathrm{IWC}_{<100}$ again (note that the unit of v_i is (m s^{-1}) in this final form).

$$v_i = 1.56 {\mathrm{IWC}_{<100}}^{0.24}$$

S3.2 Fall velocity for ice particles larger than 100 μm

Bulk fall velocity for cloud ice particles larger than 0.01 cm, v_s, was derived as follows by integration using Eq. (S4) and Eq. (S7). The function was integrated over the whole size range because the number distribution function has a value small enough in the range smaller than 0.01 cm and the contribution of that part to the derived velocity is insignificant. In this case, the integration simplifies as:

$$v_s = \int_{0.01\mathrm{cm}}^{\infty} V_t(D_m) N(D_m)\, \rho_{\mathrm{ice}} \frac{\pi}{6} {D_m}^3 dD_m / \mathrm{IWC}_{\mathrm{all}}$$

$$\approx \int_0^{\infty} V_t(D_m) N(D_m)\, \rho_{\mathrm{ice}} \frac{\pi}{6} {D_m}^3 dD_m / \mathrm{IWC}_{\mathrm{all}}$$

$$= x \left(\frac{\pi\rho_{\mathrm{ice}}}{6\alpha}\right)^{\frac{y}{\beta}} \left(D_0 e^{\left(\mu_{>100} + \frac{3}{2}\left(2 + \frac{y}{\beta}\right) {\sigma_{>100}}^2\right)}\right)^{3\frac{y}{\beta}}$$

The equation above can be modified as follows by substituting Eq. (S8) and Eq. (S9), and then by fitting using $\mathrm{IWC}_{>100}$ (note that the unit of v_s is (m s^{-1}) in this final form):

$$v_s = 2.23 {\mathrm{IWC}_{>100}}^{0.074}$$