



Supplement of

SUPECA kinetics for scaling redox reactions in networks of mixed substrates and consumers and an example application to aerobic soil respiration

Jin-Yun Tang and William J. Riley

Correspondence to: Jin-Yun Tang (jinyuntang@lbl.gov)

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Below we provide auxiliary information for the numerical benchmark of the SUPECA kinetics.

The equations for fixed-point iteration are

$$\begin{bmatrix} S_1 \end{bmatrix}_{new} = \begin{bmatrix} S_1 \end{bmatrix}_T \left(1 + \frac{\begin{bmatrix} M \end{bmatrix}_{old}}{K_{MS1}} + \frac{k_{BS1} \begin{bmatrix} B \end{bmatrix}_{old}}{k_{BS2} \begin{bmatrix} S_2 \end{bmatrix}_{old}} + \frac{k_{BS2}}{k_2^+} \begin{bmatrix} B \end{bmatrix}_{old} \frac{\begin{bmatrix} S_2 \end{bmatrix}_{old}}{\begin{bmatrix} S_1 \end{bmatrix}_{old}} + \frac{k_{BS1}}{k_2^+} \begin{bmatrix} B \end{bmatrix}_{old} \right)^{-1}$$
(S-1)

$$\begin{bmatrix} S_2 \end{bmatrix}_{new} = \begin{bmatrix} S_2 \end{bmatrix}_T \left(1 + \frac{k_{BS2} \begin{bmatrix} B \end{bmatrix}_{old}}{k_{BS1} \begin{bmatrix} S_1 \end{bmatrix}_{old}} + \frac{k_{BS2}}{k_2^+} \begin{bmatrix} B \end{bmatrix}_{old} + \frac{k_{BS1}}{k_2^+} \begin{bmatrix} B \end{bmatrix}_{old} \frac{\begin{bmatrix} S_1 \end{bmatrix}_{old}}{\begin{bmatrix} S_2 \end{bmatrix}_{old}} \right)^{-1}$$
(S-2)

$$\begin{bmatrix} B \end{bmatrix}_{new} = \begin{bmatrix} B \end{bmatrix}_{T} \left(1 + \frac{k_{BS1} \begin{bmatrix} S_1 \end{bmatrix}_{old}}{k_{BS2} \begin{bmatrix} S_2 \end{bmatrix}_{old}} + \frac{k_{BS2} \begin{bmatrix} S_2 \end{bmatrix}_{old}}{k_{BS1} \begin{bmatrix} S_1 \end{bmatrix}_{old}} + \frac{k_{BS2} \begin{bmatrix} S_2 \end{bmatrix}_{old}}{k_2^+} \begin{bmatrix} S_2 \end{bmatrix}_{old} + \frac{k_{BS1} \begin{bmatrix} S_1 \end{bmatrix}_{old}}{k_2^+} \begin{bmatrix} S_2 \end{bmatrix}_{old} + \frac{k_{BS1} \begin{bmatrix} S_1 \end{bmatrix}_{old}}{k_2^+} \begin{bmatrix} S_2 \end{bmatrix}_{old}$$
(S-3)

$$\begin{bmatrix} M \end{bmatrix}_{new} = \begin{bmatrix} M \end{bmatrix}_T \left(1 + \frac{\begin{bmatrix} S_1 \end{bmatrix}_{old}}{K_{MS1}} \right)^{-1}$$
(S-4)

The iteration starts with initial condition $[S_1]_{old} = [S_1]_T$, $[S_2]_{old} = [S_2]_T$, $[B]_{old} = [B]_T$, and $[M]_{old} = [M]_T$. The iteration stops when the relative change between two consecutive iterations is smaller than 10⁻⁴.

Supplementary figure

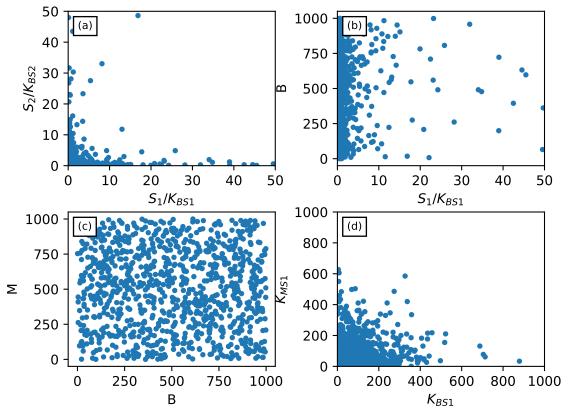


Figure S1. Scatter plots of the parameters used in the numerical benchmark of the SUPECA kinetics (which is Figure 3 in the main text).

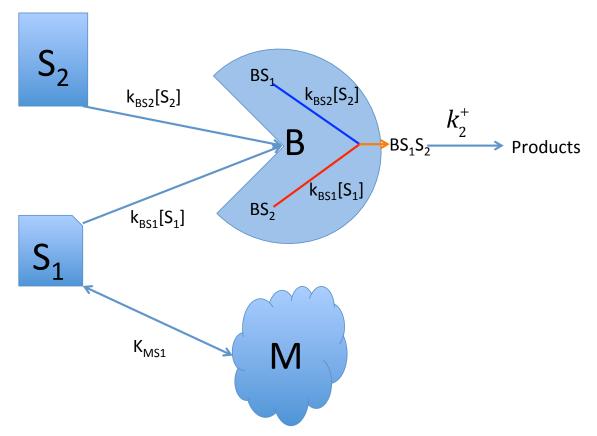


Figure S2. Graphic representation of the reaction network that is used for evaluating the numerical accuracy of SU and SUPECA. The AB-E reaction is of the form

 $S_1 + S_2 \xrightarrow{B} BS_1 S_2 \xrightarrow{k_2^+} B$ + Products, and S_1 also interacts with sorbent M through the

reaction $S_1 + M \leftrightarrow MS_1$. The equilibrium chemistry formulation of the problem is in Appendix F of the main text.