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Supplement of

Exploring precipitation pattern scaling methodologies and robustness among CMIP5 models

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1 Interpolation and Extrapolation in Time

Here we reference Supplemental Figures 1-3. Discussion of these figures is included in Section 3.2 of the main paper.

2 Comparison of Pattern Scaling Between Two Groups of Models

Supplemental Figure 4 further supports the findings in Section 2 of the main paper by showing that the patterns $P(\mathbf{x})$ are not statistically different for Groups 1 and 2 except for isolated areas. Supplemental Figures 5 and 6 show differences in the reconstructions, averaged over years 116–140. More specifically, Supplemental Figure 5 shows differences $P_{\text{Group 2}}\Delta\bar{T}_{\text{Group 1}} - \Delta B_{\text{Group 2}} = P_{\text{Group 2}}\Delta\bar{T}_{\text{Group 1}} - P_{\text{Group 2}}\Delta\bar{T}_{\text{Group 2}} = P_{\text{Group 2}}(\Delta\bar{T}_{\text{Group 1}} - \Delta\bar{T}_{\text{Group 2}})$ and Supplemental Figure 6 shows differences $P_{\text{Group 1}}\Delta\bar{T}_{\text{Group 2}} - \Delta B_{\text{Group 2}} = P_{\text{Group 1}}\Delta\bar{T}_{\text{Group 2}} - P_{\text{Group 2}}\Delta\bar{T}_{\text{Group 2}} = \Delta\bar{T}_{\text{Group 2}}$ $(P_{\text{Group 1}} - P_{\text{Group 2}})$.

The results in Supplemental Figures 5 and 6 have qualitatively more error than the results in Figure 3 in the main paper, but Supplemental Figure 5 has substantially more error than Supplemental Figure 6. This shows that errors introduced by differences in $\Delta\bar{T}$ among the two groups are larger than errors introduced by differences in P among the two groups. As discussed in Section 3.2 in the main paper, practically no region is statistically significant for the regression and epoch difference methods in Supplemental Figures 5 and 6.

3 Pattern Scaling for Non-CO_2 Forcings

In Section 4 of the main portion of the paper, we discuss splitting the RCP8.5 scaling into a CO_2 portion and a non-CO_2 portion. We also discussed why we chose not to split the RCP8.5 scaling into a greenhouse gas and non-greenhouse gas portion. Here we provide more details on the rationale for that choice.

To perform this scaling, we begin with a restatement of Equation 1 in the main paper:

$$\Delta B(\mathbf{x},t) \approx \Delta\hat{B}(\mathbf{x},t) = P(\mathbf{x})\Delta\bar{T}(t) \tag{1}$$

where $P(\mathbf{x})$ describes a time-invariant spatial pattern (the spatial dimension is denoted by $\mathbf{x}$), and $\Delta\bar{T}(t)$ describes a time-varying (the time dimension is denoted by t) series of the change in global mean temperature, starting from a reference period $t = 0$ (often the preindustrial era).

In the first case, we split ΔB into a CO_2 portion and a non-CO_2 portion:

$$\Delta B_{\mathrm{RCP8.5}} = P_{\mathrm{RCP8.5}}\Delta T^{\mathrm{RCP8.5}} = P_{\mathrm{CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} + P_{\mathrm{non-CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}} \tag{2}$$

Solving for $P_{\mathrm{non-CO_2}}$ and assuming separability of temperature change into a $\mathrm{CO_2}$ component and a non-$\mathrm{CO_2}$ component, we obtain

$$P_{\mathrm{non-CO_2}} = \frac{\Delta B_{\mathrm{RCP8.5}} - P_{\mathrm{CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}} \tag{3}$$

$$= \frac{P_{\mathrm{RCP8.5}}\Delta T^{\mathrm{RCP8.5}} - P_{\mathrm{CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}} \tag{4}$$

$$= \frac{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} + \Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}} P_{\mathrm{RCP8.5}} - \frac{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}} P_{\mathrm{CO_2}} \tag{5}$$

$$= (1+\beta)P_{\mathrm{RCP8.5}} - \beta P_{\mathrm{CO_2}} \tag{6}$$

where $\beta = \Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} / \Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}$. Because we want the pattern $P_{\mathrm{non-CO_2}}$ to be state-independent, we perform linear regression of $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}$ against $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}$ on the latter part (approximately last 50 years) of the simulation where the signal is clearest. We define the slope to be $\beta = 2.9588$; the fit has an R^2 value of 0.9883. Rounding, we obtain the expression

$$P_{\mathrm{non-CO_2}} = 4.0P_{\mathrm{RCP8.5}} - 3.0P_{\mathrm{CO_2}} \tag{7}$$

We note that this decomposition works for RCP8.5 because the relationship between $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}$ and $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}}$ is approximately linear. This may not necessarily hold for other scenarios, such as RCP2.6, so a different methodology for recovering the non-$\mathrm{CO_2}$ pattern may be necessary in that case.

In the second case, we split ΔB into a $\mathrm{CO_2}$ portion, a non-$\mathrm{CO_2}$ greenhouse gas portion (labeled "other GHG"), and a non-GHG portion:

$$\Delta B_{\mathrm{RCP8.5}} = P_{\mathrm{RCP8.5}}\Delta T^{\mathrm{RCP8.5}} = P_{\mathrm{CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} + P_{\mathrm{otherGHG}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}} + P_{\mathrm{non-GHG}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}} \tag{8}$$

Solving as above,

$$P_{\mathrm{non-GHG}} = \frac{\Delta B_{\mathrm{RCP8.5}} - P_{\mathrm{CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} - P_{\mathrm{otherGHG}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}} \tag{9}$$

$$= \frac{P_{\mathrm{RCP8.5}}\Delta T^{\mathrm{RCP8.5}} - P_{\mathrm{CO_2}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} - P_{\mathrm{otherGHG}}\Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}} \tag{10}$$

$$= \frac{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} + \Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}} + \Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}} P_{\mathrm{RCP8.5}} - \frac{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}} P_{\mathrm{CO_2}} - \frac{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}}}{\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}} P_{\mathrm{otherGHG}} \tag{11}$$

$$= (1+\gamma+\delta)P_{\mathrm{RCP8.5}} - \gamma P_{\mathrm{CO_2}} - \delta P_{\mathrm{otherGHG}} \tag{12}$$

where $\gamma = \Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}} / \Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}$ and $\delta = \Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}} / \Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}$. At this point, the above procedure fails, because according to best-fits on the data plotted in Supplemental Figure 7, $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{CO_2}}$ is approximately quadratic with $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}$, and $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{otherGHG}}$ is approximately an exponential function of $\Delta T^{\mathrm{RCP8.5}}_{\mathrm{non-GHG}}$. Because neither of these relationships is linear, any derived patterns must be state-dependent if they are to be accurate.

4 Pattern Scaling for RCP2.6

In Supplemental Section 3, we derived the approximate equivalence

$$P^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}} = 4P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}} \tag{13}$$

Using the same procedure, we can derive a similar equivalence for RCP2.6:

$$P^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}} = 2.9P_{\mathrm{RCP8.5}} - 1.9P_{\mathrm{CO_2}} \tag{14}$$

We note that the relationship between $\Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}}$ and $\Delta T^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}}$ is somewhat nonlinear, and regression revealed an R^2 value of 0.58.

We now discuss the implications of both of these formulations. First assume that $P^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}} = P^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}}$. Then

$$\Delta\hat{B}_{\mathrm{RCP2.6}} = \Delta T^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}}(4P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}}) + \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} P_{\mathrm{CO_2}} \tag{15}$$

$$(\Delta T^{\mathrm{RCP2.6}} - \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}})(4P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}}) + \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} P_{\mathrm{CO_2}} \tag{16}$$

By definition, $\hat{B}_{\mathrm{RCP2.6}} = \Delta T^{\mathrm{RCP2.6}} P_{\mathrm{RCP2.6}}$. Then

$$P_{\mathrm{RCP2.6}} = \frac{(\Delta T^{\mathrm{RCP2.6}} - \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}})}{\Delta T^{\mathrm{RCP2.6}}}(4P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}}) + \frac{\Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}}}{\Delta T^{\mathrm{RCP2.6}}} P_{\mathrm{CO_2}} \tag{17}$$

$$(1-\beta)(4P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}}) + \beta P_{\mathrm{CO_2}} \tag{18}$$

$$= 4P_{\mathrm{RCP8.5}} - 4\beta P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}} + 4\beta P_{\mathrm{CO_2}} \tag{19}$$

where $\beta = \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} / \Delta T^{\mathrm{RCP2.6}} \approx 0.75$ by regression. Then

$$P_{\mathrm{RCP2.6}} = P_{\mathrm{RCP8.5}} \tag{20}$$

If we assume this equation to be true, then by Equation 18, $P_{\mathrm{RCP8.5}} = (1-\beta)(4P_{\mathrm{RCP8.5}} - 3P_{\mathrm{CO_2}}) + \beta P_{\mathrm{CO_2}}$. Then $P_{\mathrm{RCP8.5}} = P_{\mathrm{CO_2}}$, which is clearly incorrect, invalidating the original assumption that $P^{\mathrm{RCP8.5}}_{\mathrm{non-CO_2}} = P^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}}$.

Evaluating the other expression $P^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}} = 2.9P_{\mathrm{RCP8.5}} - 1.9P_{\mathrm{CO_2}}$,

$$\Delta\hat{B}_{\mathrm{RCP2.6}} = \Delta T^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}}(2.9P_{\mathrm{RCP2.6}} - 1.9P_{\mathrm{CO_2}}) + \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} P_{\mathrm{CO_2}} \tag{21}$$

$$(\Delta T^{\mathrm{RCP2.6}} - \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}})(2.9P_{\mathrm{RCP2.6}} - 1.9P_{\mathrm{CO_2}}) + \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} P_{\mathrm{CO_2}} \tag{22}$$

By definition, $\hat{B}_{\mathrm{RCP2.6}} = \Delta T^{\mathrm{RCP2.6}} P_{\mathrm{RCP2.6}}$. Then

$$\Delta T^{\mathrm{RCP2.6}} P_{\mathrm{RCP2.6}} - 2.9P_{\mathrm{RCP2.6}}(\Delta T^{\mathrm{RCP2.6}} - \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}}) = -1.9P_{\mathrm{CO_2}}(\Delta T^{\mathrm{RCP2.6}} - \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}}) + \Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} P_{\mathrm{CO_2}} \tag{23}$$

Simplifying,

$$(P_{\mathrm{RCP2.6}} - P_{\mathrm{CO_2}})(2.9\Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} - 1.9\Delta T^{\mathrm{RCP2.6}}) = 0 \tag{24}$$

So either $P_{\mathrm{RCP2.6}} = P_{\mathrm{CO_2}}$ or $\Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} = \frac{1.9}{2.9}\Delta T^{\mathrm{RCP2.6}}$.

If the first condition is true, then $P^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}} = 0$, which by Equation 14 implies that $P_{\mathrm{RCP2.6}} = \frac{1.9}{2.9}P_{\mathrm{CO_2}}$, violating the assumption that $P_{\mathrm{RCP2.6}} = P_{\mathrm{CO_2}}$. However, if the second condition is true, then by the above derivations, $\Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} / \Delta T^{\mathrm{RCP2.6}}_{\mathrm{non-CO_2}} = 1.9$, so through some simple algebra, $\Delta T^{\mathrm{RCP2.6}}_{\mathrm{CO_2}} / \Delta T^{\mathrm{RCP2.6}} = 0.6504$. Performing regression on these two quantities yields a value of 0.7321 with an R^2 value of 0.9610. While this value is similar to the one derived above, it is sufficiently different to indicate a certain amount of nonlinearity for which pattern scaling cannot account.

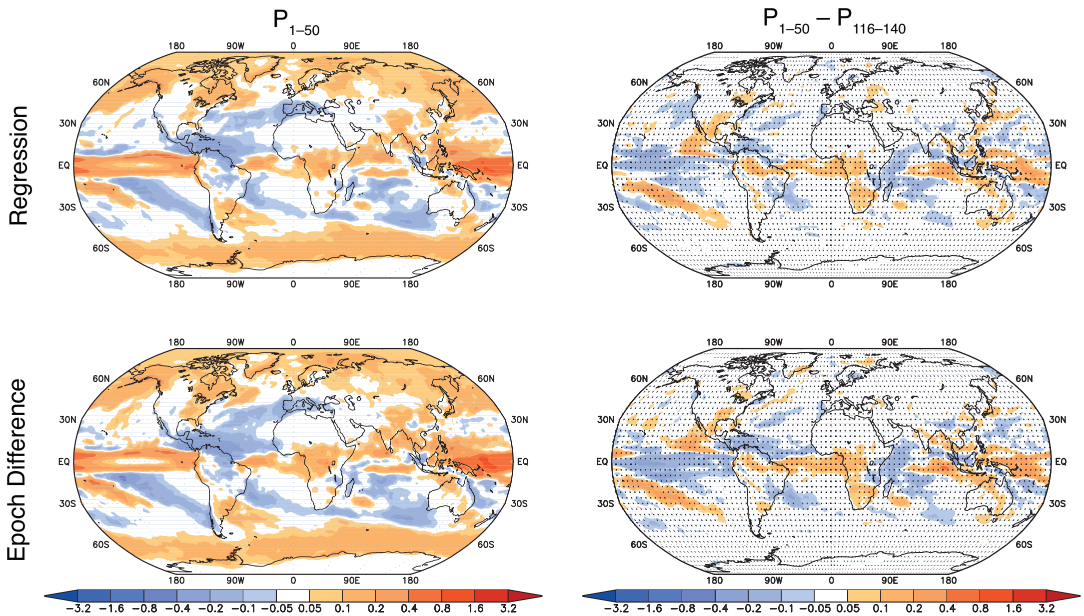

Figure 1. Absolute values (left) of and differences (right) in the precipitation scaling pattern $P(\mathbf{x})$ (Equation 1) when different time periods are used to construct the pattern (years 1–50 versus years 116–140 of the 1pctCO2 simulation). Left column shows values of P_{1-50}, and right column shows values of $P_{1-50} - P_{116-140}$ (mm day^{-1} K^{-1}). Values in subscripts denote that the associated quantities are calculated from an average over those years. Top row shows results for the regression method, and bottom row shows the epoch difference method. All values are calculated for a Group 1 multi-model average for the 1pctCO2 simulation. Stippling indicates a lack of statistical significance in the pattern of differences (Section 2.2).

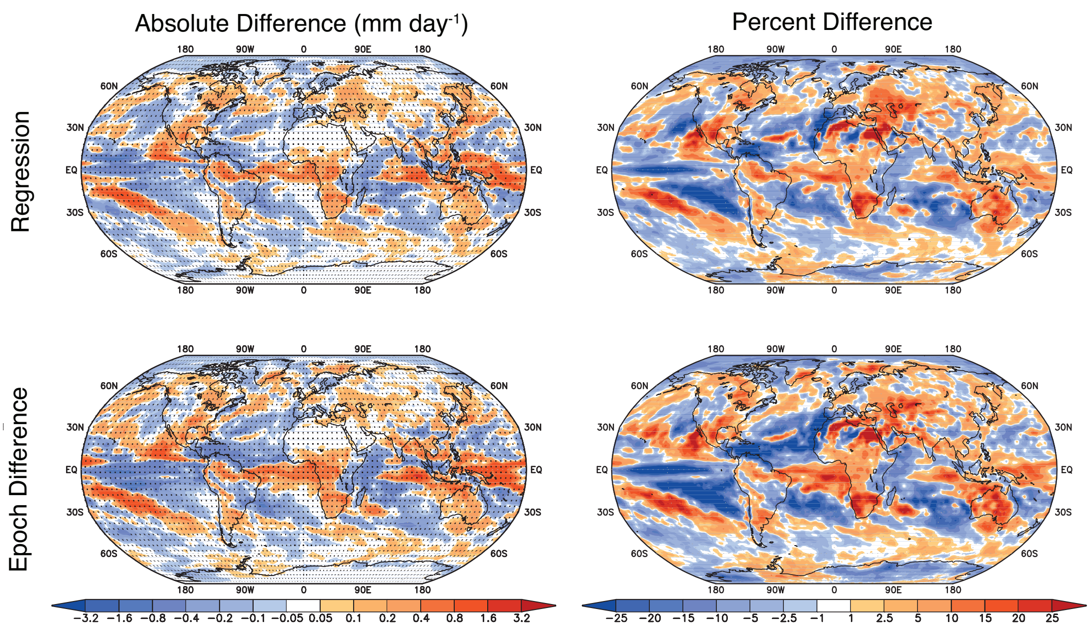

Figure 2. As in Figure 3 in the main paper but where the reconstruction $\hat{B}$ is built on the pattern P for years 1–50 (Group 1 average of the 1pctCO2 simulation), and global mean temperature $\Delta\bar{T}$ is averaged over years 116–140. That is, $\hat{B} = P_{1-50}(\mathbf{x})\Delta\bar{T}(116-140)$. Results shown are for the difference between the reconstruction and the actual model output $\hat{B} - B(\mathbf{x}, 116-140)$.

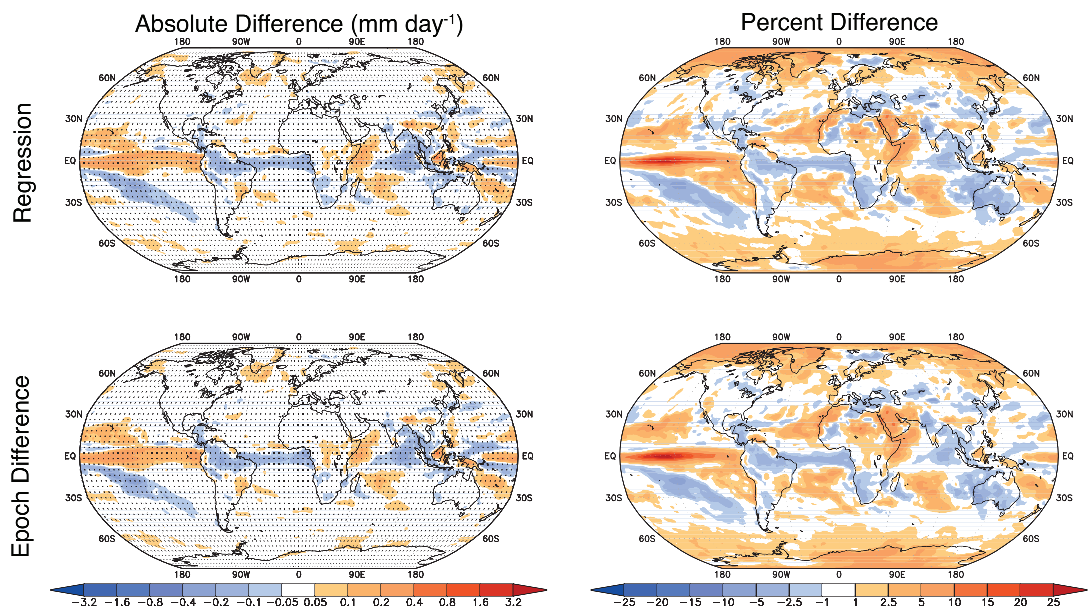

Figure 3. As in Figure 3 in the main paper but where the reconstruction $\hat{B}$ is built on the pattern P for years 116–140 (Group 1 average of the 1pctCO2 simulation), and global mean temperature $\Delta\bar{T}$ is averaged over years 58–82. That is, $\hat{B} = P_{116-140}(\mathbf{x})\Delta\bar{T}(58-82)$. Results shown are for the difference between the reconstruction and the actual model output $\hat{B} - B(\mathbf{x}, 58-82)$.

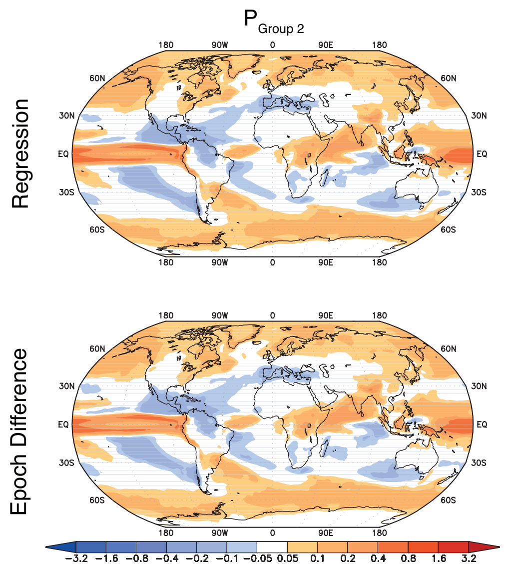

Figure 4. Absolute values (left) of and differences (right) in time-invariant patterns $P(\mathbf{x})$ among the two groups of models (Table 1), calculated for the 1pctCO2 simulation. Left column shows the multi-model average for Group 2, and right column shows the differences in multi-model averages among the two groups. All values shown have units mm day^{-1} K^{-1}. Stippling indicates a lack of statistical significance in the pattern of differences (Section 2.2).

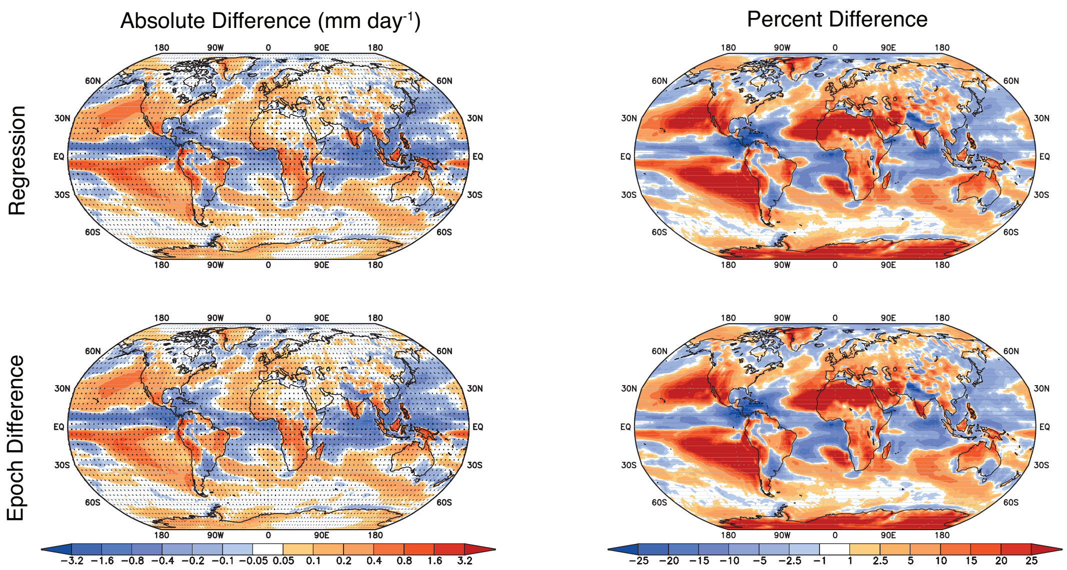

Figure 5. As in Figure 3 in the main paper but where the reconstruction $\hat{B}$ is built on the pattern P for Group 2 (average of years 116–140 of the 1pctCO2 simulation), and global mean temperature $\Delta\bar{T}$ is averaged over years 116–140 of Group 1. That is, $\hat{B} = P_{\mathrm{Group2}}(\mathbf{x})\Delta\bar{T}_{\mathrm{Group1}}(116-140)$. Results shown are for the difference between the reconstruction and the actual model output $\hat{B} - B_{\mathrm{Group2}}(\mathbf{x}, 116-140)$.

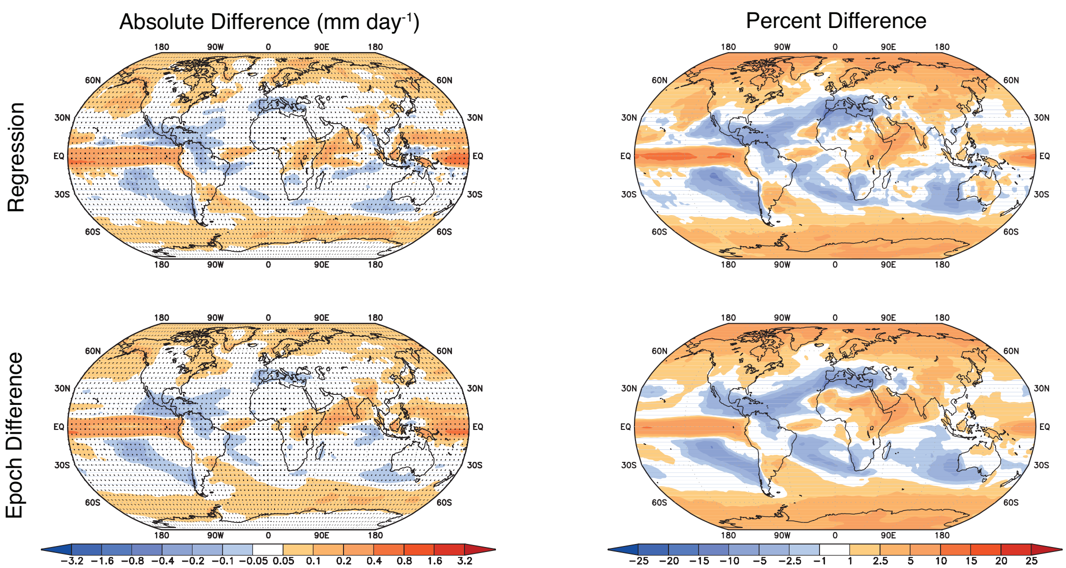

Figure 6. As in Figure 3 in the main paper but where the reconstruction $\hat{B}$ is built on the pattern P for Group 1 (average of years 116–140 of the 1pctCO2 simulation), and global mean temperature $\Delta\bar{T}$ is averaged over years 116–140 of Group 2. That is, $\hat{B} = P_{\text{Group1}}(\mathbf{x})\Delta\bar{T}_{\text{Group2}}(116-140)$. Results shown are for the difference between the reconstruction and the actual model output $\hat{B} - B_{\text{Group2}}(\mathbf{x}, 116-140)$.

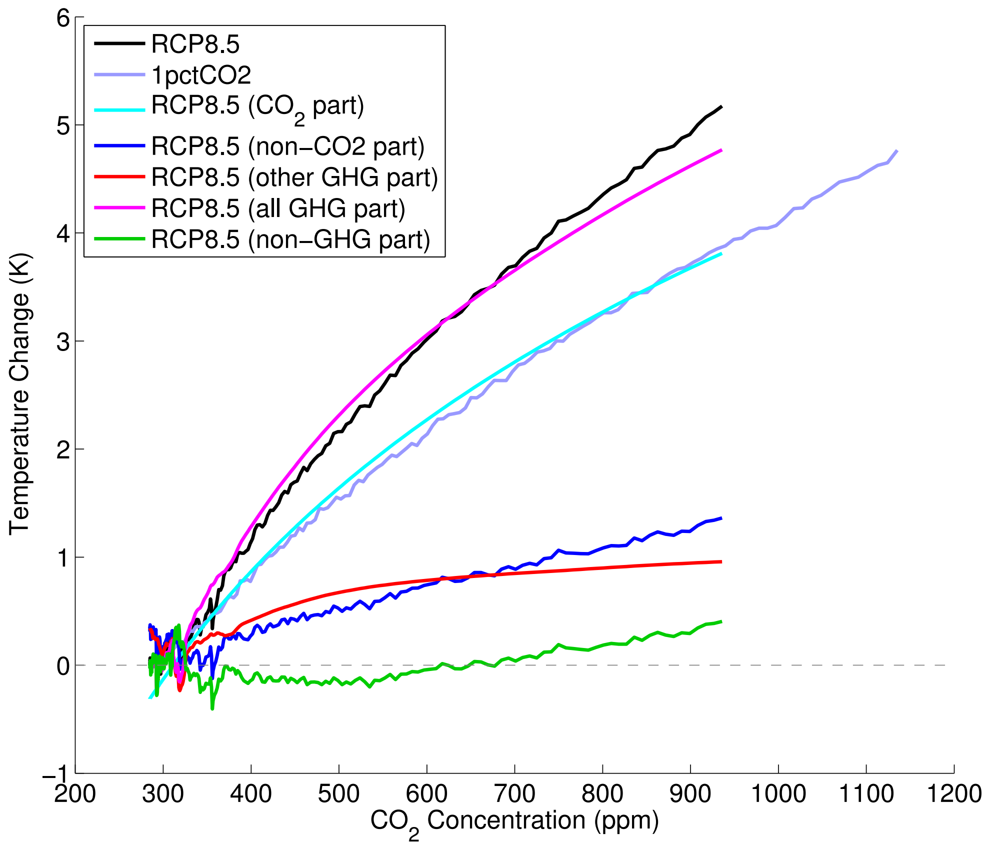

Figure 7. As in Figure 9 of the main paper but with additional forcings, per the description in Supplemental Section 3. See main Section 4.2 and Supplemental Section 3 for further details on the quantities depicted here.